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Automation and Remote Control

(The Soviet Journal *Avtomatika i Telemekhanika* in English Translation)

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SOVIET INSTRUMENTATION AND CONTROL TRANSLATION SERIES

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THE APPROACHING FIRST CONGRESS OF THE INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 657-658, June, 1960

In June of this year, the First Congress of the International Federation of Automatic Control (IFAC) will convene in Moscow. The coming Congress is a great event in world science in the area of automatic control, one recognized as exerting an influence on the development of automation in all countries.

Automation,one of the most important phenomena of our time, puts into practice the most daring dreams of many generations of mankind. Automation opens up new horizons for various branches of industry; the theory of automatic control approaches many scientific disciplines,with the raising of many problems regarding control, and modifies in principle many notions in science and technology which prevailed 10 to 15 years ago.

In recent years scientists have been faced with the most complex scientific problems of automatic control. It is difficult to overestimate the significance of these problems. Success in working them out will create the basis of rapid growth of technical progress.

It is clear that if the scientists and engineers of all countries will work in close accord on the problems of automatic control,the necessary results will be obtained very much sooner and at less expense. Such international collaboration in the field of research on automatic control is natural and necessary. Collaboration in automation is a demand of our times. Our country has made its choice; we are for the uniting of the efforts of the scientists and engineers of different countries, we are for collaboration, for close contacts of Soviet and foreign specialists in automation.

Even in 1956, Soviet scientists, along with those of other countries, were taking the initiative in this field by founding the International Federation of Automatic Control (IFAC). The formation and establishment of the IFAC has resulted in a condition of creative scientific and commercial collaboration.

The Executive Council of the IFAC, with Dr. G. Chesnut (USA) as its chairman, which was elected in September, 1957, at the First General Assembly in Paris, performed a great and very useful work. It set down the life principle of the new international scientific organization.

The Second General Assembly of IFAC was held in Chicago in September, 1959. It approved the new, improved rules and regulations, endorsed the draft of organization of the First Congress prepared by the Soviet National Committee, and heard a series of special reports.

The General Assembly elected the new staff of the Executive Council and a new President, Professor A. M. Letov, representative of the National Committee on Automative Control of the USSR.

The activity of the Federation proceeds in a condition of close scientific and commercial collaboration. To strengthen the exchange of scientific information between the national organizations of the IFAC at the present time, scientific-technical committees have been formed on theory, the technical resources of automation, on applications, terminology, education, and bibliography.

An important aspect of the activity of the IFAC is the conducting of international congresses on automation. The preparation and organization of the First Congress of the IFAC, entrusted to the National Committee of the Soviet Union, developed with the active participation of almost all the national organizations of the IFAC. In the total preparatory work a new contribution is made to the international collaboration of scientists. Reports totalling 285 were submitted for scientific consideration; they can be classified into three groups.

The reports of the first group were concerned with current problems of the theory of continuous and discrete systems of automatic control, the theory of structures and the construction of signals, stochastic, and special mathematic problems. Much attention was allotted to the theory of optimal and self-adjusting systems. A description of experimental methods of research was given in a series of reports.

Reports of the second group were concerned with the theory and experience in constructing elements of control systems, programming and computing equipment, control machines and systems of automatic control. There were reports explaining special problems in the construction of logical and digital elements and their use in digital machines.

The third group of reports tell of the experience of automation in various branches of industry — machine building, metallurgy, chemistry, energetics, petroleum production ,and so on. Much attention is given to problems in the application of digital computing technique for the control of complex technological processes.

It is expected that in June of this year a considerable number of eminent specialists in the field of automation from various countries will meet in Moscow. They will be able to exchange current ideas, scientific engineering problems and the methods of solving them. This will significantly expand the scientific and business contacts of Soviet specialists with their foreign colleagues.

For more comprehensive discussion of the reports presented to the Congress, the texts of the reports were sent beforehand to the participants, which should promote at the Congress a broad scientific discussion of the most important problems of automation. This discussion may evoke independent scientific interest. It will give the participants in the Congress the possibility of profoundly discussing with the authors of reports and other specialists the problems which interest them. Discussion is a manifestation of the creative form of contacts within the framework of the IFAC which enables exposing "white spots in the field of the theory of automatic control, imposing new scientific problems, defining the limits of application of the various methods of solving theoretical or applied problems. Soviet scientists and engineers whose work has won universal recognition are preparing with great interest for the Congress and for discussion at the Congress of actual problems of automatic control.

Soviet specialists working in the field of automation will receive in their own country a considerable number of foreign colleagues. They are doing everything they can to assure that this reception will be friendly and mutually advantageous.

The development of scientific contacts between scientists and engineers of different countries which will result from the Congress will undoubtedly have an important value for the further strengthening of peace and friendship between nations.

A MESSAGE FROM THE PRESIDENT OF THE IFAC

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 659-660, June, 1960

The occasion of the First Congress of the International Federation of Automatic Control (IFAC) is a primary event in the life of specialists in automation throughout the world.

It is expected that a very large number of eminent specialists from 30 countries will gather in Moscow in the summer of 1960 and will be able to discuss current ideas, scientific and engineering problems arising in automatics, and the methods of solving them. The principal importance of this event is that the coming Congress, like the earlier assemblies of IFAC, will demonstrate the potential possibilities of science and the acceptability for all of the form of organization of the IFAC, in which the need for cooperation, for a living exchange of scientific ideas and scientific results, is vitally important and inescapable.

The first Congress of the IFAC is a serious event of international significance, destined to exert a notable influence on the development of automation throughout the world; this explains the enormous interest in the Congress on the part of scientists, engineers, and business circles of the various countries.

The first task of IFAC is to conduct this Congress on the highest scientific and engineering level.

The Congress is being conducted with the motto: to theory – practical applicability; to technical resources – maximum reliability; to applications – the greatest effectiveness. It expresses the clear thought of the central administration regarding the creative activity of scientists and engineers throughout the world who have in mind enriching the world with products of material security. The conditions necessary for conducting the Congress have been created.

More than 280 scientific papers are to be discussed. The more active the discussion of the reports, the better IFAC will fulfill its first task. The Organizing Committee of the Congress is enlisting about 100 scientists and engineers from different countries in the posts of section chairmen of the session. They should not only know the reports well and comment on the addresses, but also foresee the possible development of discussion as regards both various aspects of the reports, and the long-range development and application of the ideas they contain.

Discussion at the Congress will be a development of the highest form of scientific contacts within the framework of the IFAC, for which we are striving so much. The IFAC has developed excellent prospects, for future scientific contacts, and in other directions.

The Executive Council of IFAC has formed scientific-technical committees which are beginning work in the following areas:

1. Theory
2. Technical resources
3. Applications of automation
4. Terminology and designations
5. Education
6. Bibliography

The basic task of the committees is to effect the international exchange of scientific information in the field of automation among the member countries of the IFAC.

Scientific connections of the IFAC with other, related international federations are also being developed.

At the present time the IFAC includes the most eminent organizations in the area of automation from 23 countries: Austria, Belgium, Great Britain, the Hungarian People's Republic, Denmark, India, Israel, Italy, the Chinese People's Republic, the Netherlands, Norway, the Polish People's Republic, the Rumanian People's Republic, the USSR, the USA, Turkey, France, Finland, Czechoslovakia, Sweden, Switzerland, Yugoslavia, and Japan.

It is expected that a number of new countries will join the IFAC in the near future.

It is a pleasure for me to mention here that a spirit of excellent, close scientific and business collaboration prevails in our international society, and that this has been shown above all by the fact that in all questions that have arisen in the IFAC, a complete and general agreement has always been reached.

All our meetings which have been held in Germany, France, Switzerland, Italy, and America have taken place with the warm hospitality of the national organizations of the IFAC and their active participation.

The spirit of collaboration in the IFAC is expressed likewise by the fact that a reciprocal exchange of books is developing among the various national organizations of the IFAC, and likewise the visitation of scientific institutions and automated enterprises. Such an exchange of visits and books is of great mutual benefit.

It is to be hoped that the spirit of collaboration will grow and be strengthened even more during the First Congress of the IFAC.

The National Committee of the Soviet Union has developed a broad program of action in preparation for the Congress, the realization of which is a contribution of the utmost importance to the strengthening and growth of the IFAC.

The spirit of collaboration in the IFAC will serve the strengthening of peace throughout the world.

A. Letov, President of the IFAC

ANALYTICAL CONTROLLER DESIGN. III

A. M. Letov

Moscow

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 661-665, June, 1960

A solution is given of the variational problem of an optimally controlled system when account is taken of servomotor speed limitations.

1. Servomotor. Speed Limitations

In the present paper we shall study the case of analytical controller design when limitations on servomotor speed are taken into account, a problem which was touched on slightly in [1, 2]. Let us pose the problem. We consider a closed autocontrolled system in which the disturbed motion is given by the equations

$$g_k = \dot{\eta}_k - \left(\sum_\alpha b_{k\alpha}\eta_\alpha + m_k\xi\right) = 0 \quad (k = 1, \ldots, n), \quad \dot{\xi} = f(\sigma) \tag{1.1}$$

Here, the η_k are the coordinates, and the $b_{k\alpha}$ are constant parameters of the object of control, while ξ is the coordinate, and the m_k are parameters of the controlling organ. The function $f(\sigma)$ is the subject of our search.

By basing ourselves on well-known attempts [3]*, we shall assume that $f(\sigma)$ lies in functions of class A', for which the following relationships hold:

$$\sigma f(\sigma) > 0, \quad \sigma \neq 0, \quad \left(\frac{df}{d\sigma}\right)_{\sigma=0} > 0. \tag{1.2}$$

The argument of the function, $\sigma = \sigma(\eta_1, \ldots, \eta_n, \xi)$ is also to be determined.

As the criterion of optimality, we consider the functional

$$I[f(\sigma)] = \int_0^\sigma (V + \dot{\xi}^2)\, dt, \tag{1.3}$$

where V is a positive-definite function of the form

$$V = \sum_k a_k \eta_k^2 + c\xi^2. \tag{1.4}$$

We consider the natural boundary conditions:

$$\eta_1(0) = \eta_{10}, \ldots, \eta_n(0) = \eta_{n0}, \quad \xi(0) = \xi_0,$$
$$\eta_1(\infty) = \ldots = \eta_n(\infty) = \xi(\infty) = 0. \tag{1.5}$$

We shall search for those continuous functions ξ, $\eta_1, \ldots, \eta_n$ of class C_1 which, while satisfying (1.1) and boundary conditions (1.5), minimize the functional in (1.3). Equations (1.1) are defined in the open region $N(\xi, \eta_1, \ldots, \eta_n)$, so that the problem just formulated is an ordinary Lagrange variational problem, the procedures for whose solution are well known.

To obtain the equations of the variational problem, we set

$$H = V + \dot{f}^2(\sigma) + \lambda[\dot{\xi} - f(\sigma)] + \sum \lambda_k g_k. \tag{1.6}$$

We then write the partial derivatives of the function H:

$$\frac{\partial H}{\partial \dot{\eta}_k} = \lambda_k, \qquad \frac{\partial H}{\partial \eta_k} = 2a_k\eta_k - \sum_\alpha \lambda_\alpha b_{\alpha k},$$
$$\frac{\partial H}{\partial \dot{\xi}} = \lambda, \qquad \frac{\partial H}{\partial \xi} = 2c\xi - \sum_\alpha m_\alpha \lambda_\alpha,$$
$$\frac{\partial H}{\partial \dot{\sigma}} = 0, \qquad \frac{\partial H}{\partial \sigma} = [2f(\sigma) - \lambda]\frac{\partial f}{\partial \sigma}.$$

The equations sought have the form

$$\dot{\eta}_k = \sum_\alpha b_{k\alpha}\eta_\alpha + m_k\xi, \quad \dot{\xi} = f(\sigma),$$
$$\dot{\lambda}_k = -\sum_\alpha b_{\alpha k}\lambda_\alpha + 2a_k\eta_k \tag{1.7}$$
$$\dot{\lambda} = 2c\xi - \sum_k \lambda_k m_k,$$
$$0 = [2f(\sigma) - \lambda]\frac{\partial f}{\partial \sigma} \qquad (k = 1, \ldots, n).$$

2. Solution Abutting the Optimal Curve's Left End

We shall study the solution of the problem for the two cases presented by the last equation of (1.7).

We consider the first case:

$$\frac{\partial f}{\partial \sigma} = 0. \tag{2.1}$$

* In autocontrol systems such as (1.1), the function σ describes the control law, and depends on $\eta_1, \ldots, \eta_n, \xi$.

If we are considering class A' functions, then there exists a positive number σ_* such that, from (2.1), we find

$$f(\sigma) = \pm \bar{f}, \quad |\sigma| \geqslant \sigma_*. \tag{2.2}$$

Thus, the controller equation is defined everywhere for $|\sigma| \geq \sigma_*$, although the number σ_* itself is still unknown.

Let

$$\Delta(\rho) = |b_{k\alpha} - \rho\delta_{k\alpha}| \tag{2.3}$$

be the determinant system (1.1) for $m_k = 0$. Then Solution (2.2) of system (1.1) is written in the following form:

$$\begin{gathered}\eta_k = \sum_{s=1}^{n} \Delta_k(\rho_s) C_s e^{\rho_s t} + M_k + N_k t,\\ \xi = \bar{C} \pm \bar{f}t,\\ \lambda_k = \sum_{s=1}^{n} \bar{\Delta}(-\rho_s) \bar{D}_s e^{-\rho_s t} + \varphi_k(t),\\ \lambda = \bar{D} + \psi(t)\end{gathered} \tag{2.4}$$

Here, ρ_k are simple roots of the equation $\Delta(\rho) = 0$, while $-\rho_k$ are simple roots of the equation

$$\bar{\Delta}(\rho) = |-b_{\alpha k} - \rho\delta_{\alpha k}| = 0; \tag{2.5}$$

$M_k + N_k t$, $\varphi_k(t)$, $\psi(t)$ are the corresponding particular solutions arising from the appearance of the line integral for the function ξ; Δ_k are the minors of the determinant in (2.3) and $\bar{\Delta}_k$ are the minors of the determinant in (2.5) corresponding to the elements with ordinal number $\underline{k}$ in the first row; $\bar{C}_1, \ldots, \bar{C}_n, \bar{C}, \bar{D}, \bar{D}_1, \ldots, \bar{D}_n$ are the $2n + 2$ arbitrary constants. We call the function defined by (2.4) a solution abutting the left (initial) end of the optimal curve ($t = 0$).

This solution will be necessary only for the verification that the boundary conditions hold.

3. Solution Abutting the Optimal Curve's Right End

We now consider the second case presented by the last of Eqs. (1.7)

$$2f(\sigma) = \lambda \tag{3.1}$$

System (1.7) becomes linear with constant coefficients:

$$\begin{gathered}\dot{\eta}_k = \sum_{\alpha} b_{k\alpha}\eta_\alpha + m_k\xi,\\ 2\ddot{\xi} = 2c\xi - \sum_{k} m_k\lambda_k,\\ \dot{\lambda}_k = -\sum_{\alpha} b_{\alpha k}\lambda_\alpha + 2a_k\eta_k \qquad (k = 1,\ldots,n)\end{gathered} \tag{3.2}$$

Let

$$\nabla(\mu) = \begin{vmatrix} b_{11}-\mu, & \ldots & b_{1n}, & m_1 & \ldots & ,0 & \ldots & ,0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ b_{n1}, & \ldots & ,b_{nn}-\mu & m_n, & \ldots & 0 & \ldots & ,0 \\ 0, & \ldots & ,0, & 2c-2\mu^2, & & -m_1, & \ldots & ,-m_n \\ 2a_1, & \ldots & ,0, & 0, & \ldots & -b_{11}-\mu, & \ldots & ,-b_{n1} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0, & \ldots & ,2a_n, & 0, & \ldots & -b_{1n}, & \ldots & ,-b_{nn}-\mu \end{vmatrix} \tag{3.3}$$

be the determinant of the system. It is easily proven that, if $\mu_1, \ldots, \mu_n$ are simple roots of the equation $\nabla(\mu) = 0$, then $-\mu_1, \ldots, -\mu_n$ will also be simple roots of this equation. For the proof of this assertion, it suffices to execute the following operations:

1) write the determinant $\nabla(-\mu)$,
2) transpose the middle row downwards,
3) transpose the first $\underline{n}$ rows downwards,
4) in the determinant thus formed, transpose the middle column to the right, making it the last column,
5) transpose the first $\underline{n}$ columns to the right.

As the result of these operations, we obtain the determinant

$$\nabla(-\mu) = \begin{vmatrix} -b_{11}+\mu, & \ldots & ,-b_{n1}, & 0, & 2a_1, & \ldots & ,0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ -b_{1n}, & \ldots & ,-b_{nn}+\mu, & 0, & 0, & \ldots & ,2a_n \\ -m_1, & \ldots & ,-m_n, & 2c-2\mu^2, & 0, & \ldots & ,0 \\ 0, & \ldots & ,0, & m_1, & b_{11}+\mu, & \ldots & ,b_{1n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0, & \ldots & ,0, & m_n, & b_{n1}, & \ldots & ,b_{nn}+\mu \end{vmatrix}$$

Now, in order to convince ourselves of the validity of our assertion, it suffices to:

a) take out the minus signs in the first n columns and make them rows;

b) multiply the last n rows by -1 and make them columns.

If, with such transpositions, the column numbers become row numbers and conversely, we obtain, after their termination, the following identity:

$$\nabla(\mu) = \nabla(-\mu). \tag{3.4}$$

To complete the argument, we recall the possibility of so numbering the roots that the following condition holds:

$$\operatorname{Re} \mu_k \leqslant 0 \quad (k = 1, \ldots n+1). \tag{3.5}$$

Then, taking the conditions at infinity into account, we write the solution of Eqs. (3.2) in the following form:

$$\begin{aligned} \eta_k &= \sum_{s=1}^{n+1} \Delta_k(\mu_s) c_s e^{\mu_s t}, \\ \xi &= \sum_{s=1}^{n+1} \Delta_{n+1}(\mu_s) c_s e^{\mu_s t}, \\ \lambda &= 2 \sum_{s=1}^{n+1} \mu_s \Delta_{n+1}(\mu_s) c_s e^{\mu_s t}, \\ \lambda_k &= \sum_{s=1}^{n+1} \Delta_{n+1+k}(\mu_s) c_s e^{\mu_s t}. \end{aligned} \tag{3.6}$$

Here, the Δ_r are the minors of determinant (3.3) with respect to the elements of r of the first row, and the c_r are constants of integration.

To obtain the controller's equation for the open region, it suffices to take the first n + 2 rows of formulas (3.6) and to eliminate time. This equation has the form

$$\begin{vmatrix} \Delta_1(\mu_1), & \ldots, & \Delta_1(\mu_{n+1}), & \eta_1 \\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot \\ \Delta_n(\mu_1), & \ldots, & \Delta_n(\mu_{n+1}), & \eta_n \\ \Delta_{n+1}(\mu_1), & \ldots, & \Delta_n(\mu_{n+1}), & \xi \\ \mu_1\Delta_{n+1}(\mu_1), & \ldots, & \mu_{n+1}\Delta(\mu_{n+1}), & \dot{\xi} \end{vmatrix} = 0. \tag{3.7}$$

After expanding the determinant, we find

$$\dot{\xi} = \sum_{\alpha=1}^{n} p_\alpha \eta_\alpha - r\xi \tag{3.8}$$

which is a linear equation with constant coefficients.

We must now convince ourselves of the possibility of determining the arbitrary constants of integration. Formulas (2.4) and (3.6) contain 3n + 3 constants of integration. For their determination we have the n + 1 initial conditions and the 2n + 2 continuity conditions for the functions $\xi, \eta_1, \ldots, \eta_n, \lambda_1, \ldots, \lambda_n, \lambda$ at the point σ_*, and also the continuity condition for the function $f(\sigma)$ which we are seeking. We now show how to find this function and the constant σ_* simultaneously.

Let

$$\frac{1}{h}\left(\sum_{\alpha=1}^{n} p_\alpha \eta_\alpha - r\xi\right) = \sigma, \tag{3.9}$$

where h is some positive constant.

Equation (3.8) gives

$$\dot{\xi} = h\sigma. \tag{3.10}$$

To determine the magnitude of σ_* we find

$$h\sigma_* = \bar{f}. \tag{3.11}$$

Thus, the final form of the controller equation is

$$\dot{\xi} = \begin{cases} +\bar{f} & \text{for } \sigma \geqslant \sigma_*, \\ h\sigma & \text{for } |\sigma| < \sigma_*, \\ -\bar{f} & \text{for } \sigma \leqslant \sigma_*. \end{cases} \tag{3.12}$$

4. Two Problems

What has been presented allows us to formulate two problems. The first of them consists of the following. Given that linear system (1.1), (3.9), (3.10) is stable for any deviations. Consequently, nonlinear optimal system (1.1), (3.9), (3.12) is stable at least for any disturbances with respect to σ which satisfy the inequality $|\sigma| \leq \sigma_*$. By virtue of the continuity property of the region of attraction, this system is extended to the limits $|\sigma| = \sigma_*$. It is required to determine the boundaries of the region of attraction.

Second problem: We consider the functional in (1.3), and remove the term ξ^2 from it. Equations (1.7) of the variational problem then give the solution $f(\sigma) = \bar{f} \operatorname{sign} \sigma$. This solution is compatible with the Weierstrass-Erdmann conditions. It is required to determine the construction of the switching function $\sigma = \sigma(\eta_1, \ldots, \eta_n, \xi)$.

5. Example

In the case n = 1, $b_1 = b$, $m_1 = m$, $a_1 = a$, we have the characteristic equation

$$\mu^4 - (b^2 + c)\mu^2 + am^2 + cb^2 = 0, \tag{5.1}$$

and the controller equation in the open region is written in the form

$$\dot{\xi} = -\frac{(\mu_1 - b)(\mu_2 - b)}{m}\eta + (\mu_1 + \mu_2 - b)\xi. \quad (5.2)$$

This equation coincides with Eq. (5.4) which was found in [1, 2].

We easily convince ourselves that, in the general case, Eq. (3.8) coincides exactly with Eq. (4.5) of work [1], which was found for an open region, and with Eq. (3.9) of work [2] for regions bounded with respect to the angle of deviation of the controlling organ.

From this there derives the interesting conclusion that, everywhere in the open region, the formula for the optimal controller is the same, and is determined only by the functional adopted as the criterion.

As in work [2], the question as to the stability of system (1.1), (3.12) must be considered separately.

The author is deeply grateful to N. N. Krasovskii for his very useful remarks and advice in regard to the problem considered here.

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† See English translation.

INVESTIGATION OF PROBABILITY STABILITY USING EXAMPLE OF AUTOMATIC CONTROL OF AIRCRAFT COURSE

V. S. Novoselov

Leningrad

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 666-673, June, 1960

Definitions are given for weak and strong probability stability of undisturbed motion. The concept of weak stability is a variation, and something of a generalization, of the definition of stability with respect to upper limits of dispersion [1]. The definition of strong stability is close to the concepts of probability stability given in [2-5]. With the example of aircraft course stabilization by means of an autopilot with a constant speed servomotor, the attempt is made to construct a methodology for the investigation of automatic control and regulation systems as regards their probability stability.

1. Let the motion of an automatic regulation or control system be defined by differential equations with random parameters and by random disturbing forces which we shall call, in their totality, random factors.

In the absence of random factors, the system's motion will be called undisturbed. We consider the random process $\{x_i(t)\}$ $(i = 1, 2, \ldots, n)$ of deviations from undisturbed motion by the system, and the n-dimensional band of width ϵ.

We denote by $\tau(\epsilon, T)$ the length of time during which, in a total time interval of duration T, some selected realization of the random process lies within the ϵ-band, i.e., $|x_i(t)| \leq \epsilon$ for $0 \leq t \leq T$.

The relative time spent by the given realization in the ϵ-band during time interval T equals

$$\nu(\epsilon, T) = \tau(\epsilon, T)/T.$$

Since the occurrence of some realization or another is a random phenomenon, then $\nu(\epsilon, T)$ is also a random variable.

Let there exist two positive numbers $Q_1 < 1$ and $Q_2 \leq 1$ such that

$$P\{\nu(\epsilon, T) > Q_1\} > Q_2, \tag{1.1}$$

where the symbol P denotes the probability of the event cited within the braces. In this case, we shall say that the random process $\{x_i\}$, during time T, is found in the ϵ-band with relative time Q_1 and with a probability not less than Q_2.

If the following relationship holds:

$$P\{\nu(\epsilon, T) = 1\} > Q \quad (0 < Q < 1), \tag{1.2}$$

than the random process $\{x_i\}$ during time interval T lies completely within the ϵ-band with a probability not less than Q.

The quantities ϵ, Q_1, Q_2 and Q give a graphic quantitative estimate in the probabilistic sense of the stability of undisturbed motion during the finite time interval T.

The following definitions of probability stability of undisturbed motion can be put forth.

We call an undisturbed motion weakly probabilistically stable with respect to the admissible set of random factors if, for all $\epsilon > 0$ and for any positive numbers arbitrarily close to unity $Q_1 < 1$, $Q_2 \leq 1$, there exists a subset of random factors for which Relationship (1.2) holds for infinitely large T.

We call an undisturbed motion strongly probabilistically stable with respect to the admissible set of random factors if, for all $\epsilon > 0$ and for a positive number $Q \leq 1$ which is arbitrarily close to unity, there exists a subset of random factors for which relationship (1.2) holds for infinitely large T.

As follows from the definitions, the property of an undisturbed motion to be weakly or strongly probabilistically stable is related to the concept of admissible random factors. A motion which is stable for some admissible random factors may be unstable for another set of them. The adoption of one or another set of admissible random factors will make concrete the concepts of stability just introduced.

Theorem 1. If the process $\{x_i\}$ has zero mathematical expectation and, for all $\delta > 0$, there exists a subset of random factors for which $\tilde{\sigma}_i < \delta$, where $\tilde{\sigma}_i$ is the time average of the dispersion of the process $\{x_i\}$, then the undisturbed motion is weakly probabilistically stable.

We give proof of this theorem, without pretending, however, to complete mathematical rigor.

For the random process $\{x_i(t)\}$ $(i = 1, 2, \ldots, n)$,

let there exist a continuous distribution function

$$F(\xi_i, t) = P\{x_i \leqslant \xi_i, t\}.$$

We introduce the random function

$$\varphi(\varepsilon, t) = \begin{cases} 1, \text{ if } \quad |x_i| \leqslant \varepsilon, \\ 0, \text{ if, for at least one subscript } j \; |x_j| > \varepsilon, \end{cases}$$

Functions of this type are used in many works [6-8]. For $\nu(\epsilon, T)$ we will have

$$\nu(\varepsilon, T) = \frac{1}{T}\int_0^T \varphi(\varepsilon, t)\, dt.$$

We set up the Chebyshev inequality

$$P\{\mu(\varepsilon, T) \geqslant 1 - Q_1\} \leqslant \frac{M[\mu(\varepsilon, T)]}{1 - Q_1}, \quad (1.3)$$

where M is the symbol for mathematical expectation and $\mu(\epsilon, T(= 1 - \nu(\epsilon, T)(0 < Q_1 < 1)$. We have

$$M[\nu(\varepsilon, T)] = \frac{1}{T}\int_0^T M[\varphi(\varepsilon, t)]\, dt =$$

$$\frac{1}{T}\int_0^T P\{|x_i| \leqslant \varepsilon, t\}\, dt. \quad (1.4)$$

where the symbol ~ denotes an average over the infinite time interval.

Since

$$\widetilde{P}\{|x_i| \leqslant \varepsilon\} > \widetilde{F}(\varepsilon) - \widetilde{F}(-\varepsilon),$$

On the basis of formulas (1.3) and (1.4), we obtain an estimate of the relative time $\nu(\epsilon)$ spent by the process in the ϵ-band during the course of an infinite interval of time:

$$P\{\nu(\varepsilon) > Q_1\} \geqslant \frac{\widetilde{P}\{|x_i| \leqslant \varepsilon\} - Q_1}{1 - Q_1},$$

where $\widetilde{F}(\epsilon)$ and $\widetilde{F}(-\epsilon)$ denote the average distribution function, taken for the values $\xi_i = \epsilon$ and $\xi_i = -\epsilon$ ($i = 1, 2, \ldots, n$) respectively, we then find that

$$P\{\nu(\varepsilon) > Q_1\} > \frac{\widetilde{F}(\varepsilon) - \widetilde{F}(-\varepsilon) - Q_1}{1 - Q_1}.$$

In the case of stationary processes, the probability functions do not depend on time, so that we shall have

$$\widetilde{P}\{|x_i| \leqslant \varepsilon\} = P\{|x_i| \leqslant \varepsilon, 0\};\ \widetilde{F}(\varepsilon) =$$

$$= F(\varepsilon, 0),\ \widetilde{F}(-\varepsilon) = F(-\varepsilon, 0).$$

For the dispersion of a random process with zero mathematical expectation, we have the obvious estimate

$$\sigma_i^2(t) \geqslant \varepsilon^2 P_i(\varepsilon, t),$$

where ϵ is an arbitrarily small positive number. Moreover,

$$P_i(\varepsilon, t) = P\{|x_i| \geqslant \varepsilon, -\infty < x_j < \infty\} \quad (j = 1, 2, \ldots, i-1, i+1, \ldots, n).$$

We therefore obtain

$$P\{|x_i| < \varepsilon, t\} \geqslant 1 - \frac{1}{\varepsilon^2}\sum_{i=1}^{n} \sigma_i^2(t).$$

By taking the time average, we find that

$$\widetilde{F}(\varepsilon) - \widetilde{F}(-\varepsilon) > 1 - \frac{1}{\varepsilon^2}\sum_{i=1}^{n} \widetilde{\sigma}_i^2.$$

By virtue of Formula (1.5), we shall have

$$P\{\nu(\varepsilon) > Q_1\} > 1 - \frac{\sum_{i=1}^{n} \widetilde{\sigma}_i^2}{\varepsilon^2(1 - Q_1)}.$$

Therefore, the random process being considered, during an infinite segment of time, will find itself in the ϵ-band for relative time Q_1 with probability Q_2, these quantities satisfying the relationships

$$Q_2 = 1 - \frac{n\sigma^2}{\varepsilon^2(1 - Q_1)}, \quad \widetilde{\sigma}_i \leqslant \sigma. \quad (1.6)$$

From whence it follows that the undisturbed motion will be weakly probabilistically stable if the time average of the dispersion can be made arbitrarily small. Indeed, for any Q_1 and Q_2 which are arbitrarily close to unity, there exists a sufficiently small σ which satisfies relationships (1.6).

Theorem 1 is thereby proven.

We note that, for stationary processes, the dispersions do not depend on time, so that $\widetilde{\sigma}_1 = \sigma_1(0)$.

Let an automatically controlled system be described by the equations

$$\frac{dx_i}{dt} = X_i(x_1, x_2, \ldots, x_n, t) + f_i(t) \quad (i = 1, 2, \ldots, n) \quad (1.7)$$

with random functions $f_i(t)$ and random initial data x_i^0.

We shall study the behavior of the process $\{x_i\}$ for small-size random factors $f_i(t)$ and x_i^0, i.e., for arbitrarily small $\alpha > 0$ and $\beta > 0$ and numbers $A > 0$ and $B > 0$ arbitrarily close to unity in the relationships

$$P\{|f_i(t)| < \alpha\} > A, \quad P\{|x_i^0| < \beta\} > B. \quad (1.8)$$

With such a choice of the random factors, the following theorem is valid.

Theorem 2. If the null solution of system (1.7) is stable for constantly acting disturbances [9], then it will be strongly probabilistically stable with respect to the upper limits of the disturbing functions and the initial data.

To find sufficient conditions for strong probability stability, both for linear and nonlinear control systems, one can use the method of constructing A. M. Lyapunov's functions [9] if the conditions of the corresponding Lyapunov theorems hold for a sufficiently large number of realizations.

It is necessary to mention that stability for constantly acting disturbances is not always decisive in establishing strong probability stability. It can happen that the solution of system (1.7) may turn out to be Lyapunov-stable for some functions $f_i(t)$ and unstable for others. If the probability of finding the realizations $\{f_i(t)\}$ in the first of these sets can be assumed to be sufficiently great, then the solution of system (1.7) will possess strong stability. The bandwidth ϵ can be made arbitrarily small by the proper choice of initial data.

We consider the system of linear differential equations

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij}x_j + f_i(t).$$

As is well known [10],the solution of this system is given in the form

$$\left\|\begin{matrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{matrix}\right\| = X(t)\left\|\begin{matrix} x_1^0 \\ x_2^0 \\ \cdot \\ \cdot \\ x_n^0 \end{matrix}\right\| + \int_0^t X_1(t, u)\left\|\begin{matrix} f_1(u) \\ f_2(u) \\ \cdot \\ \cdot \\ f_n(u) \end{matrix}\right\| du \tag{1.9}$$

Here, X(t) is the normal fundamental matrix solution of the homogeneous system.and $X_1(t, u) = X(t)X^{-1}(u)$, where $X^{-1}(u)$ is the inverse matrix of X(u).

If, for definite values of α and β, values of A and B satisfying inequalities (1.8) be known, then from formula (1.9), one can determine the quantities $\epsilon > 0$ and $Q > 0$ such that the random process $\{x_i\}$ will be found in the ϵ-band for any time interval T.

The necessary and sufficient condition for stability of a linear system for constantly acting disturbances has the form

$$\int_0^\infty |x_{ij}^1(t, u)|\, du < \infty. \tag{1.10}$$

where the x_{ij}^1 are elements of the matrix $X_1(t, u)$.

The correlation matrix of the solution of the system of linear differential equations equals [11].

$$R(t, \tau) = X(t+\tau)\, r_1 \overline{X}(t) + \int_0^{t+\tau} du_1 \int_0^t X_1(t+\tau, u_1)\, r_2(u_1, u_2)\, \overline{X}_1(t, u_2)\, du_2, \tag{1.11}$$

where r_1 is the correlation matrix of the original data, $r_2(u_1, u_2)$ is the correlation matrix of the input process; a superscript bar denotes the conjugate of the function. Let a_{ij} = const and let the roots of the characteristic equation have negative real parts; then, by transforming the equation to canonical variables, one can convince oneself that condition (1.10) holds.

In this case, for all $\mu > 0$, one can find $\nu_1 > 0$ and $\nu_2 > 0$ such that, for $|r_{ij}^1| < \nu_1$ and $|r_{ij}^2| < \nu_2$, we obtain $|R_{ij}(t, \tau)| < \mu$. Here, we have denoted by r_{ij}^1, r_{ij}^2 and R_{ij} the elements of the correlation matrices r_1, r_2 and R, respectively. Further, we have $\sigma_i = R_{ii}(t, 0)$ and, consequently, $\overset{2}{\sigma_i} < \mu$. From whence, based on Theorem 1, we are led to the proof of the following theorem:

Theorem 3. If the roots of the characteristic equation of a system of linear differential equations with constant coefficients have negative real parts, then its solution, for null initial data and disturbing functions, equal to $M[f_i(t)]$, is weakly probabilistically stable with respect to the upper bounds of the dispersions of the input process and the initial data.

For definite ν_1 and ν_2, one can determine from formula (1.11)the upper bound of the dispersion of the random process deviations of the disturbed motion from the undisturbed. After this, for selected ϵ and Q_1, one easily finds from (1.6) the definite probability of finding this process in the ϵ-band for relative time Q_1 over an infinite interval.

2. We illustrate the results of Section 1 by the example of a system of automatic aircraft course control by means of an autopilot.

With certain simplifying assumptions, the equations of the controlled system are written in the form [12-13]

$$\ddot{\varphi} + M\dot{\varphi} = -N\eta,\ \dot{\eta} = F(\psi),\ \psi = \varphi + \beta\dot{\varphi} - \frac{1}{a}\eta. \tag{2.1}$$

Here, φ is the aircraft's yaw angle, η is the angle of rudder rotation, M and N are constants characterizing, respectively, the aircraft's natural damping and the effectiveness of the rudder device. ψ is the argument of the servomotor, β is the coefficient of artificial damping, 1/a is the feedback coefficient. All coefficients in these equations are positive. The servomotor characteristic $F(\psi)$ has the form

$$F(\psi) = \begin{cases} K & \text{for} \quad \psi > \psi_0 \\ 0 & \text{for} \quad |\psi| \leqslant \psi_0 \\ -K & \text{for} \quad \psi < -\psi_0, \end{cases} \tag{2.2}$$

where $K > 0$ is the servomotor's constant speed, $2\psi_0 > 0$ is the width of the symmetric insensitivity zone.

Let the aircraft's flight, the direction of motion of which is controlled by the autopilot, occur in air layers, the density of which is a random variable. We shall therefore set $M = m\gamma$ and $N = n\gamma$. Here, m and n are positive constants, γ is the air density, taking the form of a piecewise constant random variable.

We introduce the following notation, which is con-

venient for the problem under consideration [13]:

$$x_1 = \frac{M^2}{N}\dot{\varphi} + M\eta, \quad x_2 = -M\eta,$$

$$x_3 = \frac{M^2}{N\beta}\psi = \frac{M^2}{N}\left(\frac{\varphi}{\beta} + \dot{\varphi} - \frac{\eta}{a\beta}\right), \qquad t' = Mt$$

and the dimensionless coefficients

$$\alpha = \frac{1}{\beta M}, \qquad r = \frac{M}{a\beta N},$$

$$F(\psi) = F\left(\frac{N\beta}{M^2}x_3\right) = F'(x_3).$$

By taking the derivatives with respect to t' of x_1, x_2, and x_3, we get, by virtue of (2.1),

$$\frac{dx_1}{dt'} = -x_1 + F'(x_3), \qquad \frac{dx_2}{dt'} = -F'(x_3),$$

$$\frac{dx_3}{dt'} = (\alpha - 1)x_1 + \alpha x_2 - rF'(x_3). \tag{2.3}$$

The derivative of the positive-definite function

$$V = \frac{|\alpha - 1|}{2}x_1^2 + \frac{\alpha}{2}x_2^2 + \int_0^{x_3} F'(x_3)\,dx_3,$$

computed with account taken of system (2.3), has the form

$$\frac{dV}{dt'} = -|\alpha - 1|x_1^2 + (\alpha - 1 +$$

$$+ |\alpha - 1|)x_1F'(x_3) - rF'^2(x_3).$$

We obtain from this dV/dt < 0 if the following condition holds:

$$M\left(\frac{M}{a} + N\beta\right) - N > 0. \tag{2.4}$$

Condition (2.4) is equivalent to the inequality

$$\gamma > \gamma_1, \ \gamma_1 = \frac{an}{m(m + an\beta)}.$$

The density of air can be considered bounded, so that we find such a sufficiently large number γ_2 that, with probability equal to unity, we get $\gamma < \gamma_2$.

Let the probability be larger than some number Q' that there appear a realization for which, during the entire time of motion, $\gamma_1 < \gamma < \gamma_2$. We denote by l_ϵ the least value of the function $V(x_1, x_2, x_3; \gamma)$ on the set for which $\gamma_1 \leq \gamma \leq \gamma_2$; one of the coordinates x_i is equal in absolute magnitude to ϵ; the remaining ones are less than or equal to ϵ.

Let there be known a quantity Q" such that

$$P\{|x_i^0| < \delta\} > Q''. \tag{2.5}$$

where δ will be less than ϵ, and is defined by the condition

$$\int_0^{\delta} F'(x_3)\,dx_3 + \frac{\alpha}{2}\delta^2 + \frac{|\alpha - 1|}{2}\delta^2 < l_\epsilon$$

for any α which satisfies the inequality

$$\frac{1}{\beta m\gamma_2} \leqslant \alpha \leqslant \frac{1}{\beta m\gamma_1}.$$

By assuming that the choice of initial data and the inhomogeneity of the atmosphere are statistically independent events, we obtain, by virtue of the well-known Lyapunov theorem on stability of motion [9], the expression

$$P\{\nu(\varepsilon) = 1\} > Q,$$

where

$$Q = Q' \cdot Q''. \tag{2.6}$$

Therefore, the random process of deviations of direction of motion of the aircraft from the given course is found completely in the ϵ-band with probability Q.

If the aforementioned events are dependent, then Q" in formula (2.6) must be replaced by the conditional probability of the event defined by formula (2.5), given that the event characterized by the formula $\gamma_1 < \gamma < \gamma_2$ has occurred.

Let $\gamma^* = M[\gamma]$ satisfy the condition $\gamma_1 < \gamma^* < \gamma_2$, which is equivalent to the meeting of condition (2.4) in the absence of random factors. It should be remarked that condition (2.4) is essential, since [12] it will be necessary and sufficient for the Lyapunov stability of the null solution of system (2.1).

In considering the density γ, differing slightly, probabilistically, from γ^*, we arrive at the following conclusion, based on our previous consideration. The aircraft motion with zero initial data and with atmospheric density γ^* is strongly probabilistically stable with respect to the upper bounds of the initial data and to the differences $\gamma - \gamma^*$.

We now turn to the consideration of weak probabilistic stability of the aircraft's course.

We shall assume that the aircraft is acted upon by random gusts of wind, shock waves and other stimuli whose effect on aircraft yaw may be taken into account by the introduction of the random function $f(t)$ in the right member of the first equation of system (2.1). This function equals the ratio of the torque induced by these stimuli to the aircraft's moment of inertia with respect to the yaw axis. We shall also assume that the necessary probabilistic characteristics of the one-dimensional random process $f(t)$ are given.

For action of the disturbance $f(t)$, let the nonlinear element with characteristic given by (2.2) admit the statistical linearization [14]

$$F(\psi) = k_0 M[\psi] + k_1(\psi - M[\psi]), k_0 = \text{const}, k_1 = \text{const}.$$

With this, the coefficient k_1 for the nonlinear characteristic of (2.2) is greater than zero.

The mathematical expectations $M[\varphi]$, $M[\dot{\varphi}]$, $M[\psi]$,

and M[η] will satisfy system (2.1) if, in the right number of the first equation, one adds the function of time M[f(t)] and, in the right member of the second equation, one replaces the existing function by k_0M[ψ].

The random process of deviations of the aircraft's motion from the mathematical expectation is defined by the equations

$$\ddot{\varphi}' + M\dot{\varphi}' = -N\eta' + f'(t),$$
$$\dot{\eta}' = k_1\psi', \tag{2.7}$$
$$\psi' = \varphi' + \beta\dot{\varphi}' - \frac{1}{a}\eta',$$

where the primes denote the differences of the corresponding variables from their mathematical expectations.

For the case under consideration, we introduce the notation

$$x_1 = \varphi', \quad x_2 = \dot{\varphi}', \quad x_3 = \eta'.$$

Then, system (2.7) takes the form

$$\frac{dx_1}{dt} = x_2, \quad \frac{dx_2}{dt} = -Mx_2 - Nx_3 + f'(t),$$
$$\frac{dx_3}{dt} = k_1\left(x_1 + \beta x_2 - \frac{1}{a}x_3\right).$$

We find the characteristic equation for the homogeneous system:

$$\lambda^3 + \left(M + \frac{k_1}{a}\right)\lambda^2 + k_1\left(\frac{M}{a} + N\beta\right)\lambda + k_1N = 0. \tag{2.8}$$

The coefficients of the third-degree polynomial in (2.8) are positive. Therefore, if the following condition (the second Hurwitz condition) is met:

$$\left(M + \frac{k_1}{a}\right)\left(\frac{M}{a} + N\beta\right) - N > 0 \tag{2.9}$$

the roots of characteristic Eq. (2.8) have negative real parts.

Relationship (2.9) holds for any positive k_1 if the coefficients of (2.1) satisfy condition (2.4).

On the basis of Theorem 3, we are led to the following conclusion. If condition (2.4) is met, then the undisturbed motion (M[φ], M[$\dot{\varphi}$], M[ψ] and M[η]) is weakly probabilistically stable with respect to the upper boundaries of the dispersion of the disturbing torque and of the initial data.

We note that, for M[f(t)] = 0, the null solution $\varphi = \dot{\varphi} = \psi = \eta = 0$ will be undisturbed.

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*See English translation.

THE PROBLEM OF SHAPING FILTERS AND OPTIMAL LINEAR SYSTEMS

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It is shown that, under sufficiently general conditions, a nonstationary random process can be formed from "white noise" by means of linear filters. The weight functions of the shaping filters are determined.

1. Posing of the Problem

Given a random function Y(t) ($-\infty < t < +\infty$) which is stationary "white noise" with mathematical expectation $M[Y] \equiv 0$, correlation function $k_y(u) = \delta(u)$ and spectral density function $G(\omega) = 1/2\pi$. It is necessary to show that a nonstationary random function X(t), with $M[X] \equiv 0$, ($0 < t < T$) may, under certain conditions, be presented in the form

$$X = A_X Y \tag{1}$$

where the operator A_X is defined if the function X(t) is given.*

In work [1], expression (1) is called the integral canonical representation of the random function X(t). The problem posed here is also called the problem of constructing a shaping filter.

We shall also consider here the inverse problem of presenting the "white noise" Y(t) in terms of X(t). In this case,

$$\overset{*}{Y} = A_X^{-1} X. \tag{2}$$

2. Solution of the Problem

Let Y(t) ($-\infty < t < +\infty$) be a stationary random function with correlation function $k_y(u) = \delta(u)$ and spectral density function $G(\omega) \equiv 1/2\pi$ ($-\infty < \omega < +\infty$). The function Y(t) is frequently called "white noise", since it contains all frequencies with identical dispersion density.

It is well known [1] that a random function Z(t) can be presented as a series (canonical expansion)

$$Z(t) = \sum_{k=1}^{\infty} B_k z_k(t), \tag{3}$$

where the random variables satisfy the conditions

$$M[B_j B_i] = \delta_{ji} D_j = \begin{cases} D_j & \text{for} \quad j = i \\ 0 & \text{for} \quad j \neq i \end{cases} \tag{4}$$

and the $z_k(t)$ are some regular (nonrandom) functions [1].

In order that series (3) converge in the mean† to the function Z(t), according to the basic lemma of correlation theory [2], it is necessary and sufficient that the series

$$k_z(t_1, t_2) = \sum_{k=1}^{\infty} D_k^z z_k(t_1) z_k(t_2) \tag{5}$$

converge to the correlation function in the usual sense. With this, the definition of convergence in the mean is meaningful only for random functions with finite dispersions.

To solve the basic problem, it is necessary for us to present the function Y(t) in the form of (3). We therefore, consider this problem first. In order to represent the function Y(t) by the series of (3), it is necessary to introduce a new concept of convergence. Convergence in the mean can obviously not be used, since the function Y(t) has infinite dispersion. Specifically, we shall say that the random function $U_n(t)$, as a function now of the parameter n, converges weakly in the mean to the random function U(t) if there exists, in the mean-square sense, the limit of the following integral as $n \to \infty$:

$$a_n(T) = \int_0^T R(t) U_n(t)\, dt, \tag{6}$$

where R(t) is an arbitrary "sufficiently smooth"‡ random function, uncorrelated with $U_n(t)$, and such that

$$\int_{-\infty}^{\infty} k_R^2(t, t)\, dt < \infty.$$

The random variable of (6) is basically identical to the generalized random function introduced in [4, 5]. The only difference is that here, as the space of the basic functions, we have choosen the space of random functions while, in [4, 5], the space of nonrandom functions was considered.

* For stationary random processes, such a representation is well known (cf., for example, [3]).

† Convergence in the mean is defined by the condition

$$\lim_{n\to\infty} M\left[Z(t) - \sum_{k=1}^{n} B_k z_k(t)\right]^2 = 0.$$

‡ It is assumed that the random function R(t) has a bounded dispersion, is continuous in the mean, and has the necessary number of continuous stochastic derivatives.

Below, we extend the results of correlation theory [2] to generalized random functions. Such an extension is carried out here, since it is lacking in [4, 5].

According to the basic lemma of correlation theory, it is necessary and sufficient for the convergence of (6) that there exist the limit, in the ordinary sense, of the numbers

$$p_{n,n'} = M\{a_n, a_{n'}\}$$

independently of which sequences n and n' run through as n, n' → ∞.

Thus, it is necessary and sufficient that there exist the limit of the numbers

$$p_{n,n'} = \int_0^T\int_0^T k_R(t_1, t_2) M\{U_n(t_1) U_{n'}(t_2)\} dt_1 dt_2 \quad (7)$$

independently of which sequences n and n' run through as n, n' → ∞.

We now consider the function $U_n(t)$, defined by the formula

$$U_n(t) = \sum_{k=1}^{n} C_k y_k(t), \quad (8)$$

where the random variables C_k satisfy the condition

$$M\{C_\mu C_\nu\} = \delta_{\nu\mu} N^2 \quad (9)$$

and $\{y_k\}$ is an arbitrary system of orthonormal functions on the interval (−∞, +∞), complete in L_2. We then obtain, for the numbers $p_{n,n'}$:

$$p_{n,n'} = N^2 \int_0^T\int_0^T k_R(t_1, t_2) \sum_{k=1}^{\min(n,n')} y_k(t_1) y_k(t_2) dt_1 dt_2. \quad (10)$$

The numbers $p_{n,n'}$, defined by (6), converge in the mean. Indeed,

$$p_{n,n'} = N^2 \int_0^T \sum_{k=1}^{\min(n,n')} \eta_k(t_1) y_k(t_1) dt_1, \quad (10a)$$

where $\eta_k = (k_R, y_k)$ are the Fourier coefficients of the function $k_R(t_1, t_2)$. By virtue of the fact that $k_R \in L_2$** and that the system of functions $\{y_k\}$ is complete in L_2, the sum of the integrand of (10a) converges in the mean as n → ∞ to the function $k_R(t_1, t_2)$. With these conditions, we obtain the following for p_∞:

$$p_\infty = N^2 \int_0^T k_R(t, t) dt. \quad (11)$$

But formally, on the other hand, we have

$$N^2 \int_0^T\int_0^T k_R(t_1, t_2) \delta(t_1 - t_2) dt_1 dt_2 = N^2 \int_0^T k_R(t, t) dt = p_\infty.$$

Thus, the series $N^2 \sum_{k=1}^{\infty} y_k(t_1) y_k(t_2)$, which, formally, is the correlation function of the random function

$$Y(t) = \sum_{k=1}^{\infty} C_k y_k(t), \quad (12)$$

possesses the properties of a delta-function.

Consequently, it may be said that the random function Y, defined by the series of (12), converges weakly in the mean to white noise.

We now turn to the basic problem. We present the white noise Y(t) by the series in (12), with the assumption that $\{y_k\}$ is a system of orthonormalized functions which is complete in L_2. We then determine the random variables C_k in (12) from the formula

$$C_k = \frac{V_k}{\sqrt{D_k}}, \quad (13)$$

where the random variables V_k are the coefficients in the expansion of random function X(t) in a series of the form of (3):

$$X(t) = \sum_{k=1}^{\infty} V_k x_k(t) \quad (14)$$

and $D_k = M[V_k]^2$.

We now consider the linear operator

$$AY = \int_{-\infty}^{+\infty} w_x(t, \tau) Y(\tau) d\tau, \quad (15)$$

where the weight function is defined by the series

$$w_x(t, \tau) = \sum_{k=1}^{\infty} \sqrt{D_k} x_k(t) y_k(\tau). \quad (16)$$

The functions $x_k(t)$ are the same as in (14). Then, from (12), (13), (15), and (16), we get

$$AY = \int_{-\infty}^{+\infty} w_x(t, \tau) Y(\tau) d\tau = \sum_{k=1}^{\infty} V_k x_k(t) \quad (17)$$

and, consequently, we have

$$X = AY = \int_{-\infty}^{+\infty} w_x(t, \tau) Y(\tau) d\tau. \quad (18)$$

**This follows from the condition

$$\int_{-\infty}^{+\infty} k_R^2(t, t) dt < \infty.$$

where the integral is taken in the mean, and $w_X(t, \tau)$ is defined by formula (16).

We note that (17) is a generalized random function $X(w_X)$ in the sense of [4, 5] (the notation is the same as in work [5]). Giving the function $w_X(t, \tau)$ is equivalent to defining the space of the basic functions, depending on a continuously varying parameter $\underline{t}$ (the usual functions from the space of basic functions depend on a discretely varying parameter). The necessary requirements on the function $w_X(t, \tau)$ are determined by the finitude of the dispersion of the random function X(t).

The functions $w_X(t, \tau)$ and $Y(\tau)$ in (17) can be defined in infinitely many ways. For this, it suffices to use any other expansion of the function X(t) in a series of the form of (14). As is well known [1], it is always possible to do this.

We also note that if the "white noise" Y(t) is so chosen that (18) holds, then there exists one, and only one, function $w_X(t, \tau)$.

Indeed, if it be assumed that, for one X(t), there exist two functions w_1 and w_2, then from (18), we obtain

$$\int_{-\infty}^{+\infty} \{w_1(t, \tau) - w_2(t, \tau)\} Y(\tau) d\tau \equiv 0. \quad (19)$$

By virtue of the completeness of the system of functions $\{y_k\}$, this identity is possible only if $w_1 = w_2$.

After the problem of (1) is solved, there is no difficulty in solving that of (2).

Indeed, we consider the linear operator

$$A_x^{-1} X = \int_{-\infty}^{+\infty} w_x^{-1}(t, \tau) X(\tau) d\tau, \quad (20)$$

where the weight function is defined by the series

$$w_x^{-1}(t, \tau) = \sum_{k=1}^{\infty} \frac{y_k(t) a_k(\tau)}{\sqrt{D_k}}, \quad (21)$$

the functions $a_k(t)$ satisfy the conditions

$$(a_j, x_i) = \delta_{ji} \quad (22)$$

and $\{y_k\}$ is an arbitrary system of orthonormal functions which is complete in L_2.

From (20), (21), and (22) we obtain

$$Y = A_x^{-1} X = \sum_{k=1}^{\infty} \frac{V_k}{\sqrt{D_k}} y_k(t). \quad (23)$$

On the basis of what has been previously stated, the series in (23) converges weakly in the mean to white noise. Consequently, we have

$$Y(t) = \int_{-\infty}^{+\infty} w_x^{-1}(t, \tau) X(\tau) d\tau, \quad (24)$$

where the function $w_X^{-1}(t, \tau)$ is defined by relationship (21). Thus, we have obtained the solutions to the problems of (1) and (2).

However, it is sometimes important to know which functional Hilbert space H_B it is in, whose metric, the series of (21), converges to the function $w_X^{-1}(t, \tau)$, and whether, in general, such a space exists. It is obvious that the function $w_X^{-1}(t, \tau)$ itself will also lie in the space H_B since, by definition, it is a complete space.

We now determine the space H_B. We compute the scalar product of any two functions $w_1(t, t_1)$ and $w_2(t, t_2)$ in H_B by the formula

$$(w_1, w_2)_B = \iiint k(t_1, t_2) w_1(t, t_1) w_2(t, t_2) dt_1 dt_2 dt, \quad (25)$$

where $k(t_1, t_2)$ has the form

$$k(t_1, t_2) = \sum_{k=1}^{\infty} D_k^2 x_k(t_1) x_k(t_2). \quad (26)$$

The domain of integration in (25) is defined as $t, t_1, t_2 \in T$. For simplicity of notation, the limits of integration will always be omitted. The coefficients D_k and the functions $x_k(t)$ in (26) are, respectively, the dispersions of the random variables V_k and the coordinate functions of expansion (14) of the random function X(t). The series in the right member of (26) converges to the function $k(t_1, t_2)$ since the following series converges:††

$$k_x(t_1, t_2) = \sum_{k=1}^{\infty} D_k x_k(t_1) x_k(t_2). \quad (27)$$

It is also obvious that the function $k(t_1, t_2)$ is symmetric and positive-definite. As follows from (25), the square of the norm of a function $w(t,\tau)$ in space H_B is defined by the formula

†† Indeed, if series (14) converges in the mean to random function X(t) then, in accordance with the basic lemma, series (27) must converge to $k_X(t_1, t_2)$. Consequently, the positive series $\sum_{k=1}^{\infty} D_k$ must converge, and $D_k \to 0$ for $k \to \infty$.

Thus, the series $\sum_{k=1}^{\infty} D_k^2$ is dominated by the convergent series $\sum_{k=1}^{\infty} D_k$ and, consequently, from the convergence of series (27) for any $t_1, t_2 \in T$ follows the convergence of series (25) for the same $t_1, t_2 \in T$.

$$\|w\|_B^2 = \iiint k(t_1, t_2)\, w(t, t_1)\, w(t, t_2)\, dt_1\, dt_2\, dt. \quad (28)$$

It is easily verified that the norm thus defined, by virtue of all symmetry and positive-definiteness of function $k(t_1, t_2)$, satisfies all the necessary axioms for norms.

We now define the functional Hilbert space H_B as a linear set of continuous functions which is closed in the sense of the metric introduced by (28).

We now prove that series (21) converges to the function $w_x^{-1}(t, \tau)$ in the metric of space H_B. For this, we find the square of the norm of a segment of series (21):

$$\left\| \sum_{k=n}^{n+m} \frac{y_k(t) a_k(\tau)}{\sqrt{D_k}} \right\|_B^2 = \sum_{k=n}^{n+m} D_k. \quad (29)$$

Since the series $\sum_{k=1}^{\infty} D_k$ converges, then the sum in the right member of (29) tends to zero as $n \to \infty$. It follows, thus, that series (21) converges in the metric of space H_B and, consequently, the existence of the space in which the function $w_x^{-1}(t, \tau)$ lies is proven. However, it is possible to construct, not just one, but a nondenumerable set of spaces H_B^l in whose metrics series (21) converges. It suffices for this, for example, to substitute for the function $k(t_1, t_2)$ in formulas (26) and (28) the functions

$$k_l(t_1, t_2) = \sum_{k=1}^{\infty} D_k^l x_k(t_1) x_k(t_2) \quad (l = 2, 3, 4, \ldots). \quad (30)$$

Space H_B is obtained for $l = 2$. In particular cases, it may turn out that function $w_x^{-1}(t, \tau)$ lies in subspace of H_B, for example L_2, or even the set of C of continuous functions. With this, $C \subset L_2 \subset H_B$.

3. Application of Representations (1) and (2) to Optimal Linear System Theory

The problem of determining an optimal linear operator B consists of the following. It is necessary to approximate in the very best way to a given random function Z(t) by a random function BX [an operator on X(t)] by observing the random function $X(\tau)$ on the interval $(-\infty, t)$. It is assumed with this that M[X] = = M[Z] = 0. The optimal operator B obviously lies in the class of linear operators, namely,

$$\Gamma(t) = BX = \int_{-\infty}^{t} w^*(t, \tau)\, X(\tau)\, d\tau, \quad (31)$$

where $w^*(t, \tau)$ is the weight function corresponding to the optimal operator. The closeness of the function $\Gamma(t)$ to Z(t) is defined by the mean square error

$$M\{\Gamma(t) - Z(t)\}^2. \quad (32)$$

An operator B^* which provides a minimum mean square error is considered to be optimal. It can be shown [1] that the optimal weight function must satisfy an integral equation of the form

$$\int_{-\infty}^{t} w(t, \tau)\, k_x(u, \tau)\, d\tau = k_{xz}(t, u) \quad (-\infty < u \leqslant t), \quad (33)$$

where $k_x(t_1, t_2)$ and $k_{zx}(t_1, t_2)$ are, respectively, the autocorrelation function of X(t) and the cross correlation function of X(t) and Z(t), which are assumed to be known.

Here we considered only one variant of the optimal system problem. Other variants lead to integral equations analogous to (33). We therefore limit ourselves to a consideration of the solution of Eq. (33). To solve Eq. (33), we must first make one remark, which is a simple corollary to the results of Section 2.

Let there be given the weight function $\omega(t, \tau)$ of an arbitrary linear system.

It may always be assumed that this linear system forms, from some white noise,

$$Y(t) = \sum_{k=1}^{\infty} \frac{F_k}{\sqrt{D_{F_k}}} y_k(t),$$

at its input, a random function P(t) of the form

$$P(t) = \sum_{k=1}^{\infty} F_k b_k(t).$$

According to (16), the weight function $\omega(t, \tau)$ must have the form

$$\omega(t, \tau) = \sum_{k=1}^{\infty} \sqrt{D_{F_k}}\, b_k(t)\, y_k(\tau) = \sum_{k=1}^{\infty} \eta_k(t)\, y_k(\tau), \quad (34)$$

Then, the weight function $\omega^{-1}(t, \tau)$ of the inverse operator, in accordance with (21), is written in the form

$$\omega^{-1}(t, \tau) = \sum_{k=1}^{\infty} \frac{y_k(t)\, d_k(\tau)}{\sqrt{D_{F_k}}} = \sum_{k=1}^{\infty} y_k(t)\, \zeta_k(\tau), \quad (35)$$

where the functions $b_k(t)$ and $d_k(t)$ satisfy the biorthogonality condition

$$(b_j, d_i) = \delta_{ji}. \quad (36)$$

On the basis of the remark just made, the solution of Eq. (33), if it exists, is determined in an elementary

way. Indeed, we present the function $k_x(u, \tau)$, the weight function of the linear operator in Eq. (33), by the series

$$k_x(u, \tau) = \sum_{k=1}^{\infty} \eta_k(u) y_k(\tau), \tag{37}$$

where $\eta_k = (k_x, y_k)$ and $\{y_k\}$ is any complete system of orthonormalized functions in L_2. Then, in accordance with the previous remark, the weight function of the inverse operator has the form

$$k_x^{-1}(\tau, u) = \sum_{1}^{\infty} y_k(\tau) \zeta_k(u). \tag{38}$$

By applying the inverse operator to both members of Eq. (33) and by taking (38) into account, we obtain the solution in the form

$$w^*(t, \tau) = \sum_{k=1}^{\infty} y_k(\tau) \int k_{zx}(t, u) \zeta_k(u)\, du. \tag{39}$$

If we use the expansion of the correlation function in a series such as (5), we then obtain

$$k_x(u, \tau) = \sum_{k=1}^{\infty} D_k x_k(u) x_k(\tau)$$

and, for the inverse operator,

$$k_x^{-1}(\tau, u) = \sum_{k=1}^{\infty} \frac{a_k(\tau) a_k(u)}{D_k}.$$

Then, for the weight function of the optimal operator, we get

$$w^*(t, \tau) = \sum_{k=1}^{\infty} \frac{a_k(\tau)}{D_k} \int k_{zx}(t, u) a_k(u)\, du. \tag{40}$$

If we use the expansion of the correlation function in a series with respect to orthonormalized eigenfunctions, we obtain

$$k_x(u, \tau) = \sum_{k=1}^{\infty} D_k \phi_k(u) \phi_k(\tau),$$

where D_k and $\psi_k(t)$ are, respectively, the eigenvalues and eigenfunctions of the equation

$$\int k_x(u, \tau) \phi(\tau)\, d\tau = D\phi(u). \tag{41}$$

Formula (39) is then written in the form

$$w^*(t, \tau) = \sum_{k=1}^{\infty} \frac{1}{D_k} \phi_k(\tau) \int k_{zx}(t, u) \phi_k(u)\, du. \tag{42}$$

Results (40) and (42) were obtained in [1].

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THE OPTIMAL CONTROL OF SYSTEMS WITH DISTRIBUTED PARAMETERS

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The paper poses the problem of the optimal control of systems with distributed parameters for certain classes of objects described by systems of first-order partial differential equations and by the heat conduction equation. The solution is adduced for one optimal control problem for an object described by a first-order partial differential equation, and the functional block schematic is given for an optimal control system for objects of this type.

The recently developed optimal system theory allowed new approaches to the solution of the control problem to be developed. The posing of the problem, in all the generally known works on optimal control theory, derived from the tendency to take into account limiting conditions imposed on the controlling stimuli and on the coordinates of the given portions of systems described by systems of ordinary differential equations (for example, [1-4]).

The solutions of these problems can be based on L. S. Pontryagin's maximum principle [3-7], R. Bellman's dynamic programming method [8], or on the use of isosurfaces in phase space [9, 10]. However, in many engineering applications, one has to do with objects with spatially distributed parameters which are impossible to replace by delay lines. This can give rise to the circumstance that the controlling stimuli can be applied to various points or regions of the distributed system or to the position that the control problem is the attainment of a given distribution of object states.

The optimal control problems for such objects cannot be solved by a direct employment of the well-known methods, and require a new approach to their posing and to their solution.

1. Posing of the Problem

We shall consider systems with distributed parameters which are described by partial differential equations.

Many objects of control can be described by systems of first-order partial differential equations:

$$\frac{\partial Q_i}{\partial t} = f_i\Big(x,\ t,\ Q_1 \ldots, Q_n,\ \frac{\partial Q_1}{\partial x}, \ldots$$
$$\ldots, \frac{\partial Q_n}{\partial x},\ u_1, \ldots, u_k,\ v_{k+1}, \ldots, v_r,\ w_{r+1}, \ldots, w_s\Big) \tag{1}$$
$$(i = 1, 2, \ldots n),$$

where $Q_i = Q_i(x, t)$ ($i = 1, \ldots, n$) are functions of the two arguments $\underline{x}$ and $\underline{t}$, defined in the rectangle $l_0 \leq x \leq l_1$, $t_0 \leq t \leq t_1$ and characterizing the object's state; $u_j = u_j(t)$ ($j = 1, 2, \ldots, k$) are controlling functions of the variable $\underline{t}$, constrained by the condition

$$F(u_1, \ldots, u_k) \leqslant C. \tag{2}$$

Here, F is some given functional, and C is a constant.

Further, $v_j = v_j(x, t)$ ($j = k + 1, \ldots, r$) are controlling functions of two variables, constrained by the conditions

$$\left|\frac{\partial^{\gamma+\delta} v_j}{\partial x^{\gamma} \partial t^{\delta}}\right| \leqslant A_j, \tag{3}$$

where γ and δ depend on $\underline{j}$, $w_j = w_j(x)$ ($j = r + 1, \ldots, s$) are controlling functions of one variable $\underline{x}$, constrained by the condition

$$\left|\frac{d^{\alpha} w_j(x)}{dx}\right| \leqslant B_j, \tag{4}$$

where α depends on $\underline{j}$.

In the given case, we can formulate three optimal control problems.

I. To vary the controlling stimuli u_j, v_j, w_j ($j = 1, \ldots, s$) constrained by conditions (2), (3), (4), so that, with initial conditions

$$Q_i(x,\ t_0) = Q_{in}(x) \tag{5}$$

and boundary conditions

$$Q_i(l_0,\ t) = Q_{i0}(t)$$

some functional

$$I = I\left(x,\ t,\ Q_i,\ \frac{\partial Q_i}{\partial x},\ \frac{\partial Q_j}{\partial t},\ u_j,\ v_j,\ w_j\right)$$
$$(i = 1, \ldots, n;\ j = 1, \ldots, s) \tag{6}$$

will be minimized. Here, Q_i, u_j, v_j, w_j are vectors with the corresponding coordinates.

II. To vary u_j, v_j, w_j ($j = 1, 2, \ldots, s$) so that a function of the type of (6), for fixed values of $\underline{x}$ lying on the segment $[l_0, l_1]$ (for example, for $x = l_1$), attains its minimum value.

III. Finally, to vary u_j, v_j, w_j ($j = 1, 2, \ldots, s$) so that a functional of the type of (6), for fixed values of

$\underline{t}$ lying on the segment $[t_0, t_1]$, attains its minimum value.

This is the problem with "free ends," i.e., in the case when no limitations are imposed on the functions $Q_i = Q_i(x, t)$ for $t = t_1$ ($i = 1, 2, \ldots, n$). However, it is meaningful in some cases to require that the state functions $Q_i = Q_i(x, t)$ ($i = 1, 2, \ldots, n$) differ in some known sense from certain given functions $Q_i^* = Q_i^*(x)$ by a magnitude not exceeding a permissible deviation ϵ. In other words, it is necessary to require that the vector function $Q = Q(x, t) = (Q_1(x,t), \ldots, Q_n(x, t))$, for $t = t_1$, lie in some given ϵ-neighborhood of the vector function $Q^* = Q^*(x) = (Q_1^*(x), \ldots, Q_n^*(x))$, i.e., that the following condition holds:

$$\max |Q_i^*(x) - Q_i(x, t_1)| \leqslant \varepsilon \qquad (l_0 \leqslant x \leqslant l_1,\ i = 1, 2, \ldots n). \tag{7}$$

In connection with this, one might also pose the problem: to vary the controlling stimuli u_j, v_j, w_j ($j = 1, 2, \ldots, s$) so that there shall be minimized, in some sense, the deviation of the vector function $Q(x, t)$ from a given vector function $Q^*(x)$ for a fixed time t_1, for example, to attain

$$\min \max |Q_i^*(x) - Q_i(x, t_1)| \qquad (l_0 \leqslant x \leqslant l_1, \quad i = 1, 2, \ldots, n). \tag{8}$$

Many controlled objects are described by partial differential equations of second and higher orders. Here, we shall consider one of the fundamental problems of mathematical physics, namely, the heat conduction equation. We first consider the one-dimensional heat conduction equation in the simplest case:

$$\frac{\partial Q}{\partial t} = a \frac{\partial^2 Q}{\partial x^2} + g(x, t, u_1(t)) \tag{9}$$

with initial conditions

$$Q(x, t_0) = Q_0(x) \tag{10}$$

and boundary conditions

$$\left.\frac{\partial Q}{\partial x}\right|_{x=l_0} + \beta_0(l_0, t) = \phi_1(t, u_2(t)),$$
$$\left.\frac{\partial Q}{\partial x}\right|_{x=l_1} + \beta_1(l_1, t) = \phi(t_3, u_3(t)). \tag{11}$$

Here, $\underline{g}$, ψ_1 and ψ_2 are given functions of their arguments; $\underline{a}$, β_0 and β_1 are coefficients which, in the general case, depend on $\underline{x}$, $\underline{t}$ and $Q(x, t)$; $Q = Q(x, t)$ is a function of the two arguments $\underline{x}$ and $\underline{t}$, defined in the region $l_0 \le x \le l_1$, $t_0 \le t \le t_1$ and characterizing the state of the controlled object, and $u_1(t)$, $u_2(t)$ and $u_3(t)$ are controlling functions constrained by conditions (2) for $k = 3$.

The optimal control problem for such objects can consist of the determination of the law of variation of the controlling stimuli $u_1(t)$, $u_2(t)$, and $u_3(t)$, constrained by conditions of the type of (2), for which the functional

$$I = I\left(t, x, Q, \frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial t}, u_1, u_2, u_3\right) \tag{12}$$

attains its minimum value. In other cases, instead of minimizing functional (12) with the same conditions (2), it is necessary to choose the controlling stimuli so that functional (12) will be minimized but, for fixed $\underline{x}$ from $[l_0, l_1]$ or for fixed $\underline{t}$ from $[t_0, t_1]$.

Additional conditions, analogous to condition (7) can be imposed on the function $Q = Q(x,t)$, i.e., the function $Q = Q(x, t)$, at time $t = t_1$, must lie in some given ϵ-neighborhood of a given function $Q^* = Q^*(x)$, for example,

$$\max |Q^*(x) - Q(x, t_1)| \leqslant \varepsilon \quad (l_0 \leqslant x \leqslant l_1). \tag{13}$$

An analogous problem can be posed for a three-dimensional object, described, for example, by the heat conduction equation

$$\frac{\partial Q}{\partial t} = a\left(\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} + \frac{\partial^2 Q}{\partial z^2}\right) + v_1 \frac{\partial Q}{\partial x} + v_2 \frac{\partial Q}{\partial y} + v_3 \frac{\partial Q}{\partial z} + g(P, t, u_1, \ldots, u_k) \tag{14}$$

with initial condition

$$Q(P, 0) = Q_0(P) \tag{14'}$$

and boundary condition

$$\left.\frac{\partial Q}{\partial n}\right|_{\Gamma} + \beta Q\Big|_{\Gamma} = \phi(P, t, u_{k+1}, \ldots, u_r)\Big|_{\Gamma}. \tag{15}$$

Here, P is a point of the body D bounded by surface Γ; $\underline{n}$ is the external normal; $\underline{a}$, ν_1, ν_2, ν_3, β are given functions of the point P and time $\underline{t}$; $\underline{g}$ and ψ are given functions of their arguments. Limitations of the type of (2) are imposed on the controlling stimuli $u_1, \ldots, u_r$.

It might be important here to solve the problem of finding such controlling stimuli which, in conjunction with limitations (2), will give the functional

$$I = I(P, t, Q(P, t); u_1, \ldots, u_r) \tag{16}$$

its minimum value.

With this there may still be imposed the further condition that the function $Q = Q(P, t)$ for $t = t_1$ be found in some ϵ-neighborhood of a given function $Q^*(P)$, for example, that the following condition hold:

$$\max_{P \in D} |Q^*(P) - Q(P, t_1)| \leqslant \varepsilon. \qquad (17)$$

Here there might also arise the problem: to so vary the controlling stimuli $u_1, \ldots, u_r$ that the deviation of the function $Q(P, t_1)$ from the given function $Q^*(P)$ is minimal in some sense — for example, that during time $t = t_1 - t_0$ the following integral be a minimum:

$$\iiint_D [Q^*(P) - Q(P, t_1)]^2 \, dP.$$

Moreover, the following problem is also of interest: to determine those controlling stimuli $u_1, \ldots, u_r$ so that the functional of (16) will attain a minimum, where the point $P \in \Gamma' \subset \Gamma$, , where Γ' is some definite part of the boundary.

It is necessary to mention that here, as in the case when the object is described by equations of the type of (1), the controlling functions may depend, not only on time t, but also on the spatial coordinates x, x and y, or x, y, z, or only on the space coordinates.

A large number of engineering problems, related to the creation of optimal control systems for objects with distributed parameters, lead to the mathematical problems which we have just enumerated.

Such objects, for example, are flow-through furnaces for the heating or thermal processing of materials. If, with this, we are limited to the accuracy of solution which the theory of regular regimens [11] can give, then objects of this type can be described by first-order partial differential equations of the type of (1). In this case, the limitations of (2) flow from the limitations on range of variation of the controlling stimuli, limitations on their speed of variation, etc. Limitations (3) and (4) are imposed on spatially distributed controlling actions as occur, for example, in velocity heating furnaces, deriving from the impossibility of practical implementation in the furnaces of temperature fields with very high gradients. For such objects, functional (6) to be minimized might stand for the time to compensate disturbing stimuli, the mean square deviation of the material's temperature at the furnace output from a given temperature, etc. As disturbing stimuli here, there might be adduced variations of temperature at the input to the furnace (or in one of its zones), variations in the tempo at which the material is fed through the furnace, variations in thickness of the layer of material to be heated. Introducing the quantity Q, which characterizes the material's state, as a vector rather than a scalar, is due to the fact that the requirements on the processing might include, not one, but several indicators, such as, for example, temperature, chemical constitution, crystal structure, etc.

The problem of optimizing a thick body in a flow-through or chamber furnace leads to a posing of the problem wherein the object is described by second-order partial differential equations, such as (9). It might be necessary here to attain a given temperature distribution, in minimal time or with minimal expenditure of energy [functional (12)], in the body to be heated, or to guarantee that the nonuniformity of heating does not exceed some given magnitude.

It is with such types of problems that one has to deal in creating automatic control systems for a broad class of frequently occurring practical production processes. In addition to the aforementioned heating and thermal flow-through and chamber furnaces, this class of objects includes rotating kilns for sintering free-flowing materials, continuous dessicators for ribbon and free-flowing materials, blast and open-hearth furnaces, ribbon-sintering furnaces, machines for continuous plating, etc.

2. Example of an Object with Distributed Parameters

For a certain class of objects, such as continuous furnaces for heating "fine" billets, heat exchangers, and many others, the equations describing the process can be reduced to first-order partial differential equations.

We consider, for example, the one-sided heating of a "fine" lamina (Biot criterion, $Bi \leq 0.25$) which (Fig. 1) moves through the furnace of length L with a velocity of $\nu = \nu(t) \geq 0$ for $0 \leq t \leq T$. We use the notation: thermal capacity of the metal is c, the specific weight is γ, the lamina thickness is s, the heat transfer coefficient to the metal in the given range of temperature variation is α.

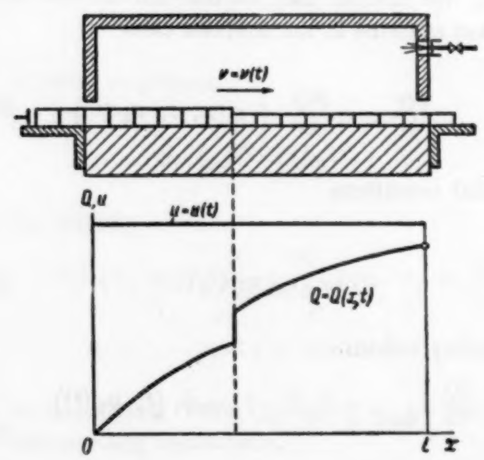

Fig. 1. Single zone flow-through heating furnace. $Q = Q(x, t)$ is the metal temperature as a function of the coordinate x and the time t; $u(t)$ is the furnace temperature as the controlling stimulus; $\nu = \nu(t)$ is the speed at which the metal passes through the furnace.

We assume that the temperature of the furnace's working space (Fig. 1) is the same at all points and that, for $0 \leq t \leq T$, the following limitation is imposed on it:

$$u_1 \leqslant u(t) \leqslant u_2, \qquad (18)$$

where u_1 and u_2 are, respectively, the upper and lower admissible or attainable limits of temperature variation of the furnace's working space.

With the aforementioned assumptions taken into account, one can write the equation of the metal heating process in the furnace in the form

$$a\frac{\partial Q}{\partial t} + av\frac{\partial Q}{\partial x} + Q - u = 0. \qquad (19)$$

Here, $Q = Q(x, t)$ is the metal temperature at point x at time t; $a = c\gamma s/\alpha$ is the coefficient characterizing the metal's thermal properties. We shall assume that this coefficient depends only on the quantity

$$\xi = x - \int_0^t v(p)\,dp.$$

In the case of radiant heat transfer, this equation can be considered the same as an equation in increments.

The initial conditions have the form

$$Q(x, 0) = Q_0(x), \; 0 \leqslant x \leqslant L,$$

where $Q_0(x)$ is a given function.

The boundary conditions have the form

$$Q(0, t) = 0, \; 0 \leqslant t \leqslant T.$$

In Eq. (19) we carry out the nondegenerate change of variables given by the formulas

$$x = \xi + \int_0^\tau v(p)\,dp, \quad t = \tau. \qquad (20)$$

The inverse transformation has the form

$$\xi = x - \int_0^t v(p)\,dp, \quad \tau = t. \qquad (21)$$

After the change of variables, Eq. (19) takes the form

$$a(\xi)\frac{\partial Q(\xi, \tau)}{\partial \tau} + Q(\xi, \tau) = u(\tau). \qquad (22)$$

In this equation, the term with the partial derivative with respect to ξ drops out. By solving this equation for fixed ξ and, thereafter, transforming back to the original variables by means of Formulas (21), we get

$$Q(x, t) = e^{-\frac{t}{a}}\Big[Q_0\Big(x - \int_0^t v(p)\,dp\Big) + \frac{1}{a}\int_0^t u(p)\,e^{\frac{p}{a}}dp\Big], \qquad (23)$$

where Q_0 and a are given functions of the single argument $\xi = x - \int_0^t v(p)\,dp$.

3. Optimal Control of Systems with Distributed Parameters

For many objects described by equations of the type of (19), the optimal control problem amounts to finding controlling stimuli, constrained by a condition $u_1 \leq u(t) \leq u_2$ for $0 \leq t \leq T$, such that the functional

$$I = \int_0^T [Q_g - Q(L, t)]^2\,dt \qquad (24)$$

attains its minimum value. Here, Q_g is some given function of time t or is a constant.

For the continuous furnace of the example of Section 2, this requirement signifies the requirement that the mean square deviation of the metal's temperature at the furnace output from a given temperature be minimized.

By setting $x = L$ in formula (23) and substituting in (24), we get

$$I = \int_0^T \Big[Q_g - e^{-\frac{t}{a}} Q_0\Big(L - \int_0^t v(p)\,dp\Big) - \frac{1}{a}e^{-\frac{t}{a}}\int_0^t u(p)\,e^{\frac{p}{a}}dp\Big]^2 dt. \qquad (25)$$

With the notation

$$\Theta(t) = Q_g - e^{-\frac{t}{a}} Q_0\Big(L - \int_0^t v(p)\,dp\Big),$$

Expression (25) can be written in the following equivalent form:

$$I = \int_0^T [Q(t) - \Delta(t)]^2\,dt, \qquad (26)$$

where $\Delta(t)$ is defined by the equation

$$a\Delta'(t) + \Delta(t) = u(t) \;\text{ for }\; \Delta(0) = 0.$$

Here, the optimal control u = u(t) can be found by means of L. S. Pontryagin's maximum principle [6].

The function H has the form

$$H=\psi_0[\theta(t)-\Delta(t)]^2-\frac{1}{a}\psi_1\Delta(t)+\frac{1}{a}\psi_1 u(t), \tag{27}$$

where ψ_0 = const < 0, and ψ_1 satisfies the equation

$$\dot{\psi}_1=2[\theta(t)-\Delta(t)]\psi_0+\frac{1}{a}\psi_1. \tag{28}$$

We determine from (28) that

$$\psi_1(t)=e^{\frac{t}{a}}\left\{C+\int_0^t[\theta(p)-\Delta(p)]e^{-\frac{p}{a}}dp\right\}, \tag{29}$$

where C is an arbitrary constant.

The maximum of the function H is attained for

$$u(t)=\begin{cases} u_2, & \text{if } \psi_1>0,\\ u_1, & \text{if } \psi_1<0.\end{cases}$$

However, based on the maximum principle, it is impossible to find an optimal control u = u(t) in the case when $\psi_1(t) \equiv 0$ on some segment of the extremals. Such segments, following Rozonoér ([12], Pt. II), we shall call singular. On singular segments, function H does not depend on u, and the control u = u(t) with the condition $u_1 \leq u(t) \leq u_2$ must be so chosen that

$$\theta(t)-\Delta(t)\equiv 0. \tag{30}$$

We assume that condition (30) is met on the portions of the optimal process for $0 \leq t \leq t_1$ and $t_2 \leq t \leq T$. Then, in accordance with [12], Pt. I, in the conditions of problems with "free ends," it is necessary to have $\psi_1(T) = 0$. From this we find that, in expression (29), the constant C equals zero and that, during the course of the optimal process, the following condition must hold:

$$\int_0^T[\theta(t)-\Delta(t)]e^{-\frac{p}{a}}dp=\int_{t_1}^{t_2}[\theta(t)-\Delta(t)]e^{-\frac{p}{a}}dp=0. \tag{31}$$

Thus, the optimal control has the form

$$u(t)=\begin{cases} u_1 \text{ for } \int_0^t[\theta(p)-\Delta(p)]e^{-\frac{p}{a}}dp>0,\\ u_2 \text{ for } \int_0^t[\theta(p)-\Delta(p)]e^{-\frac{p}{a}}dp<0.\end{cases} \tag{32}$$

We now assume that it is necessary to smooth, in some optimum way in the sense of minimizing the functional of (24), step disturbances in the coefficient a. The function $a = a(\xi)$ has the form*

$$a=a(\xi)=\begin{cases} a_1 & \text{for } \xi\leqslant 0,\\ a_2 & \text{for } \xi>0,\end{cases}$$

where a_1 and a_2 are constants and $\xi = x - \int_0^t \nu(p)\,dp$.

In this case, $\theta(t)$, at some moment of time t', $t_1 \leq t' \leq t_2$, will have a jump. Then, in accordance with (32), the optimal control corresponds to the maintenance of u during the interval (t_1, t_2) at one of the limiting levels, the choice of which depends on whether the coefficient a decreased or increased.

The moment t_1 at which there occurs the transition from the singular segment to the nonsingular one is defined by condition (31). The optimal process thus obtained is shown in Fig. 2.

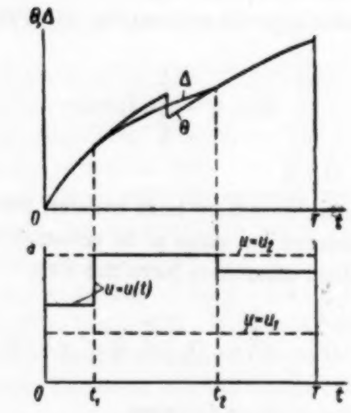

Fig. 2. Optimal process for a disturbance in the coefficient a.

We now determine the optimal control u = u(t) on the singular segments where condition (30) holds. For this, we multiply both members of expression (30) by $ae^{t/a}$ and differentiate them with respect to t. Then, if the function $Q_0(x)_t$ is differentiable with respect to t, we obtain for $0\leqslant L-\int_0^t\nu(p)\,dp\leqslant L$,

$$u(t)=Q_g+a\nu(t)e^{-\frac{t}{a}}Q_0'\Big(L-\int_0^t\nu(p)\,dp\Big), \tag{33}$$

where

$$a=\begin{cases} a_1 & \text{for } t\in[0,t_1],\\ a_2 & \text{for } t\in[t_2,T].\end{cases}$$

*This corresponds to a variation in thickness of the billets fed to the continuous furnace.

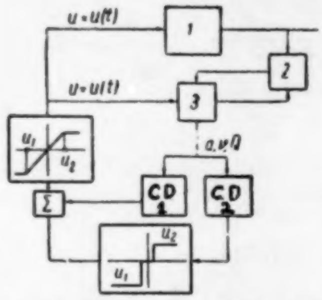

Fig. 3. Block schematic for the optimal control system for an object with distributed parameters. R is a three-position relay: u_1, 0, u_2 as a function of the signal $\delta a = a_2 - a_1$. 1 is the object, 2 is the optimizer, 3 is a model of the object.

It is obvious that condition (33) will also be sufficient for the holding of condition (30). Thus, expression (33) gives the sought-for control on the singular segments of the optimal trajectory.

Implementation of the optimal control process we have found may be achieved, for example, by means of the system whose block schematic is shown on Fig. 3. As is obvious from formulas (31) and (33), it is necessary, for the implementation of the optimal process, to have the function $Q_0 = Q_0(x)$, the coefficient a, and the velocity function $\nu = \nu(t)$.

These data can be obtained from a model of the object, the parameters of which are tuned automatically by means of the optimizer. The initial data, obtained from the model, are fed to computing devices CD1 and CD2. Device CD1 determines the moments for the initial transition from the singular portion of the trajectory to the nonsingular one, and from the nonsingular one to the singular one. With this, there is first opened, and then closed, a key which passes the sign of the increment of coefficient a which is subject to the disturbance. Relay element R has the output values u_2 or u_1 as a function of the sign of the increment of a. Device CD2, using the initial data in correspondence with formula (33), determines the value of controlling parameter $u = u(t)$.

SUMMARY

1. In automating objects with distributed parameters, it is necessary, in seeking an optimal control, to go to a variational problem of a new type, formulated in Section 1 of the present work.

2. The standard works on optimal control theory do not directly give a general method for solving the problem posed here.

3. The characteristic special features of the objects of control considered here are: the presence of both lumped and spatially distributed controlling stimuli and the limitations associated with them, which are imposed, not only on the time derivatives of the controlling stimuli, but also on their derivatives with respect to the spatial coordinates and their combinations. It is an essential point that these controlling stimuli can enter, not only into the equations of the process, but also into the boundary conditions.

4. Control of the objects of the type considered may be directed towards the achievement, either of a minimal deviation of the object's state from a given one, or to a given distribution of object states with certain conditions being met. There becomes very important, with this, the question of the attainability of the given states.

5. In the present paper we cited one of the ways in which the problem posed here can be reduced to a problem which can be solved by means of L. S. Pontryagin's maximum principle.

6. For the optimal control of the objects considered, the control system must contain a sufficiently large operational memory in which there can be stored the disturbing and controlling stimuli for a quite prolonged period of object operation.

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† See English translation.

THEORY OF CERTAIN INDIRECT CONTROL SYSTEMS WITH SEVERAL ESSENTIAL NONLINEARITIES

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In various domains of technology there are employed single-pulse systems for controlling pressure, level, temperature, speed, etc., with hydraulic, electromechanical or pneumatic amplifiers. The presence of moving parts and linkages in the controllers gives rise to the appearance of dry friction, free play, etc., that is, to factors which occasion essential nonlinearities of the characteristics. As shown both by theoretical investigations and by practical experience, these nonlinearities frequently have a decisive effect on dynamics.

One of the first works devoted to the analysis of the dynamics of nonlinear systems of indirect control is Léauté's classic work [1]. The most significant results in this direction were obtained by A. A. Andronov and his coworkers ([2, 3] and others). These, and many other, investigations explained the influence, principally, of one or another nonlinearity individually. In actual conditions, there are usually several characteristic nonlinearities. The problem thus arises of singling out systems with standard complexes of essential nonlinearities and, on the basis of an analysis of their dynamics, of constructing nomograms or tables which might facilitate the design and adjustment of these systems. The solution of this problem, despite the significant number of investigations, remains in its initial stages [4].

Below, we consider the simplest systems with account taken of nonlinearities of the types of hysteresis loops, dead zones and saturation. Each of these nonlinearities is given a physical interpretation, but other factors can be reduced to them.

The System to Be Considered

For the disturbed motion of the indirect control schemes [5, 6] to be considered, we have:

the equation of the object of control

$$T_a\dot{\varphi} + k\varphi = \mu; \tag{1}$$

the equation of the measurer and valve, with account taken of Coulomb friction in the valve

$$\gamma\eta + b\,\mathrm{sign}\,\dot{\eta} = \varphi + \delta\mu \begin{cases}\text{for} & \dot{\eta} \neq 0\\ \text{for} & \dot{\eta} = 0,\end{cases} \tag{2}$$

$$\text{but } |\gamma\eta - \varphi - \delta\mu| = b,$$

$$\eta = \text{const} \quad \text{for } |\gamma\eta - \varphi - \delta\mu| < b;$$

the equation of the servomotor with a dead zone and power limitations

$$\begin{aligned} T_s\dot{\mu} - c\,\mathrm{sign}\,\eta &= -\eta \quad \text{for} \quad c < |\eta| < c_1\\ \dot{\mu} &= 0 \quad \text{for} \quad |\eta| \leqslant c,\\ \dot{\mu} &= \text{const for} \quad |\eta| \geqslant c_1. \end{aligned} \tag{3}$$

As $c_1 \to \infty$, we have, from (3), the equation of a proportional control servomotor with a dead zone:

$$\begin{aligned} T_s\dot{\mu} - c\,\mathrm{sign}\,\eta &= -\eta \quad \text{for} \quad |\eta| > c,\\ \dot{\mu} &= 0 \quad \text{for} \quad |\eta| \leqslant c. \end{aligned} \tag{4}$$

As $c_1 \to c$ and $(c_1 - c)/T_s = 1/T_c = \text{const}$, we obtain the equation of a constant speed servomotor with a dead zone:

$$\begin{aligned} T_s\mu - c\,\mathrm{sign}\,\eta &= -c_1\,\mathrm{sign}\,\eta \quad \text{for} \quad |\eta| > c,\\ \dot{\mu} &= 0 \quad \text{for} \quad |\eta| \leqslant c, \end{aligned} \tag{5}$$

Here, φ, η, μ are dimensionless coordinates of the object, the valve and the servomotor (the measurer's coordinate is eliminated by means of the valve equation); T_a and T_s are the time constants of the object and servomotor, respectively; k is the self-alignment coefficient; γ is the measurer's nonuniformity coefficient; δ is the feedback factor; b and c are the insensitivity coefficients of the valve and servomotor, respectively; c_1 is the value of coordinate η for which $\dot{\mu}$ attains its greatest magnitude.

We shall consider all the coefficients in Eqs. (1)-(5) to be constant, while of them, in particular cases, b, c, k and δ may be zero, and, moreover, k and δ can be less than zero. For $\delta = 0$, we have a controller without feedback.

As an example, Fig. 1 shows the schemes of controllers whose dynamics, with well-known assumptions, are described by the equations just given. Some of the controllers with such schemes are obtained from the I. I. Polzunov TsKTI, the F. É. Dzerzhinskii VTI, the A. N. Krylov TsNII (SSSR), the English Kent and Bailey companies, the German Askania Company (controllers with jet pipes) and others. It is important to mention that the most widely used schemes in practice of those cited are the controller schemes with feedback of Fig. 1, b and c, in which the measurer and valve are rigidly

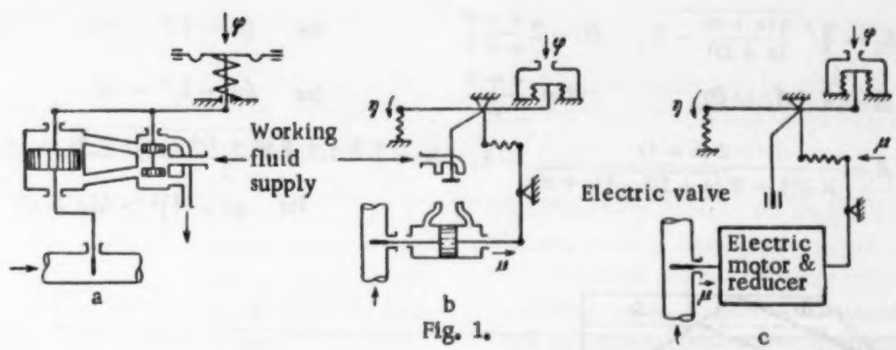

Fig. 1.

connected, and the frictional forces in these elements are inseparable. For such schemes, the foregoing equations take into account the dry friction in all the elements – in the measurer, valve and servomotor.

Method of Investigation

For the investigation of the aforementioned systems, the concept of phase space is applicable, this concept having achieved great popularity in control theory, both in the USSR and other countries, after the well-known works of A. A. Andronov and his school.

We note the specific peculiar features of constructing the phase images of the systems with which we are dealing. First, these systems are piecewise linear, which allows us to obtain the equations of the phase trajectories in the usual form. In considering spaces composed of regions with different trajectories, the concept of virtual singular points is useful [7]. Second, in the three-dimensional phase spaces of the systems under investigation, the trajectories are laid out on a definite form of oriented plane, which permits the investigation of the projections of such spaces on one plane; to eliminate nonsingle-valuedness we employ multisheeted phase surfaces [8, 9] which are analogous to the Riemann surfaces in the theory of functions of a complex variable.

The representation in phase space of a topological map leads, in essential points, to quantitative results by means of the method of point transformations of Poincaré – Brauer – Birkhoff – Andronov. We note that the presence of the Kronecker function in [2] allows the representative point to move only in definite directions on parts of the segments of the characteristics of the equivalent nonlinear link $\dot{\mu} = f(\varphi + \delta\mu)$. We denote these as the basic directions. As the basic directions of the remaining segments, we shall understand such directions in which the representative point can move in its cyclical circuit of the designated characteristic. If the point does not move in a basic direction, then it will either move during a finite time only in basic directions, or fall on a resting segment (or at infinity), without completing an oscillation. Therefore, for an employment of the method of point transformations, it suffices to take into account only those portions of the phase surface sheets which correspond to the aforementioned basic directions which allow one to obtain a simplified multisheet surface with linked (joined) phase trajectories immediately.

Parameter Space of the Systems Considered

For system (1)-(3), the regions corresponding to the different types of motion are located in the space of parameters

$$a = \frac{\delta T_a}{\gamma k T_s} \gtrless 0, \quad d = \frac{T_a}{\gamma k^2 T_s} > 0 \ \ (\text{for } k > 0),$$

$$d_1 = \frac{T_a}{\gamma k^2 T_s} > 0 \ \ (\text{for } k < 0), \quad G = \frac{\gamma c}{b} \geqslant 0$$

and are defined in the following way. In the region for which $a > -1$, $a + d > 0$, i.e., $\delta T_a + \gamma k T_s > 0$, $1 + \delta k > 0$, $k > 0$ (we call it region I), there lies surface T whose coordinates are determined, after elimination of the parameter V_1, from the equations

$$F_1(V_1) = F(V_1), \quad \left(\frac{\partial F_1}{\partial V}\right)_{V_1} = \left(\frac{\partial F}{\partial V}\right)_{V_1}, \tag{6}$$

where

$$F_1 = G\left(B \ln \frac{B}{B-V} - V\right) + \begin{cases} \sqrt{1 + A_1^2}\exp\left(\frac{1}{A_1}\operatorname{arc\,tg} A_1\right) & \text{for} \quad (a-1)^2 < 4d, \\ e & \text{for} \quad (a-1)^2 = 4d, \\ \exp\left(\frac{A}{A-1}\ln A\right) & \text{for} \quad (a-1)^2 > 4d, \end{cases}$$

$$F = \begin{cases} \sqrt{A_1^2 + (V-1)^2}\exp\left(-\frac{1}{A_1}\operatorname{arc\,tg}\frac{A_1}{V-1}\right) & \text{for} \quad (a-1)^2 < 4d, \\ (V-1)\exp\frac{1}{1-V} & \text{for} \quad (a-1)^2 = 4d, \\ (V-1)\exp\left(\frac{A}{1-A}\ln\frac{V-1}{V-A}\right) & \text{for} \quad (a-1)^2 > 4d, \end{cases}$$

$$A_1 = \sqrt{\frac{4(a+d)}{(a+1)^2} - 1}, \quad B = 2\,\frac{a+d}{a+1} \qquad \text{for} \quad (a-1)^2 < 4d,$$

$$A = 1 \ (A_1 = 0), \qquad B = \frac{a+1}{2} \qquad \text{for} \quad (a-1)^2 = 4d,$$

$$A = \frac{2(a+1)}{a+1-\sqrt{(a+1)^2-4(a+d)}} - 1, \quad B = \frac{a+1+\sqrt{(a+1)^2-4(a+d)}}{2}$$

$$\text{for} \quad (a-1)^2 > 4(a+d)$$

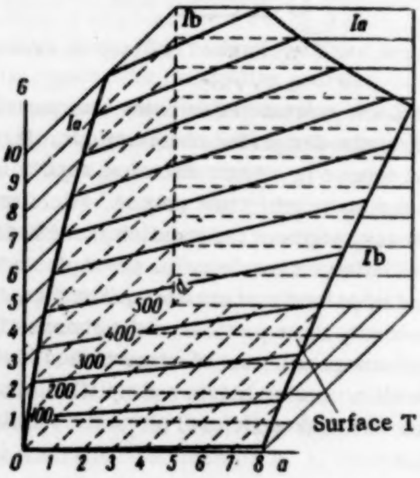

Fig. 2.

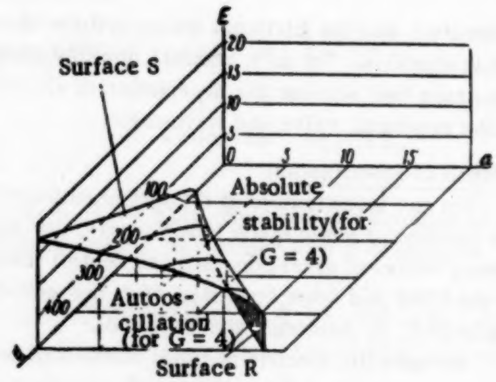

Fig. 3.

Surface T, part of which is shown on Fig. 2, divides region I into the subregions of absolute stability Ia and of rigid autooscillation mode Ib. In subregion Ia lie the points for which $d < d^0$ (for $a = a^0 = \text{const}$, $G = G^0 = \text{const}$), where a^0, d^0 and G^0 are the coordinates of surface T. For $G = 0$ (i.e., $c = 0$), subregion Ib degenerates to the region of soft autooscillation mode and, for $G = \infty$ (i.e., $b = 0$), disappears entirely due to the coalescence of surface T and the plane $a = -1$.

In the absence of a dead zone of the servomotor ($c = 0$), the boundary of stability coincides with that found previously in [3], and is defined by the trace of surface T on the plane $G = 0$ passing through the points $a = -1$, $d = 1$, and $a = 0$, $d = 3{,}04$. The remaining regions of the space being considered are of lesser practical interest; without giving the formulas for them, we fall back on Table 1, in which the case $c_1 = \infty$ is referred to system (1)-(3).

For system (1), (2), (4), the corresponding regions are constructed in the space of parameters a, d, d_1, G, $E = b/\gamma\,(c_1 - c) \geq 0$. If the point lies in the aforementioned subregion Ib, the given system, for $G > 0$, can be absolutely stable or autooscillatory (rigid mode). The parameter values on the boundary between stability and autooscillations in subregion Ib are determined by eliminating parameter V_2 from the system

$$F_1(V_2) = F(V_2), \quad \left(\frac{\partial F_1}{\partial V}\right)_{V_2} > \left(\frac{\partial F}{\partial V}\right)_{V_2},$$

$$E = \begin{cases} \dfrac{B}{2(1+A_1^2)}\left(B \ln \dfrac{B}{B-V_2} - V_2\right) & \text{for} \quad (a-1)^2 < 4d, \\ \dfrac{B}{2}\left(B \ln \dfrac{B}{B-V_2} - V_2\right) & \text{for} \quad (a-1)^2 = 4d, \\ \dfrac{B}{2A}\left(B \ln \dfrac{B}{B-V_2} - V_2\right) & \text{for} \quad (a-1)^2 > 4d. \end{cases} \tag{7}$$

When $V_2 = V_1$, the inequality in (7) becomes an equality. Stability corresponds to values of $E > E^0$ (for $a = a^0 = \text{const}$, $d = d^0 = \text{const}$, $G = G^0 = \text{const}$), where a^0, d^0, G^0 and E^0 are defined in accordance with (7). For fixed G, the parameter space of system (1), (2), (4) for region I can be constructed in the coordinates a, d, E. With this, the region of autooscillation will be bounded by the plane $E = 0$, the cylindrical surface R and by some surface S (Fig. 3). An increase in G leads to a decrease in the region of autooscillation. A number of the com-

TABLE 1

Form nonlin. considered		Character of poss. syst. motions in reg. of param. values			
Values of nonlin. parameters	Name of nonlin.	I $a > -1$ $a + d > 0$ (for $k > 0$)	II $a < -1$ $a + d > 0$ (for $k > 0$)	III $a < -1$ $a + d_1 > 0$ (for $k < 0$)	IV 1) $a + d < 0$ (for $k > 0$) or 2) $a + d_1 < 0$ (for $k < 0$) or 3) $a + d_1 > 0$ $a > -1$(for $k < 0$)
$c_1 = \infty$ $c = 0$ $b = 0$	Linear system	Abs. stability	Abs. instability	Abs. stability	Abs. instability
$c_1 \neq \infty$ $c = 0$ $b = 0$	Saturation nonlinearity	Absolute stability	Autooscillation (soft mode)	Stability bounded instab. "in the large"	Absolute instability
$c_1 = \infty$ $c \neq 0$ $b = 0$	Servomotor dead zone	Absolute stability	Stab., bounded instab. "in the large"	Autooscillation (soft mode)	Instability
$c_1 = \infty$ $c = 0$ $b \neq 0$	Coulomb friction in valve	Abs. stability or autooscillations (soft mode)	Absolute instability	Autooscillations (soft mode)	Absolute instability
$c_1 \neq \infty$ $c = 0$ $b \neq 0$	Satur. nonlin., Coulomb fric. in the valve	Abs. stability or autooscillations (soft mode)	Autooscillations (soft mode)	Autoosc. (soft mode) bounded instab. "in the large"	Absolute instability
$c_1 = \infty$ $c \neq 0$ $b \neq 0$	Coulomb fric. in valve and servom. dead zone	Abs. stability or autooscillations (rigid mode)	Stab. bounded instability "in the large"	Autooscillations (soft mode)	Instability
$c_1 \neq \infty$ $c \neq 0$ $b \neq 0$	Satur. nonlin., Coulomb fric. in valve and servom. dead zone	Abs. stability or autooscillations (rigid mode)	Autooscillations (rigid mode) or abs. stability	Autooscillations (soft mode), bounded instab. "in the large" or abs. instab.	Instability

TABLE 2

G=0	a	0,000	1,000	2,148	3,867	6,472	8,180	11,01
	d	3,045	4,441	5,905	7,977	10,98	12,89	16,05
	E	∞	∞	∞	∞	∞	∞	∞
	a	14,90	25,02	55,48	106,0	156,3	206,4	256,8
	d	20,21	30,92	63,50	114,2	164,9	215,4	266,3
	E	∞	∞	∞	∞	∞	∞	∞
	a	306,8	356,9	407,1	507,2	1008	1508	2008
	d	316,6	366,9	417,5	517,7	1020	1520	2022
	E	∞	∞	∞	∞	∞	∞	∞
G=0,2	a	1,054	3,992	6,679	10,75	13,28	13,42	13,45
	d	9,857	16,26	22,80	32,40	39,19	38,53	38,61
	E	5,158	9,411	13,83	20,36	24,35	24,57	24,62
	a	13,27	16,34	23,90	31,68	39,67	55,51	87,21
	d	39,80	45,31	62,18	79,19	97,04	113,6	200,6
	E	25,45	29,24	40,92	52,82	65,18	89,33	137,6
	a	165,6	244,6	322,4	635,0	947,1	1260	1571
	d	368,2	538,4	703,1	1368	2030	2692	3352
	E	255,5	374,8	491,1	959,4	1426	1894	2359
G=1,0	a	0,9120	4,337	9,561	15,17	25,68	29,01	29,22
	d	22,85	47,29	81,06	115,5	166,7	198,4	199,1
	E	2,026	4,702	8,456	12,33	18,09	21,42	21,74
	a	29,36	30,45	35,50	51,21	66,48	84,20	117,3
	d	200,5	206,9	236,4	327,4	420,4	515,7	703,1
	E	21,88	22,62	25,95	36,25	47,73	57,62	78,94
	a	183,6	347,6	512,6	675,5	1329	1982	2635
	d	1077	1989	2913	3810	7429	11,04·10^3	14,65·10^3
	E	121,4	225,3	330,4	432,9	845,5	1258	1669
G=2,0	a	0,1526	2,138	7,707	16,19	25,32	39,14	47,73
	d	33,39	61,85	129,7	223,9	321,0	464,5	552,0
	E	0,9529	2,038	4,631	8,376	12,19	17,86	21,32
	a	48,28	50,08	58,31	83,83	110,3	137,3	119,1
	d	557,4	576,0	659,6	915,5	1175	1444	1971
	E	21,56	22,29	25,58	35,72	46,09	56,74	77,73
	a	298,6	564,8	832,7	1096	2158	3216	4278
	d	3022	5590	8192	10,71·10^3	20,92·10^3	31,05·10^3	41,26·10^3
	E	119,5	221,7	325,1	426,0	833,6	1235	1644
G=3,0	a	0,6062	3,353	11,04	22,75	35,38	54,43	66,99
	d	65,54	119,8	251,5	435,4	626,6	905,8	1089
	E	0,9683	2,048	4,662	8,329	12,19	17,78	21,49
	a	69,55	80,91	116,0	152,9	190,0	264,2	412,4
	d	1125	1288	1785	2305	2824	3856	5909
	E	22,20	25,51	35,48	45,97	56,53	77,23	118,7
	a	781,1	1148	1515	2981	4443	5908	7364
	d	10,98·10^3	15,99·10^3	21,02·10^3	41,05·10^3	60,95·10^3	80,94·10^3	100,7·10^3
	E	221,4	322,3	424,3	828,6	1232	1636	2035
G=4,0	a	1,056	4,563	14,37	29,30	45,44	69,65	85,53
	d	105,7	196,6	413,6	716,8	1033	1490	1788
	E	0,9743	2,054	4,663	8,345	12,19	17,74	21,35
	a	88,88	103,4	148,3	195,3	242,4	336,9	525,6
	d	1850	2123	2948	3305	4652	6353	9728
	E	22,13	25,42	35,54	46,07	56,28	76,97	118,2
	a	996,0	1466	1932	3796	5667	7532	9397
	d	18,11·10^3	26,48·10^3	34,71·10^3	67,56·10^3	100,7·10^3	133,6·10^3	166,6·10^3
	E	220,6	323,4	423,8	824,2	1232	1634	2038

TABLE 2 (continued)

$G=5{,}0$	a	1,507	5,760	17,70	35,84	55,52	85,10	104,6
	d	157,2	291,2	616,1	1067	1542	2231	2682
	E	0,9797	2,054	4,674	8,337	12,24	17,86	21,56
	a	108,4	126,0	180,8	237,6	234,8	411,0	639,4
	d	2764	3165	4409	5675	6934	9536	14,53·10^3
	E	22,19	25,48	35,62	46,06	56,25	77,77	118,3
	a	1208	1779	2345	4602	6880	9148	11,41·10^3
	d	26,88·10^3	39,35·10^3	51,60·10^3	100,3·10^3	149,9·10^3	199,1·10^3	248,0·10^3
	E	219,3	320,6	419,8	822,5	1228	1630	2031
$G=6{,}0$	a	1,952	6,968	21,06	42,24	65,40	100,5	122,0
	d	218,1	405,7	860,7	1477	2139	3117	3700
	E	0,9748	2,044	4,703	8,251	12,16	17,92	21,32
	a	123,1	123,3	128,2	148,5	212,7	279,6	347,5
	d	3727	3729	3881	4412	6130	7893	9690
	E	21,41	21,45	22,40	25,44	35,53	45,98	56,40
	a	483,0	752,0	1420	2097	2763	5418	8104
	d	13,24·10^3	20,22·10^3	37,40·10^3	55,08·10^3	72,12·10^3	140,0·10^3	209,4·10^3
	E	77,20	117,9	218,0	322,9	422,5	818,0	1224
$G=7{,}0$	a	0,0112	2,392	8,148	24,37	48,83	75,40	115,6
	d	160,0	288,0	535,8	1142	1969	2843	4136
	E	0,4305	0,9713	2,038	4,697	8,297	12,15	17,87
	a	141,8	148,1	170,3	244,9	323,8	399,0	553,9
	d	4958	5184	5818	8151	10,62·10^3	12,83·10^3	17,48·10^3
	E	21,36	22,37	25,40	35,54	46,67	56,00	76,33
	a	867,0	1643	2411	3180	6254	9319	12,45·10^3
	d	26,99·10^3	50,29·10^3	73,13·10^3	95,99·10^3	187,4·10^3	278,3·10^3	373,0·10^3
	E	119,2	222,2	322,5	422,5	823,0	1223	1636
$G=8{,}0$	a	0,1451	2,850	9,431	27,99	55,30	85,41	130,8
	d	205,1	371,5	697,8	1495	2520	3648	5296
	E	0,4313	0,9826	2,092	4,821	8,265	12,16	17,82
	a	160,8	166,6	193,2	275,6	364,3	450,2	626,5
	d	6383	6617	7505	10,35·10^3	13,49·10^3	16,38·10^3	22,44·10^3
	E	21,60	22,17	25,36	34,98	46,13	55,55	76,37
	a	978,3	1849	2723	3590	7060	10,52·10^3	14,00·10^3
	d	34,49·10^3	63,94·10^3	93,66·10^3	122,8·10^3	239,8·10^3	356,1·10^3	473,8·10^3
	E	117,3	218,6	319,4	418,5	825,8	1227	1628
$G=9{,}0$	a	0,2777	3,290	10,62	31,01	61,93	95,20	145,2
	d	255,0	461,3	867,0	1826	3156	4523	6531
	E	0,4289	0,9762	2,079	4,697	8,323	12,09	17,50
	a	176,8	187,5	215,8	307,4	405,5	503,9	702,2
	d	7723	8340	9378	12,91·10^3	16,75·10^3	20,56·10^3	28,27·10^3
	E	20,95	22,40	25,34	34,92	45,40	56,05	77,47
	a	1091	2058	3034	3997	7870	11,74·10^3	15,59·10^3
	d	43,03·10^3	79,42·10^3	116,6·10^3	152,7·10^3	298,9·10^3	444,6·10^3	589,0·10^3
	E	118,4	216,0	322,0	421,5	821,5	1220	1615
$G=10{,}0$	a	0,4121	3,722	11,80	33,92	68,43	106,1	160,5
	d	310,7	559,4	1054	2177	3849	5630	7986
	E	0,4281	0,9861	2,067	4,634	8,308	12,42	17,56
	a	198,6	205,1	239,0	341,2	447,0	554,6	773,4
	d	9765	9988	11,52·10^3	15,92·10^3	20,38·10^3	24,96·10^3	34,36·10^3
	E	21,74	22,13	25,66	35,73	44,93	55,60	76,97
	a	1212	2266	3352	4421	8701	12,96·10^3	17,21·10^3
	d	53,20·10^3	96,52·10^3	142,7·10^3	187,3·10^3	366,2·10^3	543,0·10^3	720,0·10^3
	E	120,0	215,5	323,5	423,5	826,0	1223	1621

TABLE 3

$G = 0,2$	$a=0$	d	100	200	300	400	500
		E	220,4	459,6	698,8	943,4	1187
	$a=5$	d	100	200	300	400	500
		E	209,2	448,2	689,0	931,4	1176
	$a=10$	d	100	200	300	400	500
		E	197,0	437,1	678,8	921,8	1165
	$a=15$	d	100	200	300	400	500
		E	183,7	421,2	665,4	909,8	1154
	$a=25$	d	100	200	300	400	500
		E	154,7	397,6	640,4	886,1	1131
	$a=50$	d	500	1000	1500	2000	2500
		E	1070	2296	3527	4762	5995
	$a=100$	d	500	1000	1500	2000	2500
		E	936,8	2174	3407	4644	5883
	$a=200$	d	500	1000	1500	2000	2500
		E	589,5	1904	3151	4409	5637
	$a=300$	d	1000	1500	2000	2500	3000
		E	1601	2881	4131	5378	6612
	$a=400$	d	1000	1500	2000	2500	3000
		E	1218	2584	3853	5104	6352
	$a=500$	d	1000	1500	2000	2500	3000
		E	0	2256	3562	4828	6084
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$110\cdot 11^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	$117,9\cdot 10^3$	$242,3\cdot 10^3$	$367,1\cdot 10^3$	$492,1\cdot 10^3$	$616,4\cdot 10^3$
	$a=5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	$111,5\cdot 10^3$	$236,1\cdot 10^3$	$361,0\cdot 10^3$	$485,2\cdot 10^3$	$609,7\cdot 10^3$
	$a=10\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	$91,08\cdot 10^3$	$223,1\cdot 10^3$	$347,9\cdot 10^3$	$472,0\cdot 10^3$	$597,4\cdot 10^3$
	$a=15\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	$83,16\cdot 10^3$	$208,8\cdot 10^3$	$334,5\cdot 10^3$	$459,8\cdot 10^3$	$585,4\cdot 10^3$
$G = 1,0$	$a=0$	d	100	200	300	400	500
		E	31,99	69,09	106,8	144,9	183,1
	$a=5$	d	100	200	300	400	500
		E	26,59	64,29	102,2	140,4	178,7
	$a=10$	d	100	200	300	400	500
		E	19,34	58,98	97,36	135,7	174,2
	$a=15$	d	100	200	300	400	500
		E	0	53,14	92,08	131,3	169,3
	$a=25$	d	100	200	300	400	500
		E	0	37,74	80,39	120,3	159,3
	$a=50$	d	500	1000	1500	2000	2500
		E	129,7	329,8	525,3	721,4	917,7
	$a=100$	d	500	1000	1500	2000	2500
		E	0	271,4	473,6	672,3	868,9
	$a=200$	d	1000	1500	2000	2500	3000
		E	0	343,2	560,0	761,4	959,5
	$a=300$	d	1000	1500	2000	2500	3000
		E	0	0	403,2	633,7	845,4
	$a=400$	d	1000	1500	2000	2500	3000
		E	0	0	0	455,3	703,9
	$a=500$	d	1000	1500	2000	2500	3000
		E	0	0	0	0	499,8
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10,^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	$17,34\cdot 10^3$	$37,29\cdot 10^3$	$57,24\cdot 10^3$	$77,19\cdot 10^3$	$97,05\cdot 10^3$
	$a=5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	$14,48\cdot 10^3$	$36,77\cdot 10^3$	$54,77\cdot 10^3$	$74,72\cdot 10^3$	$94,64\cdot 10^3$

TABLE 3 (continued)

$G=1{,}0$	$a=10\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	$29{,}00\cdot10^3$	$49{,}47\cdot10^3$	$69{,}58\cdot10^3$	$89{,}71\cdot10^3$
	$a=15\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	$21{,}40\cdot10^3$	$43{,}50\cdot10^3$	$64{,}12\cdot10^3$	$84{,}39\cdot10^3$
$G=2{,}0$	$a=0$	d	100	200	300	400	500
		E	9,938	22,94	36,30	49,83	63,49
	$a=5$	d	100	200	300	400	500
		E	4,487	19,39	32,98	46,63	60,36
	$a=10$	d	100	200	300	400	500
		E	0	14,90	29,28	43,14	57,12
	$a=25$	d	100	200	300	400	500
		E	0	0	0	30,32	45,57
	$a=50$	d	1000	1500	2000	2500	3000
		E	98,28	170,6	242,2	313,4	384,9
	$a=100$	d	1000	1500	2000	2500	3000
		E	0	130,0	205,6	278,9	351,8
	$a=200$	d	1000	1500	2000	2500	3000
		E	0	0	0	190,0	268,7
	$a=2{,}5\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	$5{,}493\cdot10^3$	$12{,}85\cdot10^3$	$20{,}15\cdot10^3$	$27{,}42\cdot10^3$	$34{,}72\cdot10^3$
	$a=5\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	$2{,}778\cdot10^3$	$11{,}03\cdot10^3$	$18{,}45\cdot10^3$	$25{,}77\cdot10^3$	$33{,}07\cdot10^3$
	$a=10\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	$5{,}594\cdot10^3$	$14{,}49\cdot10^3$	$22{,}10\cdot10^3$	$29{,}55\cdot10^3$
	$a=15\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	0	$8{,}431\cdot10^3$	$17{,}77\cdot10^3$	$25{,}64\cdot10^3$
$G=3{,}0$	$a=0$	d	100	200	300	400	500
		E	4,195	10,71	17,39	24,24	31,16
	$a=5$	d	100	200	300	400	500
		E	0	7,554	14,67	21,66	28,67
	$a=10$	d	100	200	300	400	500
		E	0	0	11,26	18,72	25,95
	$a=25$	d	100	200	300	400	500
		E	0	0	0	0	14,30
	$a=50$	d	1000	1500	2000	2500	3000
		E	37,09	76,54	113,9	147,0	187,9
	$a=200$	d	1000	1500	2000	2500	3000
		E	0	0	0	0	59,40
	$a=2{,}5\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	$2{,}352\cdot10^3$	$6{,}250\cdot10^3$	$10{,}06\cdot10^3$	$13{,}84\cdot10^3$	$17{,}64\cdot10^3$
	$a=5\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	$4{,}731\cdot10^3$	$8{,}701\cdot10^3$	$12{,}54\cdot10^3$	$16{,}36\cdot10^3$
	$a=10\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	0	$5{,}011\cdot10^3$	$9{,}510\cdot10^3$	$13{,}53\cdot10^3$
	$a=15\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	0	0	0	0	$10{,}01\cdot10^3$
$G=4{,}0$	$a=0$	d	100	200	300	400	500
		E	1,918	5,776	9,754	13,83	17,97
	$a=5$	d	100	200	300	400	500
		E	0	0	7,311	11,60	15,84
	$a=10$	d	100	200	300	400	500
		E	0	0	0	8,825	13,42
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	38,22	61,90	84,80	107,5
	$a=2{,}5\cdot10^3$	d	$50\cdot10^3$	$100\cdot10^3$	$150\cdot10^3$	$200\cdot10^3$	$250\cdot10^3$
		E	$1{,}048\cdot10^3$	$3{,}555\cdot10^3$	$5{,}905\cdot10^3$	$8{,}226\cdot10^3$	$10{,}55\cdot10^3$

TABLE 3 (continued)

$G=4,0$	$a=5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	$2,135\cdot 10^3$	$4,745\cdot 10^3$	$7,148\cdot 10^3$	$9,503\cdot 10^3$
	$a=10\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	0	0	$4,298\cdot 10^3$	$7,039\cdot 10^3$
$G=5,0$	$a=0$	d	100	200	300	400	500
		E	0	3,363	5,977	8,644	11,38
	$a=5$	d	100	200	300	400	500
		E	0	0	3,494	6,599	9,476
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	14,01	36,03	51,71	67,04
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	$2,212\cdot 10^3$	$3,816\cdot 10^3$	$5,382\cdot 10^3$	$6,961\cdot 10^3$
	$a=5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	0	$2,771\cdot 10^3$	$4,441\cdot 10^3$	$6,051\cdot 10^3$
$G=6,0$	$a=0$	d	100	200	300	400	500
		E	0	2,010	3,853	5,712	7,618
	$a=5$	d	100	200	300	400	500
		E	0	0	0	3,730	5,887
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	0	20,79	32,94	44,18
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 11^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	$1,440\cdot 10^3$	$2,611\cdot 10^3$	$3,749\cdot 10^3$	$4,891\cdot 10^3$
	$a=5\cdot 10^2$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	0	$1,630\cdot 10^3$	$2,907\cdot 10^3$	$4,087\cdot 10^3$
$G=7,0$	$a=0$	d	1000	1500	2000	2500	3000
		E	12,72	20,32	28,08	35,95	43,80
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	0	0	21,66	29,96
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	$0,9578\cdot 10^3$	$1,866\cdot 10^3$	$2,732\cdot 10^3$	$3,594\cdot 10^3$
$G=8,0$	$a=0$	d	1000	1500	2000	2500	3000
		E	9,490	15,36	21,32	27,40	33,54
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	0	0	12,67	20,43
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	$0,6344\cdot 10^3$	$1,374\cdot 10^3$	$2,049\cdot 10^3$	$2,736\cdot 10^3$
$G=9,0$	$a=0$	d	1000	1500	2000	2500	3000
		E	7,240	11,87	16,62	21,42	26,32
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	0	0	0	13,49
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	$0,3726\cdot 10^3$	$1,031\cdot 10^3$	$1,582\cdot 10^3$	$2,133\cdot 10^3$
$G=10,0$	$a=0$	d	1000	1500	2000	2500	3000
		E	5,620	9,408	13,26	17,24	21,09
	$a=50$	d	1000	1500	2000	2500	3000
		E	0	0	0	0	8,134
	$a=2,5\cdot 10^3$	d	$50\cdot 10^3$	$100\cdot 10^3$	$150\cdot 10^3$	$200\cdot 10^3$	$250\cdot 10^3$
		E	0	0	$0,7817\cdot 10^3$	$1,244\cdot 10^3$	$1,692\cdot 10^3$

puted values of the coordinates of surface R are given in Table 2, and those for surface S, in Table 3. The values of parameter E in Table 2 are given for the points of contact of surfaces R and S. Surface T consists of the traces of surface R on the plane E = 0, so that Table 1 also contains the boundary values of the parameters for the previous system (1)-(3). For G = 0 in subregion Ib, the system (1), (2), (4), will always be autooscillatory (soft mode). With no feedback in the controller ($\delta = 0$), the boundary of stability is defined by the traces of surfaces R and S on the plane a = 0, and coincides with that constructed previously in [2]. The behavior of the system in the other regions is clear from Table 1, where the case $c_1 \neq \infty$ (i.e., $E \neq 0$) has been referred to system (1), (2), (4). We note that in region IV, for $k > 0$, $c > 0$, for some initial conditions there is a subregion of attraction of the rest segment. This last remark also applies to system (1)-(3).

In the previously cited cases of rigid modes of autooscillation, the threshold values of the initial deviations, bounding the subregions of stability of systems (1)-(3) and (1), (2), (4) "in the small," may be expressed by the formulas

$$\varphi(0)+\delta\mu(0)=b+\gamma c+\begin{cases}\dfrac{2b\,(1+A_1^2)}{B\left(B\ln\dfrac{B}{B-V_2}-V_2\right)} & \text{for}\quad (a-1)^2<4d,\\[2ex] \dfrac{2b}{B\left(B\ln\dfrac{B}{B-V_2}-V_2\right)} & \text{for}\quad (a-1)^2=4d,\\[2ex] \dfrac{2bA}{B\left(B\ln\dfrac{B}{B-V_2}-V_2\right)} & \text{for}\quad (a-1)^2>4d,\end{cases}\tag{8}$$
$$\dot\varphi(0)+\delta\dot\mu(0)=0,$$

where the parameter V_2 is defined by the first equation and inequality in (7).

For $k = 0$, the systems under consideration are autooscillatory only if $\delta > 0$ and $b \neq 0$.

From the spaces constructed it follows that the Coulomb friction in the valve has a negative effect on stability. The servomotor's dead zone and the saturation nonlinearity affect stability positively for stable objects ($k > 0$) and negatively for unstable objects ($k < 0$). On the whole, the effect of the nonlinearities treated here is so significant that, due to them, an absolutely stable linear system may become absolutely unstable, and conversely. In the most important region practically, $k > 0$, $\delta > 0$, the dependence of the boundary of stability on the magnitude of the Coulomb friction and on the magnitude of the saturation nonlinearity parameter c_1 is only observed when several nonlinearities are taken into account jointly. In this region, the deleterious influence on stability of the Coulomb friction in the valve can be compensated by an increase in the servomotor's dead zone, or by limiting its maximum power. These stability requirements are in conflict with requirements for rapidity of system response. Increasing the stability margin at the cost of increasing δ is contrary to the requirement on static accuracy. It is therefore advantageous to have quantities estimates.

For relay system (1), (2), (5), the parameter space was previously constructed in [3]. It follows, from a comparison of the stability boundaries of these three systems, that the region of stability of system (1)-(3) is bounded upon the introduction of a saturation nonlinearity; with a transition from system (1), (2), (4) to a relay system, no increase in the region of stability occurs.

Other Questions of Dynamics

In analyzing a system by our designated method, the "amplitudes" of autooscillations are defined by the coordinates of the stable fixed points of the corresponding point transformations, and the periods of autooscillations are defined by the sums of the times spent by the representative point on the segments of the trajectories making up closed cycles. Thus, for system (1)-(3), in subregion Ib, the "amplitude" of the autooscillation $x_n = (\varphi + \delta\mu)_{max}$ is expressed by formulas (8) with V_3 replacing V_2 in them, where the parameter V_3 is determined from the system

$$F_1(V_3)=F(V_3),\quad \left(\frac{\partial F_1}{\partial V}\right)_{V_3}<\left(\frac{\partial F}{\partial V}\right)_{V_3}.\tag{9}$$

With this, the period of autooscillation, for example, for $(a-1)^2 > 4d$, will be

$$T_{\text{p}}=\frac{2T_a}{k}\left[\frac{A}{B(A-1)}\ln A\,\frac{V_3-1}{V_3-A}+\frac{A/B\,\dfrac{2b}{x_{\text{p}}-b-\gamma c}+V_3}{B}+\right.$$
$$\left.+\ln\frac{(V_3-A)\exp\left(\dfrac{1}{1-A}\ln\dfrac{V_3-1}{V_3-A}\right)}{(V_3-A)\exp\left(\dfrac{1}{1-A}\ln\dfrac{V_3-1}{V_3-A}\right)-\dfrac{A}{B}\,\dfrac{2\gamma c}{x_{\text{p}}-b-\gamma c}}\right].\tag{10}$$

Autooscillations are ordinarily not the working modes of the systems being considered, so that we shall not give all the corresponding formulas here. But, for $k = 0$, $\delta > 0$, $b > 0$, all three systems are autooscillatory, and may be used as stabilization systems of certain processes only under the condition that the maximum deviation φ_{max} of the controlled quantity be small in the autooscillatory mode. In this case, the relative "amplitudes"

of the autooscillation $\Phi = (\varphi_{max} - \gamma c_1)/\gamma(c_1 - c)$, defining the value of φ_{max}, are expressed by the formulas:
for system (1)-(3)

$$\Phi_1 = E + E\frac{1-2h}{E^*} - 1, \tag{11}$$

for system (1), (2), (4),

$$\Phi_2 = E + 2h\left(\frac{U_p^2}{4} - 1\right) \quad \text{for} \quad E > E^*,$$
$$\Phi_2 \equiv \Phi_1 \quad \text{for} \quad E < E^*, \tag{12}$$

for system (1), (2), (5),

$$\Phi_3 = \frac{E^2}{8h} - 1. \tag{13}$$

Here, $h = a^2/4d$; E^* and U_p are defined by the equations

$$(L)_{U=0,\, E=E^*} = 0, \quad (L)_{U=U_p} = 0,$$

$$L = \sqrt{A_1^2 + [\sqrt{U^2 + 4E(1+A_1^2)} - 1]^2}\exp\left[-\frac{1}{A_1}\operatorname{arc\,tg}\frac{A_1}{\sqrt{U^2+4E(1+A_1^2)}-1}\right] -$$
$$-\sqrt{A_1^2 + (U+1)^2}\exp\left(\frac{1}{A_1}\operatorname{arc\,tg}\frac{A_1}{U+1}\right) \quad \text{for} \quad h < 1,$$
$$L = (\sqrt{U^2+4E} - 1)\exp\left(-\frac{1}{\sqrt{U^2+4E}-1}\right) - (U+1)\exp\frac{1}{U+1} \quad \text{for } h = 1,$$
$$L = (\sqrt{U^2 + 4AE} - 1)\exp\left(\frac{A}{1-A}\ln\frac{\sqrt{U^2+4AE}-1}{\sqrt{U^2+4AE}-A}\right) -$$
$$-(U+1)\exp\left(\frac{A}{A-1}\ln\frac{U+A}{U+1}\right) \quad \text{for} \quad h > 1,$$

where, in accordance with our previous notation,

$$A_1 = \sqrt{\frac{1}{h} - 1}, \quad A = \frac{1 + \sqrt{1 - 1/h}}{1 - \sqrt{1 - 1/h}}.$$

Formulas (11)-(13) isolate, in coordinates E, h, the regions for which one of the inequality signs $\Phi_1 \gtrless \Phi_3$, $\Phi_2 \gtrless \Phi_3$, is valid, and it follows from them, that systems (1)-(3) and (1), (2), (4), when compared with system (1), (2), (5) in terms of the magnitude of Φ, gain with a decrease in h and an increase in E, and lose for the inverse changes of the parameters.

Analysis of phase spaces permits one to establish the conditions for monotonic (without overshoot) flow of the transient response. We note that the region of monotonicity of System (1)-(3) is broadened upon the introduction of a saturation nonlinearity. For system (1), (2), (4), the region in which the transient response, with initial conditions $-\infty < \varphi(0) + \delta\mu(0) < \infty$, $\dot{\varphi}(0) + \delta\dot{\mu}(0) = 0$, occurs without overshoot and which lies in parameter space inside the region of stability, is defined by the inequalities

$$\sqrt{A_1^2 + (B-1)^2}\exp\left(-\frac{1}{A_1}\operatorname{arc\,tg}\frac{A_1}{B-1}\right) \leqslant \frac{2(1+A_1^2)}{B}GE \quad \text{for} \quad (a-1)^2 < 4d,$$
$$B \leqslant 1 + \frac{2}{B}GE\exp\frac{1}{B-1} \quad \text{for} \quad (a-1)^2 = 4d,$$
$$B \leqslant A + \frac{2A}{B}GE\exp\left(\frac{1}{A-1}\ln\frac{B-1}{B-A}\right) \quad \text{for} \quad (a-1)^2 > 4d.$$

Practical Applications of the Theory

An important assumption in the theory presented is the neglect of inertial forces in the controlling devices. However, as was shown by experimental investigation of a number of control schemes for naval steam energy stands, this assumption was justified in the corresponding cases (low inertia measuring system, hydraulic piston servomotor).

As an example of the use of the results presented, we determine the magnitude of the feedback factor necessary for stabilizing the steam pressure regulator [10] of the A. N. Krylov TsNII system, one of which was demonstrated at the All-Union Industrial Exhibition in 1957. For this example, it was determined that: $T_a = 60$ seconds; $k = 0.25$; $\gamma = 0.03$; $T_s = 40$ seconds; $c = 0.2$; $b = 0.001$; $c_1 = 0.9$; i.e., $a = 200\delta$; $d = 800$; $G = 6$; $E = = 0.0476$. From the tables we find that, for stability, the adjustment must provide $\delta > 0.09$, which is borne out by practical testing.

The data given are applied in an analogous way to some other practical problems.

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ON THE THEORY OF OSCILLATIONS OF QUASI-LINEAR SYSTEMS WITH CONSTANT LAGS

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The conditions are given for the existence and stability of almost periodic oscillations in quasi-linear systems with time delays.

The problem of studying the conditions for the appearance of autooscillations, and also forced oscillations, in control systems is of significant interest. The study of oscillations in control systems described by ordinary differential equations was the subject-matter of works by B. V. Bulgakov [1], A. M. Letov [2], A. I. Lur'e [3], M. A. Aizerman, Ya. Z. Tsypkin, and others. In particular, very effective use was made in the works cited of the method of small parameters in the study of oscillations in certain controlled systems.

It is of interest to develop the method of small parameters in the direction of its use in the study of the appearance and stability of periodic and almost periodic modes in nonlinear control systems. The present paper presents the author's results in the theory of almost periodic oscillations in general quasi-linear systems with constant lags.

The method of N. N. Bogolyubov [4] is used in this work. The paper is a development of results obtained by I. G. Malkin [5] in solving the problem of the existence of almost periodic modes in quasi-linear systems whose motions are described by ordinary differential equations.

In clearing up the question of stability, the methods of A. M. Lyapunov, developed in the works of N. G. Chetaev [6], N. N. Krasovskii [7], and others, turned out to be very useful.

1. We consider a system whose motions are described by a set of differential-difference equations of the form

$$\frac{dx_s(t)}{dt} = a_{s1}x_1(t) + \ldots + a_{sn}x_n(t) + b_{s1}x_1(t-\tau) + \ldots + b_{sn}x_n(t-\tau) + \\ + f_s(t) + \mu F_s[t;\ x_1(t), \ldots, x_n(t);\ x_1(t-\tau), \ldots, x_n(t-\tau);\ \mu] \quad (s = 1, \ldots, n) \tag{1}$$

Here, a_{si}, b_{si} are constant coefficients; τ is a constant which characterizes the temporal delay; $f_s(t)$ are finite trigonometric sums; $F_s(t, x_1, \ldots, x_n, x_1', \ldots, x_n', \mu)$ are polynomials in $x_1, \ldots, x_n, x_1', \ldots, x_n'$ and trigonometric finite sums in $\underline{t}$ with frequencies which do not depend on the variables x, x', and μ is a small parameter on which the coefficients of the F_s depend analytically; $[x_1' = x_1(t-\tau), \ldots, x_n' = x_n(t-\tau)]$.

We consider the characteristic equation

$$\Delta(\lambda) \equiv |a_{si} - \delta_{si}\lambda + b_{si}\exp(-\lambda\tau)| = 0. \tag{2}$$

Theorem 1. If Eq. (2) has no roots on the imaginary axis of the λ plane, then system (1) admits a unique almost periodic solution $x_s^*(t, \mu)$ for sufficiently small $|\mu|$.

The proof of this theorem is carried out by the method of successive approximations on the basis of the following lemma.

Lemma 1. We consider the system of linear differential-difference equations

$$\frac{dx_s(t)}{dt} = a_{s1}x_1(t) + \ldots + a_{sn}x_n(t) + b_{s1}x_1(t-\tau) + \ldots + b_{sn}x_n(t-\tau) + f_s(t) \\ (s = 1, \ldots, n), \tag{3}$$

This system is obtained from (1) for $\mu = 0$.

If characteristic Eq. (2) has no roots on the imaginary axis of the λ plane, then Eq. (3) admits one, and only one, almost periodic solution $x_s^*(t)$, and this solution has the estimate

$$|x_s^*(t)| < AM, \tag{4}$$

where A does not depend on the f_s, and is determined only by the coefficients a, b and the lag τ.

We construct the solution $x_s^*(t)$ by Bückner's method in the following way. Let $\Delta_{jk}(i\lambda)$ be the cofactor of the element in the kth column and jth row of the matrix $\|\Delta(i\lambda)\|$. We introduce the notation $\Gamma_{kj}(i\lambda) = \Delta_{jk}(i\lambda)/\Delta(i\lambda)$.

Let
$$K_{js}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\lambda t}\Gamma_{js}(i\lambda)\,d\lambda.$$

It can be shown that the integral

$$\int_{-\infty}^{+\infty} |K_{js}(t)|\,dt \tag{5}$$

has a finite value.

Then the almost periodic solution of system (3) is determined by the formulas

$$x_s^* = \sum_{j=1}^{n}\int_{-\infty}^{+\infty} f_j(\xi)K_{sj}(\xi - t)\,d\xi \equiv \sum_{j=1}^{n}\int_{-\infty}^{+\infty} f_j(t+\xi)K_{sj}(\xi)\,d\xi \quad (s = 1, \ldots, n).$$

From this expression, for the solutions x_s^*, with account taken of (5), the estimate of (4) is rapidly obtained.

In the course of the proof of convergence of the successive approximations for the almost periodic solution of system (1), a rough estimate is obtained for the region of existence of an almost periodic solution in terms of the parameter μ.

Theorem 2. If all the roots of Eq. (2) have negative real parts, then the solution $x_s^*(t, \mu)$ of system (1) will be asymptotically stable for $t \to +\infty$ and for sufficiently small, by modulus, initial functions, characterizing the disturbance μ for sufficiently small $|\mu|$.

2. We consider the case when, among the roots of Eq. (2), there are m purely imaginary ones: $\omega_1 i, \omega_2 i, \ldots, \omega_m i$. We assume that to these roots there will correspond m periodic solutions $\varphi_{s1}, \varphi_{s2}, \ldots, \varphi_{sm}$ of homogeneous system (3). The periods of these solutions will be, respectively: $2\pi/|\omega_1|, \ldots, 2\pi/|\omega_m|$. In case some of the ω equal zero, the periods of the solutions will be arbitrary, and we shall take them as equal to, for example, 2π. We assume that $\omega_m \neq 0$.

We again consider system (3), in which we shall assume the $f_s(t)$ to be arbitrary, almost periodic functions.

Lemma 2. We consider the auxiliary system

$$\begin{aligned}\frac{dx_s(t)}{dt} &= a_{s1}x_1(t) + \ldots + a_{sn}x_n(t) + b_{s1}x_1(t-\tau) + \ldots + b_{sn}x_n(t-\tau) + \\ &+ f_s(t) + \varphi_{s1}W_1(t) + \ldots + \varphi_{sm}W_m(t) \quad (s = 1\ldots, n)\end{aligned} \tag{6}$$

No matter what the almost periodic functions $f_s(t)$, it is always possible to construct almost periodic functions $W_1, \ldots, W_m$ with frequencies whose moduli do not exceed 2ϵ (ϵ is an arbitrarily small positive number) such that system (6) will admit an almost periodic solution of the form

$$x_s^* = M_1\varphi_{s1} + \ldots + M_m\varphi_{sm} + \varphi_s(t), \tag{7}$$

where $M_1, \ldots, M_m$ are arbitrary constants, and $\varphi_s(t)$ is an almost periodic function. The functions $\varphi_s(t)$ will be completely uniquely defined if it is required that, for example, frequencies equal to the frequencies $\omega_1, \ldots, \omega_m$ not enter in φ_n. The functions φ_s admit estimates of the type of (4).

Lemma 3. We consider an auxiliary system of the form

$$\begin{aligned}\frac{dx_s(t)}{dt} &= a_{s1}x_1(t) + \ldots + a_{sn}x_n + b_{s1}x_1(t-\tau) + \ldots + b_{sn}x_n(t-\tau) + f_s(t) + \\ &+ \mu F_3[t, x_1(t), \ldots, x_n(t-\tau), \mu] + \varphi_{s1}W_1 + \ldots + \varphi_{sm}W_m \quad (s = 1, \ldots, n).\end{aligned} \tag{8}$$

System (8) permits almost periodic solutions of the form

$$x_s^*(t, M_1, \ldots, M_m, \mu) = \varphi_{s1}M_1 + \ldots + \varphi_{sm}M_m + \varphi_s + \mu\overline{x}_s(t, M, \mu) \quad (s = 1, \ldots, n). \tag{9}$$

where $M_1, \ldots, M_m$ are arbitrary parameters, defined in some neighborhood of the point $\overline{M}_1, \ldots, \overline{M}_m, \mu = 0$, and the $\overline{x}_s^*$ are analytical functions of the parameters $M_1, \ldots, M_m$, in the neighborhood of the point $\overline{M}_1, \ldots, \overline{M}_m, \mu = 0$.

With respect to t, the function x_s^* is almost periodic. Solution (9) exists for completely determined functions $W_1, \ldots, W_m$ with low frequencies, less than some 2ϵ, and are analytic with respect to the parameters $M_1, \ldots, M_m$ and μ in the neighborhood of the same

point $\bar{M}_1, \ldots, \bar{M}_m, 0$. With our assumptions as to F_s and as to the polynomials $\underline{x}$, the point $\bar{M}_1, \ldots, \bar{M}_m$ is arbitrary.

The proof is carried out on the basis of Lemma 2 by the method of successive approximations.

The lemma generalizes a result of the author [8] obtained earlier for ordinary equations and functions F_s which are periodic in $\underline{t}$.

3. We assume that we have found the almost periodic solutions of auxiliary system (8) and the corresponding slowly varying functions $W_1, \ldots, W_m$. We note that, for the sequel, it suffices to find, for example, only several terms in the series expansions of these functions in powers of μ.

Thanks to the fact that the F_s are assumed to be polynomials in $\underline{x}$ and finite trigonometric sums in $\underline{t}$, it is always possible to choose a number ϵ sufficiently small that the functions W_i will have the expressions

$$W_i \equiv \mu Q_i(M_1, \ldots, M_m) + \mu^2 W_i'(t, M_1, \ldots, M_m, \mu) \quad (i = 1, \ldots, m),$$

where the Q_i are determined in the following manner.

Let $\psi_{s1}, \ldots, \psi_{sm}$ be periodic solutions of the linear homogeneous system "adjoint" to system (3):

$$\frac{dx_s(t)}{dt} = -a_{1s}x_1(t) - \ldots - a_{ns}x_n(t) - b_{1s}x_1(t+\tau) - \ldots - b_{ns}x_n(t+\tau).$$

Let $\lim\limits_{t\to+\infty} (1/t) \int\limits_0^t \sum\limits_{s=1}^n \varphi_{sj}\, \psi_{sk}\, dt = d_{jk}$. It can be shown that $|d_{jk}| \neq 0$. Then

$$d_{j1}Q_1 + \ldots + d_{mj}Q_m + P_j = 0 \quad (j = 1, \ldots, m),$$

where

$$P_j \equiv \lim_{t\to+\infty} \frac{1}{t} \int\limits_0^t \sum_{s=1}^n F_s[t, x_1^0(t), \ldots, x_n^0(t), x_1^0(t-\tau), \ldots, x_n^0(t-\tau), 0]\, \psi_{sj}(t)\, dt.$$

$$(j = 1, \ldots, m).$$

In the last expression,

$$x_s^0 = M_1\varphi_{s1} + \ldots + M_m\varphi_{sm} + \varphi_s \quad (s = 1, \ldots, n)$$

are solutions of system (1) with $\mu = 0$.

Theorem 3. Let the system of equations

$$P_j(M_1, \ldots, M_m) = 0 \quad (j = 1, \ldots, m) \tag{10}$$

admit the solutions $M_j = \bar{M}_j$, and let system (1), for $\mu = 0$, admit almost periodic solutions.

If the equation

$$\left| \frac{\partial (P_j(\bar{M}))}{\partial \bar{M}_k} - \delta_{jk}\lambda \right| = 0 \tag{11}$$

has roots with negative real parts, and the roots of (2), not lying on the imaginary axis, have negative real parts, then system (1) has an almost periodic solution in a sufficiently small region $|\mu| \leq \mu^*$, asymptotically stable for $t \to \infty$ and initial disturbances of sufficiently small moduli. This solution, to a first approximation, is defined by the formulas

$$x_s = \bar{M}_1\varphi_{s1} + \ldots + \bar{M}_m\varphi_{sm} + \varphi_s(t) \quad (s = 1, \ldots, n). \tag{12}$$

SUMMARY

It was shown in this work that, for quasi-linear systems with time delays, it is possible to establish the conditions for existence and stability of almost periodic motions analogous to the well-known conditions for systems described by ordinary differential equations.

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TRANSIENT RESPONSES IN CONTROL SYSTEMS WITH LAGS

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A method is given for constructing the transient response in systems with lags. Transfer functions are found for systems with lags, where these lags may be in the forward circuit, in the basic feedback path and in auxiliary feedback paths. For the transient response under consideration, an integral equation is set up and solved, its solution being given in the form of a series which converges absolutely and uniformly on any given finite interval of time.

Estimates of overshoot and control time (duration of the transient response) are given, based on the parameters of the real and imaginary frequency characteristics of the system with lags.

Relationships are derived which permit one to compare different integral criteria of control quality for systems with, and without, lags.

1. Representation and Construction of the Transient Response

We consider a model of a control circuit with a lag (Fig. 1), where $\Phi(z)$ is the link's transfer function when the lag is not taken into account, and θ is the lag time which, in the sequel, we shall take to be constant. By assuming that, to construct the representation function $\varphi_{out}(t)$, we should take into account the initial function $\varphi^*_{in}(t)$, given in the interval $-\theta \leq t \leq 0$, which precedes the initial moment of time,* we may write

$$L[\varphi_{out}(t)] = \Phi(z)e^{-\theta z}L[\varphi_{in}(t)] + \Phi(z)\varepsilon(\theta, z), \quad (1)$$

where L is the symbol of the Laplace transform,

$$\varepsilon(\theta, z) = e^{-\theta z}\int_{-\theta}^{0} \varphi^*_{in}(t)e^{-zt}\,dt.$$

Formula (1) shows that, for nonzero initial functions, the representation of the transient response output signal consists of two terms, whereby the effect of the initial functions can be taken into account independently by assuming that, at the input of the ordinary link with transfer function $\Phi(z)$, there acts an external disturbance, given by the representation $\epsilon(\theta, z)$. By taking this circumstance into account, we shall henceforth consider a standard representation of the transient response in a closed linear control system with lags (Fig. 2) in the form

$$W(z) = \frac{W_0(z)e^{-\theta_1 z}}{1 + W_k(z)e^{-\theta_2 z}}, \quad (2)$$

where $W_0(z) = \Phi_1(z)L[\varphi_{in}(t)]$, $W_k(z) = \Phi_1(z)\Phi_2(z)$, $\theta_2 = \theta_1 + \theta_3$. It is assumed with this that the functions $\Phi_1(z)$ and $\Phi_2(z)$ must take into account the lags in the auxiliary feedback paths in those cases when this is necessary.

In work [2], for representations of the type given in (2), there is given a method of constructing the transient response based on the expansion of W(z) in an infinite series in powers of $W_k(z)$ which converges for $|W_k(z)| < 1$, i.e., for small values of the gains.

It is preferable to give a method of constructing the transient response which does not entail the investigation of convergences of an infinite series. Directly from formula (2) we obtain

$$W(z) = W_0(z)e^{-\theta_1 z} - W(z)W_k(z)e^{-\theta_2 z}. \quad (3)$$

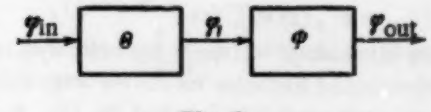

Fig. 1.

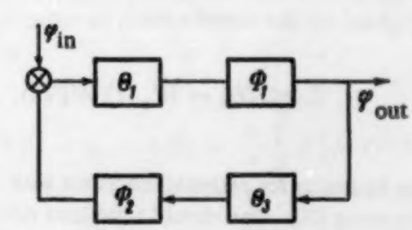

Fig. 2.

*Cf., for example, [1].

By applying the convolution and lag theorems [3] to expression (3), we obtain an integral equation for the transient response being sought:

$$\varphi_{out}(t) = \tag{4}$$

$$= \varphi_0(t-\theta_1) - \int_0^t \varphi_{out}(\tau)\varphi_k(t-\theta_2-\tau)\,d\tau,$$

where $\varphi_{out}(t)$, $\varphi_0(t)$, and $\varphi_k(t)$ are the time domain functions corresponding to the transforms $W(z)$, $W_0(z)$, and $W_k(z)$. The solution of Eq. (4) is written in the form of a series:

$$\varphi_{out}(t) = \varphi_0(t-\theta_1) - \varphi_1(t-\theta_1-\theta_2) + \\ + \varphi_2(t-\theta_1-2\theta_2) - \ldots + \\ + (-1)^n\varphi_n(t-\theta_1-n\theta_2), \tag{5}$$

which converges absolutely and uniformly in t on any finite interval of time. The sufficient conditions which are imposed on the initial function $\varphi_0(t)$ and the kernel $\varphi_k(t)$ of integral Eq. (4) amount, as is well known [4], to the holding of the conditions

$$|\varphi_0(t)| < Ae^{at}, \quad |\varphi_k(t)| < Be^{bt},$$

where A, B, a, b are positive constants.

In computing by formula (5), it must be taken into account that all the functions entering into it, $\varphi_m(t-\theta_1-m\theta_2)$ (m = 0, 1, 2, . . . n), equal zero for negative values of the argument. This important circumstance allows one to find the values of $\varphi_{out}(t)$ exactly for any previously given finite interval of time t which, at the same time, can be considered as an approximation for a large interval.

To construct the time domain functions entering into formula (5), we take into account that the transform of any of them,

$$L[\varphi_m(t-\theta_1-m\theta_2)] = \\ = W_0(z)W_k^m(z)e^{-(\theta_1+m\theta_2)t}, \tag{6}$$

is easily obtained if we apply the ordinary Laplace transformation to the formulas for the successive determinations of the $\varphi_m(t-\theta_1-m\theta_2)$ by integral Eq. (4). In practice, the problem reduces to the determination of the functions $\varphi^*_m(t)$, given by the transforms

$$L[\varphi^*_m(t)] = W_0(z)W_k^m(z), \tag{7}$$

from the formulas for rational fractions with known multiple poles [3]. One should take into account with this that $W_0(z)$ and $W_k(z)$ are defined by the given transform of the input stimulus $W_{in}(z)$ and the transfer functions $\Phi_1(z)$ and $\Phi_2(z)$ of the open-loop circuits (Fig. 2).

As an example of the application of the formulas we have derived, we construct the transient response for the representation

$$W(z) = \frac{ke^{-\theta z}}{z[(T_1z+1)(T_2z+1)+ke^{-\theta z}]}.$$

Since

$$\lim_{z\to\infty} zW(z) = \varphi_{out}(0) = 0,$$

we shall find it preferable to consider the transform of the transient response's velocity,

$$L\varphi'_{out}(t) = zW(z) = \frac{ke^{-\theta z}}{(T_1z+1)(T_2z+1)+ke^{-\theta z}}.$$

In accordance with our notation $\theta_1 = \theta_2 = \theta$,

$$W_0(z) = W_k(z) = \frac{k}{(T_1z+1)(T_2z+1)}.$$

The transforms of the successive functions, in accordance with formula (7), are found from the expression

$$L[\varphi^*_m(t)] = \\ = \frac{k^{m+1}}{(T_1z+1)^{m+1}(T_2z+1)^{m+1}} \quad (m=0,1,2,\ldots,n). \tag{8}$$

Since

$$L^{-1}\left[\frac{1}{(T_kz+1)^{m+1}}\right] = \frac{t^m e^{-\frac{t}{T_k}}}{T_k^{m+1}m!} \quad (k=1,2),$$

then all the functions given by the transforms of (8) are defined by the convolution formula

$$\varphi^{**}_m(t) = \frac{k^{m+1}e^{-\frac{t}{T_2}}}{(m!)^2(T_1T_2)^{m+1}}\int_0^t \tau^m(t-\tau)^m e^{-\left(\frac{\tau}{T_1}-\frac{\tau}{T_2}\right)}d\tau.$$

Consequently,

$$\dot\varphi_m(t) = \tag{9}$$

$$\frac{k^{m+1}}{(m!)^2(T_1T_2)^{m+1}}\int_0^t e^{-\frac{t}{T_2}}dt\int_0^t \tau^m(t-\tau)^m e^{-\left(\frac{\tau}{T_1}-\frac{\tau}{T_2}\right)}d\tau.$$

The solution being sought is written in the form

$$\varphi_{out}(t) = \varphi_0(t-\theta) - \varphi_1(t-2\theta) + \ldots + \\ + (-1)^{n-1}\varphi_n(t-n\theta),$$

where all the functions φ^*_m are computed from formula (9).

For representations of more general types, the method just presented for constructing the transient responses leads to arduous computations, particularly when the gains are large.

2. Frequency Characteristics of Systems with Lags

By taking into account that, in representation (2), the factor $e^{-\theta_1 z}$ leads only to a shift of the corresponding time domain characteristic by the amount θ_1, we shall find it more advantageous in the sequel to consider the following function as the representation of a transient response with lags:

$$W(z) = \frac{W_0(z)}{1 + W_k(z)\, e^{-\theta z}}. \tag{10}$$

As will be shown later, the frequency characteristics corresponding to representation (10) ordinarily have a finite number of points of intersection with the frequency axis, and do not differ essentially in their main features from the analogous characteristics of systems without lags. This allows us to carry over to systems with lags the estimating methods which have been previously developed for systems without lags [5, 6, 7].

If all the poles of representation (10) lie in the left half-plane, then the abscissa of convergence of the integral

$$W(z) = \int_0^\infty \varphi(t)\, e^{-zt}\, dt,$$

is $\sigma < 0$, and we can set $z = i\omega$ in it. Then,

$$W(i\omega) = S(\omega) + iQ(\omega) = \int_0^\infty \varphi(t)\, e^{-i\omega t}\, dt.$$

Consequently,

$$S(\omega) = \int_0^\infty \varphi(t) \cos \omega t\, dt. \tag{11}$$

$$Q(\omega) = -\int_0^\infty \varphi(t) \sin \omega t\, dt. \tag{12}$$

The functions $S(\omega)$ and $\varphi(t)$, $Q(\omega)$ and $\varphi(t)$ are Fourier transform pairs. Based on their duality property [8], one can write

$$\varphi(t) = \frac{2}{\pi} \int_0^\infty S(\omega) \cos t\omega\, d\omega, \tag{13}$$

$$\varphi(t) = -\frac{2}{\pi} \int_0^\infty Q(\omega) \sin t\omega\, d\omega. \tag{14}$$

If representation (10) has a simple pole at the origin:

$$W(z) = \frac{\Phi(z)}{z},$$

we then set

$$\Phi(i\omega) = R(\omega) + iI(\omega)$$

and find the transient response corresponding to W(z) from the convolution formula, taking into account that the transient response for $\Phi(z)$ is determined from formulas (13) and (14):

$$\varphi(t) = \frac{2}{\pi} \int_0^t \int_0^\infty R(\omega)\, 1(t - \tau) \cos \tau\omega\, d\omega\, d\tau.$$

By changing the order of integration, we get

$$\varphi(t) = \frac{2}{\pi} \int_0^\infty \frac{R(\omega)}{\omega} \sin t\omega\, d\omega. \tag{15}$$

Analogously,

$$\begin{aligned} \varphi(t) &= -\frac{2}{\pi} \int_0^t \int_0^\infty I(\omega)\, 1(t - \tau) \sin \tau\omega\, d\omega\, d\tau = \\ &= \frac{2}{\pi} \int_0^\infty \frac{I(\omega)}{\omega} \cos t\omega\, d\omega - \frac{2}{\pi} \int_0^\infty \frac{I(\omega)}{\omega}\, d\omega. \end{aligned} \tag{16}$$

Since the first of these integrals equals zero for $t \to \infty$, then

$$\varphi(\infty) = \varphi_v = -\frac{2}{\pi} \int_0^\infty \frac{I(\omega)}{\omega}\, d\omega = \frac{2}{\pi} \int_0^\infty S(\omega)\, d\omega, \tag{17}$$

where

$$S(\omega) = -\frac{I(\omega)}{\omega}, \tag{18}$$

and φ_s is the steady-state value of the output coordinate. Consequently,

$$\varphi(t) = -\frac{2}{\pi} \int_0^\infty S(\omega) \cos t\omega\, d\omega + \varphi_v. \tag{19}$$

By shifting the origin of coordinate, we shall henceforth consider as the transient response the time domain function

$$\phi(t) = \varphi_v - \varphi(t) = \frac{2}{\pi} \int_0^\infty S(\omega) \cos t\omega\, d\omega. \tag{20}$$

A comparison of formulas (13) and (20) shows that the function $S(\omega)$, given by formula (18), is the frequency characteristic of the system for the transient response $\phi(t)$.

3. Some Properties of the Frequency Characteristics W(z)

We limit our consideration to the case when the representation has a simple pole at the origin. Since, for linear systems and a broad class of disturbing actions $W_0(z)$ and $W_k(z)$ are rational fractions, then, by introducing the following notation into formula (10):

$$zW_0(z) = \frac{d_0(z)}{d_1(z)}, \quad W_k(z) = \frac{d_2(z)}{d_3(z)},$$

$$D_0(z) = d_0(z)\, d_3(z),$$

$$D_1(z) = d_1(z)\, d_3(z), \quad D_2(z) = d_1(z)\, d_2(z),$$

where $d_k(z)$ and $D_k(z)$ are polynomials in z, we present the function $\Phi(z)$, for $z = i\omega$, in the form

$$\Phi(i\omega) = \frac{D_0(i\omega)}{D_1(i\omega) + D_2(i\omega)\, e^{-i\theta\omega}} = R(\omega) + iI(\omega).$$

By setting $D_k(i\omega) = x_k + iy_k$ ($k = 0, 1, 2$), we get

$$R(\omega) = \frac{Ax_0 + By_0}{A^2 + B^2},$$

$$I(\omega) = \frac{Ay_0 - Bx_0}{A^2 + B^2},$$

$$A = x_1 + x_2 \cos\theta\omega + y_2 \sin\theta\omega,$$

$$B = y_1 + y_2 \cos\theta\omega - x_2 \sin\theta\omega.$$

The zeros of the frequency characteristics $R(\omega)$ and $I(\omega)$ are defined, respectively, by the roots of the transcendental equations:

$$\begin{gathered} x_0x_1 + y_0y_1 + (x_0x_2 + y_0y_2)\cos\theta\omega + \\ + (x_0y_0 - y_0x_2)\sin\theta\omega = 0, \\ y_0x_1 - x_0y_1 + (y_0x_2 - x_0y_2)\cos\theta\omega + \\ + (y_0y_2 + x_0x_2)\sin\theta\omega = 0. \end{gathered} \tag{21}$$

Let the stimulus $\varphi_{in}(t) = 1(t)$ be applied to the measuring device, and let $W_k(z) = zW_0(z)$. Then $D_0(z) = D_2(z)$ and Eqs. (21) for $R(\omega)$ and $I(\omega)$ are rewritten in the form, respectively,

$$\cos\theta\omega = -\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}, \quad \sin\theta\omega = \frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}. \tag{22}$$

Let the polynomials $D_2(z)$ and $D_1(z)$ be of degrees m and n, respectively, where $m \le n$. Then, a detailed analysis of formulas (22) leads to the following results.

The frequency characteristic $R(\omega)$ has a finite number of points of intersection with the ω axis for $m < n-1$, independently of the system parameters and the lag θ. For $m \ge n-1$, the number of points of intersection with the ω axis will be finite only if the following inequality holds:

$$\lim_{\omega\to\infty}\left|\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}\right| > 1. \tag{23}$$

Frequency characteristic $I(\omega)$, and with it frequency characteristic $S(\omega)$, has a finite number of points of intersection with the ω axis for $m < n$ independently of the system parameters and the lag θ. Since the case $m = n$ is not possible for $\varphi(0) = 0$, the characteristics $I(\omega)$ and $S(\omega)$, for sufficiently large ω, do not change sign, and tend to zero for $\omega \to \infty$. The same behavior occurs for characteristic $R(\omega)$ for $m < n-1$ or, if inequality (23) holds, for $m < n$. Consequently, for the particular case being investigated, one can neglect the high-frequency parts of the characteristics $R(\omega)$, $I(\omega)$ and $S(\omega)$ and can estimate the errors thus introduced by means of the formulas derived in [5, 6], for characteristics without lags. For the characteristics of systems with lags we thereby establish the concept of the essential interval of frequency on which they should be constructed. Thus, for example, considering $S(\omega) > 0$, for definiteness, in the high-frequency interval $\omega_c \le \omega \le \infty$, and by then discarding this segment of the characteristic, we obtain, by using the results of [6], the following estimate of the transient response error:

$$|\Delta\psi| \leqslant \psi(0) - \frac{2}{\pi}F, \tag{24}$$

where F is the area bounded by the frequency characteristic $S(\omega)$ in the chosen essential interval of frequency $0 \le \omega \le \omega_c$, $\psi(0) = \varphi_s$. Since φ_s is easily computed from the given representation of the transient response, then formula (24) allows the essential frequency interval (by area) to be so chosen that the error in the transient response will not exceed the previously given quantity $|\Delta\psi|$.

4. Estimate of Overshoot and Control Time

To determine overshoot and control time from frequency characteristics, we construct these latter from a large number of computed ordinates in the essential frequency interval, and then replace them by the sum of standard characteristics for which the formulas of the corresponding transient responses have been previously obtained. After this, one computes the ordinates of the transient response being investigated and, from the curve thus constructed, one fins the overshoot and the control time. If one takes as standards the natural frequency characteristics of simple systems without lags, then the constructions and computations involved in estimating overshoot and control time are significantly simplified [7]. We limit ourselves to the consideration of just two

standard cases, referring the reader to the literature [7] for a more complete analysis. As standards, we shall choose the frequency characteristics corresponding to the representation

$$W(z) = \frac{b_1 z + a_2}{z(z^2 + a_1 z + a_2)}. \tag{25}$$

By setting $z = i\omega$ in (25) we obtain, for $zW(z)$,

$$R(\omega) = \frac{c_2\omega^2 + c_4}{\omega^4 + d_2\omega^2 + c_4}, \tag{26}$$

where $c_2 = a_1 b_1 - a_2$, $c_4 = a_2^2$, $d_2 = a_1^2 - 2a_2$ are three new essential parameters of characteristic $R(\omega)$ by which it is completely defined. Among the various possible types of curves for $R(\omega)$, we consider in detail the characteristic which is nonnegative in the essential frequency interval (Fig. 3). Let the characteristic $R(\omega)$ of the system with lags under investigation be nonnegative in the essential interval $0 \leq \omega \leq \omega_2$ and have one maximum $R = R_1$ for $\omega = \omega_1$. We choose the parameters c_2, c_4 and d_2 such that the characteristic of the system with lags shall coincide with that given by (26) in the three points for $\omega = 0$, $\omega = \omega_1$ and $\omega = \omega_2$ (Fig. 3). Since, by (26), $R(0) = 1$, then coincidence of the ordinates at the point $\omega = 0$

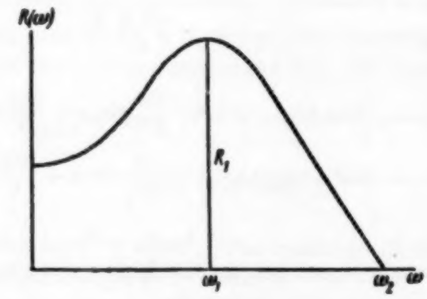

Fig. 3.

is attained by a simple change of scale for the investigated characteristic of the system with lags. After this, the parameters being constructed are determined from the system of equations

$$R_1 = \frac{c_2\omega_1^2 + c_4}{\omega_1^4 + d_2\omega_1^2 + c_4} = \frac{c_2}{2\omega_1^2 + d_2}, \qquad c_2\omega_2 + c_4 = 0 \tag{27}$$

by the formulas

$$c_2 = \frac{R_1\omega_1^4}{\omega_2^2(1 - R_1)}, \quad c_4 = \frac{R_1\omega_1^4}{R_1 - 1}, \tag{28}$$

$$d_2 = \frac{\omega_1^4}{\omega_2^2(1 - R_1)} - 2\omega_1^2.$$

We then compute the initial parameters:

$$a_2 = \sqrt{c_4}, \quad a_1 = \sqrt{d_2 + 2a_2}, \quad b_1 = \frac{a_2 + c_2}{a_1}. \tag{29}$$

By the formulas derived in [9], the quantities just found permit one to compute the overshoot and control time exactly for the transient response corresponding to the representation of (25).

We now give the computational scheme:

$$\Delta\phi = \sqrt{\frac{F(-b_1)}{a_2}}\, e^{-\frac{\alpha}{\beta}\gamma}, \qquad \operatorname{tg}\gamma = -\frac{\beta b_1}{a_2 - \alpha b_1}, \quad e^{\alpha T} = s\sqrt{\frac{F(-b_1)}{a_2}}. \tag{30}$$

In the formulas just given, $\Delta\psi$ is the overshoot, T is the control time, s is a parameter defining the zone of admissible deviations (s = 10 to 20), $2\alpha = a_1$, $\beta = \sqrt{a_2 - \alpha^2}$, $F(-b_1) = b_1^2 - a_1 b_1 + a_2$.

As an example, we estimate the quality of control in a static system with lags by setting, in the representation of (10),

$$W_k(z) = zW_0(z) = \frac{k(T_1 z + 1)}{(T_2 z + 1)(T_3 z + 1)}$$

and by taking $T_1 = 0.1$ second, $T_2 = 1$ second, $T_3 = 0.2$ second, $k = 4$.

By using the method developed in [2], we can determine that the system is stable if the lag time is $\theta < 0.48$ second. For the following computations, we take $\theta = 0.3$ second.

The closed system's real frequency characteristic is

$$R(\omega) = \frac{(400 + 4\omega^2)\cos\theta\omega + 100 - 8\omega^2}{A^2 + B^2}, \tag{31}$$

where $A = 5 - \omega^2 - 20\cos\theta\omega + 2\omega\sin\theta\omega$, $B = 6\omega + 2\omega\cos\theta\omega - 20\sin\theta\omega$.

The points of intersection of characteristic $R(\omega)$ with the ω axis are defined by the equation

$$\cos\theta\omega = \frac{2\omega^2 - 25}{\omega^2 + 100},$$

which has the sole real solution $\omega_2 = 4.7$.

In addition, we find from formula (31) that $R(0) = 0.8$ and $R_1 = 1.7$ for $\omega = 4$ at the maximum point. We change the scale of the axis of ordinates, setting $R(0) = 1$, $R_1 = 2.12$. Then, by formulas (28) and (29), we find: $a_1 = 21.3$, $a_2 = 22$, $b_1 = 0.069$, $\beta^2 = 21.6$. By substituting

TABLE 1

k	1.6	1.2	1.0	0.8	T_i	5	4	3	2
S_0	8	6	5	4	S_0	8.0	6.4	4.8	3.2
ω_2	0.54	0.64	0.74	0.82	ω_2	0.54	0.54	0.57	0.6
ω_1	0.27	0.32	0.37	0.41	ω_1	0.27	0.27	0.285	0.3
S_1	1.05	1	0.97	0.95	S_1	1.05	1.25	1.7	2.0

these values in formulas (30), we obtain: $\tan\gamma = -0.0145$, $\gamma = \pi - 0.01 = 3.13$, $\Delta\psi = 0.64$, $T = 4.6$ seconds for $s = 20$. From the exact curve of the transient response, constructed by the method presented in Section 1 to verify our example, $\Delta\psi = 0.6$ (with the scales of the variables taken into account), which differs little from the estimated value. In astatic systems with variable loads, $\varphi_s = 0$, and formulas (18) to (30) cannot be applied. For such systems, one may take as standard the frequency characteristics corresponding to the representation

$$W(z) = \frac{b_1}{z^2 + a_1 z + a_2}.$$

By setting $z = i\omega$, we obtain, for the real frequency characteristic,

$$S(\omega) = \frac{b_1(a_2 - \omega^2)}{(a_2 - \omega^2)^2 + a_1^2\omega^2}. \tag{32}$$

The three essential parameters b_1, a_1 and a_2 of the characteristic of (32) are defined by the ordinates in three chosen points of the analogous characteristic of the system with lags under investigation as a function of its behavior. If it be required that the characteristics coincide for $\omega = 0$, $\omega = \omega_1$ and $\omega = \omega_2$ for $S(\omega_2) = 0$, then the parameters being sought are determined from the formulas

$$a_2 = \omega_2^2,\ b_1 = S_0 a_2 = S_0\omega_2^2,\ a_1^2 = \tag{33}$$
$$= \frac{S_0\omega_2^2(\omega_2^2 - \omega_1^2) - S_1(\omega_2^2 - \omega_1^2)^2}{S_1\omega_1^2},$$

where S_0 and S_1 are the ordinates of the investigated system's frequency characteristic at the points $\omega = 0$ and $\omega = \omega_1$, respectively.

As an example of the use of formulas (33), we take the analysis of the control quality of concentrating sulfur dioxide (SO_2) for an external disturbance $\varphi_{in} = 1(t)$ applied to the object of control. With some simplifying assumptions, we give the basic functions entering into the representation of (2):

$$W_0(z) = \frac{k_1k_2k_3}{z(T_1z+1)(T_2z+1)(T_3z+1)},$$
$$W_k(z) = \frac{(T_iz+1)\,k_1k_2k_3k_6k_8}{(T_1z+1)(T_2z+1)(T_3z+1)(T_6z+1)(T_8z+1)\,kT_iz} \tag{34}$$

In the sequel, we shall measure time in minutes, taking into account the corresponding scale changes in representation of the transient response. Taking the controller adjustment parameters T_i and $\underline{k}$ as variables, we present the analysis for the following values of the remaining system parameters: $\theta_1 = 0.25$ minutes, $\theta_2 = 1$ minute, $T_1 = 0.25$ minutes, $T_2 = 0.5$ minutes, $T_3 = 0.33$ minutes, $T_8 = 0.25$ minutes, $T_6 = 1$ minute, $k_1k_2k_3 = 1$, $k_6k_8 = 1$.

By substituting the parameters' numerical values in formulas (34) and discarding the factor $e^{-0.25z}$, the effect of which is taken into account by shifting the time domain characteristic by $\theta = 0.25$ minutes, we obtain

$$W(z) = \frac{24(z+1)(z+4)}{z(z+4)^2(z+3)(z+2)(z+1) + \frac{96}{k}\left(z + \frac{1}{T_i}\right)e^{-z}}.$$

By setting $z = i\omega$ in this representation, we find that

$$W(i\omega) = \frac{(96 - 24\omega^2)A + 120\omega B + i\,[1200A - B(96 - 24\omega^2)]}{A^2 + B^2},$$

where

$$A = -\omega^6 + 75\omega^4 - 224\omega^2 + \frac{96}{kT_i}\cos\omega + \frac{96\omega}{k}\sin\omega,$$
$$B = -14\omega^5 - 190\omega^3 + 96\omega - \frac{96}{kT_i}\sin\omega + \frac{96\omega}{k}\cos\omega.$$

We separate out, for the further computations, the real frequency characteristic

$$S(\omega) = \frac{(96 - 24\omega^2)A + 120\omega B}{A^2 + B^2}. \tag{35}$$

Analysis of stability, carried out in [2], showed that the system is stable in the following range of parameter variations: $1.8 \le T_i \le 5$, $0.8 \le k \le 1.6$.

Since, within the given limits of parameter variation, the characteristics $S(\omega)$, found from formula (35), do not increase in the interval $0 \le \omega \le \omega_2$, then, for the subsequent estimates, we took $\omega_2 = 2\omega_1$. The values of the ordinates of $S(\omega_k)$, computed from formula (35) for $T_i = 5$ in the range $0.8 \le k \le 1.6$, and for $k = 1.6$ in the range $2 \le T_i \le 5$, are given in Table 1.

Substitution of the values thus found into formulas (33) allowed the computation of a_1, a_2 and b_1 for the accepted values of controller adjustment, T_i and $\underline{k}$. After this, the greatest deviation of the controlled coordinate, and the control time, were computed by the

TABLE 2

k	1.6	1.2	1.0	0.8	T_i	5	4	3	2
ψ max	0.83	0.85	0.89	0.87	ψ max	0.83	0.82	0.85	0.82
T	19	14	11	8.3	T	19	15	10.3	7.4

formulas derived in [9]. If $a_1^2 - 4a_2 > 0$, then

$$\psi_{max} = S_0\omega_2 e^{-\alpha t_{el}},\ t_{el} = \frac{1}{\alpha^2 - \omega_2^2} \ln \frac{\alpha + \sqrt{\alpha^2 - \omega_2^2}}{\omega_2}, \quad (36)$$

where $2\alpha = a_1$. The control time T was defined by the transcendental equation

$$\frac{S_0\omega_2^2}{2\sqrt{\alpha^2 - \omega_2^2}}\left(e^{-z_1 T} - e^{-z_2 T}\right) = \frac{1}{s} \quad (37)$$

for $s = 10$, $z_1 = \alpha - \sqrt{\alpha^2 - \omega_2^2}$, $z_2 = \alpha + \sqrt{\alpha^2 - \omega_2^2}$.

If $z_2 \gg z_1$, we may then use the following formula for the determination of the control time:

$$T \approx \frac{1}{z_1} \ln \frac{sS_n\omega_2^2}{2\sqrt{\alpha^2 - \omega_2^2}}. \quad (38)$$

In the case $a_1^2 - 4a_2 < 0$, the greatest deviation, and the control time, were determined from the formulas

$$\psi_{max} = S_0\omega_2 e^{-\frac{\alpha}{\beta}\gamma}, \quad \operatorname{tg}\gamma = \frac{\beta}{\alpha}, \quad (39)$$

$$T = \frac{1}{\alpha} \ln (sS_0\omega_2),$$

where $2\alpha = a_1$, $\beta^2 = \omega_2^2 - \alpha^2$. The results of all these computations are given in Table 2.

Analysis of the results obtained shows that the isodrome controller time T_i and the parameter k, corresponding to the range of the throttled controller, have very little influence on the greatest deviation of the controlled parameter. The control time is essentially decreased by decreases in T_i and k. The optimal adjustment of the controller corresponds to the lowest values of k and T_i which are compatible with the stability requirements. Verification of the computations made by more exact methods showed that the deviations of the estimated values from the more exact ones did not go beyond the limits of computational accuracy.

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LYAPUNOV FUNCTIONS FOR RELAY CONTROL SYSTEMS

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The idea of a trajectory is introduced, and the stability in the large is investigated of the equilibrium state of one class of relay control systems.

1. In many cases, the dynamic properties of automatic-control relay systems are well described by a mathematical model consisting of a combination of a linear part having a transfer function

$$W(p)=\frac{\sigma(p)}{y(p)}=\frac{b_np^{n-1}+b_{n-1}p^{n-2}+\cdots+b_1}{a_{n+1}p^n+a_np^{n-1}+\cdots+a_1}k_p \tag{1}$$

$$(a_1,\ldots,a_{n+1},b_1,\ldots,b_n=\text{const},\ b_1,a_{n+1},k_p>0)$$

and a relay element without a zone of insensitivity having the characteristic form $y=\varphi(\sigma)$.

In actual fact, both the input $\sigma(t)$ and the output $\varphi(\sigma(t))$ of any real unit approximating such a relay element (a kinematic couple with dry friction, an amplifier with a small zone of linearity, a relay, etc.) are continuous functions of the time. If the coordinate $\sigma(t)$ passes rapidly through its threshold value $\sigma=0$, the function $\varphi(\sigma)$ can be assumed to be discontinuous, and we can set $\varphi(\sigma)=\operatorname{sign}\sigma$, for example. Such an idealization becomes unsatisfactory for the investigation of those relay systems (sliding and partially sliding regimes, motion with stops [1-4]) for which $\sigma(t)$ remains for long periods of time near to its change-over value. The continuous character of the function $\varphi(\sigma)$ is taken into account most simply in the description of such motions by the Coulomb characteristic:

$$y=\varphi(\sigma)=\begin{cases}-(1-\varphi_0) & \text{for } \sigma<0,\\ \xi & \text{for } \sigma=0,\\ +(1+\varphi_0) & \text{for } \sigma>0,\end{cases} \tag{2}$$

where $\varphi_0=\text{const}$, $|\varphi_0|\ll 1$, and ξ can be any number from the interval $[-(1-\varphi_0),\ 1+\varphi_0]$, and with $\xi=\xi(t)$ expressed as in [3], so that the equations (1)-(2) for $y(t)=\varphi(0)=\xi(t)$ are compatible with the equality $\sigma(t)=0$.

In order to derive and investigate the phase space of the system (1)-(2), we write its equations in the normal Cauchy form

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{b}\varphi(\sigma),\quad \sigma=\mathbf{k}'\mathbf{x}, \tag{3}$$

using for brevity of notation the usual vector-matrix notation of [5]: **A** is a square matrix, and **x**, **b**, and **k** are n-dimensional column matrices. The prime indicates the transpose of a matrix, so that, according to the rule for multiplying matrices ([5], p. 14) **k'x** is a scalar, and **kb'** is a square matrix. The transformation of the equations (1)-(2) to the normal form is particularly simple, and yields

$$\begin{gathered}\dot{x}_k=x_{k+1}\ (k=1,\ \ldots,\ n-1),\\ \dot{x}_n=-a_{n+1}^{-1}\Big(\sum_{i=1}^{n}a_ix_i+k_p\varphi(\sigma)\Big),\\ \sigma=\sum_{i=1}^{n}b_ix_i.\end{gathered} \tag{4}$$

In several works (for example, see [1]) the trajectories of the system (2)-(3) have been considered close to a certain periodic motion with simple switching conditions, and in [6], close to the origin. In [1, 3, and 4] sliding regimes of relay systems were investigated in detail. Many authors have laid out the whole **x** phase space of the system (2)-(3) in trajectories for n = 2, 3. This is done below for any n under the assumption that

$$a_{n+1}^{-1}k_pb_n=-\mathbf{k}'\mathbf{b}>0 \tag{5}$$

(the case

$$b_n=0,\quad b_{n-1}>0 \tag{6}$$

was considered in [7], the violation of the conditions (5)-(6) is connected [1, 6] with the loss of stability). The second Lyapunov method is applied to the dynamic system obtained. Effective algorithms are obtained for deriving a wide class of sufficient conditions for stability in the large of the equilibrium of the system (2)-(3) with a stable or unstable linear part. The stability criteria given in this paper are expressed in terms of the coefficients of the transfer function (1), but the method of constructing the Lyapunov functions is described in invariant form,

and it can be applied in obtaining criteria expressed in terms of the elements of the matrices $\mathbf{A}$, $\mathbf{b}$, and $\mathbf{k}$ in the usual form. In the case

$$\operatorname{Re}\lambda_i < 0 \quad (i = 1, \ldots, n), \tag{7}$$

where λ_i are the roots of the equation $\det(\mathbf{A} - \lambda\mathbf{E}) = 0$, the Lyapunov functions are obtained by the interlacing method, and can be used to estimate the control time. The case

$$\lambda_1 = 0, \ \operatorname{Re}\lambda_k < 0 \ (k = 2, \ldots, n). \tag{8}$$

is also discussed briefly.

It should be mentioned that the unsymmetric characteristic (2) must also be considered [1] in the investigation of systems

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}\varphi\,(\mathbf{k}'\,\mathbf{z}), \quad \varphi(\sigma) = \begin{cases} -1 & \text{for } \sigma < 0, \\ +1 & \text{for } \sigma > 0 \end{cases} \tag{9}$$

with a symmetric relay link when, for example, in the case (7) we are investigating the effect of a discontinuous external action:

$$\sigma(t) = \mathbf{k}'\,\mathbf{z}(t) + \mathbf{1}(t)\,\sigma_0, \tag{10}$$

and in the case (8) for motion with a linear input:

$$\sigma(t) = \mathbf{k}'z(t) + \mathbf{1}(t)\,(\sigma_0 + \sigma_1 t). \tag{11}$$

The system (9)-(10) and (11) reduces to the form (2)-(3) by a nonsingular linear transformation of the coordinates $\mathbf{z}$. The value of φ_0 is determined here from the parameters σ_0 and σ_1 of the external disturbance.

2. We denote by $\mathbf{x}_+(\mathbf{x}^0, t)$, and $\mathbf{x}_-(\mathbf{x}^0, t)$ the trajectories of the systems

$$\dot{\mathbf{x}} = \mathbf{f}_+(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{b}\,(1 + \varphi_0), \tag{12}$$

$$\dot{\mathbf{x}} = \mathbf{f}_-(\mathbf{x}) = \mathbf{A}\mathbf{x} - \mathbf{b}\,(1 - \varphi_0), \tag{13}$$

passing for $t = 0$ through the point $\mathbf{x}^0$ of the space $\mathbf{x}$. For the investigation of the system (2)-(3) and (7), it is convenient to write Equations (12) and (13) in the form

$$\dot{\mathbf{x}}_+ = \mathbf{A}\mathbf{x}_+, \ \mathbf{x}_+ = \mathbf{x} + \mathbf{A}^{-1}\mathbf{b}\,(1 + \varphi_0), \tag{12'}$$

$$\dot{\mathbf{x}}_- = \mathbf{A}\mathbf{x}\ , \ \mathbf{x}\ = \mathbf{x} - \mathbf{A}^{-1}\mathbf{b}\,(1 - \varphi_0). \tag{13'}$$

We denote by $\dot{\sigma}_+(\mathbf{x})$ and $\dot{\sigma}_-(\mathbf{x})$ the derivatives with respect to time of the function $\sigma(\mathbf{x}) = \mathbf{k}'\mathbf{x}$ according to the systems (12) and (13). Thus,

$$\dot{\sigma}_+(\mathbf{x}) = \mathbf{k}'\mathbf{A}\mathbf{x} + \mathbf{k}'\mathbf{b}\,(1 + \sigma_0),$$

$$\dot{\sigma}_-(\mathbf{x}) = \mathbf{k}'\mathbf{A}\mathbf{x} - \mathbf{k}'\mathbf{b}\,(1 - \varphi_0),$$

and, according to (5), we have

$$\dot{\sigma}_+(\mathbf{x}) < \dot{\sigma}_-(\mathbf{x}). \tag{14}$$

Using the method in [1-4, 6, 7], we give the determination of the trajectory $\mathbf{x}(\mathbf{x}^0, t)$ of system (2)-(3) satisfying the condition (5).

(a) In the space $\sigma(\mathbf{x}) > 0$ ($\sigma(\mathbf{x}) < 0$) we assume that $\mathbf{x}(\mathbf{x}^0, t) = \mathbf{x}_+(\mathbf{x}^0, t)$, $(\mathbf{x}(\mathbf{x}^0, t) = \mathbf{x}_-(\mathbf{x}^0, t))$.

(b) At any point $\mathbf{q}_1$ of the set $\sigma(\mathbf{x}) = 0$, $\dot{\sigma}_+(\mathbf{x})\,\dot{\sigma}_-(\mathbf{x}) > 0$, representing two half-planes $(\mathbf{k}'\mathbf{b})^{-1}\mathbf{k}'\mathbf{A}\mathbf{x} < -(1 + \varphi_0)$, $\mathbf{k}'\mathbf{x} = 0$ and $(\mathbf{k}'\mathbf{b})^{-1}\mathbf{k}'\mathbf{A}\mathbf{x} > (1 - \varphi_0)$, $\mathbf{k}'\mathbf{x} = 0$, the trajectories $\mathbf{x}_+(\mathbf{q}_1, t)$, $\mathbf{x}_-(\mathbf{q}_1, t)$ intersect the switching surface $\sigma(\mathbf{x}) = 0$ in the same direction. We determine $\mathbf{x}(\mathbf{q}_1, t)$ by linking continuously at $\mathbf{q}_1$ those half-trajectories $\mathbf{x}_+(\mathbf{q}_1, t)$ and $\mathbf{x}_-(\mathbf{q}_1, t)$ which lie close to $\mathbf{q}_1$ in the regions $\sigma(\mathbf{x}) > 0$ and $\sigma(\mathbf{x}) < 0$, respectively.

(c) According to (14), at every point $\mathbf{q}_0$ of the set $\sigma = 0$, $\dot{\sigma}_+(\mathbf{x})\,\dot{\sigma}_-(\mathbf{x}) < 0$ the trajectories $\mathbf{x}_+(\mathbf{q}_0, t)$, and $\mathbf{x}_-(\mathbf{q}_0, t)$ intersect the surface $\sigma(\mathbf{x}) = 0$ in the same direction. In accord with what was said in Sec. 1, we will assume that in the set consisting of the zone

$$(1 - \varphi_0) > (\mathbf{k}'\mathbf{b})^{-1}\mathbf{k}'\mathbf{A}\mathbf{x} > -(1 + \varphi_0), \ \mathbf{k}'\mathbf{x} = 0, \tag{15}$$

the representative point of the system (2)-(3) moves according to the equations

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\varphi\,(0) = \mathbf{A}\mathbf{x} + \mathbf{b}\xi(t), \tag{16}$$

where $\xi(t)$ is chosen so that the derivative with respect to time $\dot{\sigma}_0(\mathbf{x})$ of the function $\sigma(\mathbf{x})$ obtained as in (16), satisfies the equality

$$\dot{\sigma}_0(\mathbf{x}) = \mathbf{k}'\mathbf{A}\mathbf{x}(t) + \mathbf{k}'\mathbf{b}\xi(t) = 0. \tag{17}$$

If we combine (16) and (17), we obtain

$$\sigma(\mathbf{x}) = \mathbf{k}'\mathbf{x} = 0, \tag{18}$$

$$\dot{\mathbf{x}} = \mathbf{f}_0(\mathbf{x}) = \mathbf{B}\mathbf{x}, \ \mathbf{B} = \mathbf{A} - (\mathbf{k}'\mathbf{b})^{-1}\mathbf{b}\mathbf{k}'\mathbf{A}.$$

Various forms of the scalar representation of (18) are given in [1, 3, and 4]. For trajectories of the system (18) we will use the notation $\mathbf{x}_0(\mathbf{x}^0, t)$.

(d) We consider finally the boundary $\sigma(\mathbf{x}) = 0$, $\dot{\sigma}_+(\mathbf{x})\,\dot{\sigma}_-(\mathbf{x}) = 0$ of the zone (15) of the sliding regime, represented by the two hyperlines

$$\mathbf{k}'\mathbf{x} = 0, \ \dot{\sigma}_+(\mathbf{x}) = \mathbf{k}'\mathbf{A}\mathbf{x} + \mathbf{k}'\mathbf{b}\,(1 + \varphi_0) = 0. \tag{19}$$

and

$$\mathbf{k}'\mathbf{x} = 0, \ \dot{\sigma}_-(\mathbf{x}) = \mathbf{k}'\mathbf{A}\mathbf{x} - \mathbf{k}'\mathbf{b}\,(1 - \varphi_0) = 0. \tag{20}$$

According to (12), (13), (14), and (18), for all the points q_+ and q_-, belonging, respectively, to the straight lines (19) and (20), the relations

$$\mathbf{f}_+(\mathbf{q}_+) = \mathbf{f}_0(\mathbf{q}_+),\ \mathbf{f}_-(\mathbf{q}_-) = \mathbf{f}_0(\mathbf{q}_-), \tag{21}$$

and

$$\dot{\sigma}_-(\mathbf{q}_+) > 0,\ \dot{\sigma}_+(\mathbf{q}_-) < 0, \tag{22}$$

are valid, so that, for example, for the straight line (19), we have

$$\frac{d^m}{dt^m}[\sigma_+(\mathbf{q}_+)]_+ = \frac{d^m}{dt^m}[\dot{\sigma}_+(\mathbf{q}_+)]_0,\ m \geqslant 1, \tag{23}$$

where $\frac{d^m}{dt^m}[\dot{\sigma}_+(\mathbf{q}_+)]_+$ and $\frac{d^m}{dt^m}[\dot{\sigma}_+(\mathbf{q}_+)]_0$ are the derivatives of order m with respect to the time of the function $\dot{\sigma}_+(\mathbf{x})$, evaluated at the point $\mathbf{q}_+$ by (12) and (18). It is evident from (23) that in a sufficiently small neighborhood of the point $\mathbf{q}_+$ the positive half-trajectories $\mathbf{x}_+(\mathbf{q}_+, t)$, and $\mathbf{x}_0(\mathbf{q}_+, t)$ lie simultaneously either in the region $\dot{\sigma}_+(\mathbf{x}) \geqslant 0$, or in the region $\dot{\sigma}(\mathbf{x}) < 0$. In this neighborhood for $t > 0$, in the first case of the similar location of the trajectories $\mathbf{x}_+(\mathbf{q}_+, t)$, and $\mathbf{x}_0(\mathbf{q}_+, t)$, we set $\mathbf{x}(\mathbf{q}_+, t) = \mathbf{x}_+(\mathbf{p}_+, t)$; in the second $\mathbf{x}(\mathbf{q}_+, t) = \mathbf{x}_0(\mathbf{q}_+, t)$. The positive half-trajectory $\mathbf{x}(\mathbf{p}_+, t)$ will be unique since, according to (22), the trajectories $\mathbf{x}_-(\mathbf{q}_+, t)$ for $t > 0$ start in the half-space $\sigma(\mathbf{x}) < 0$. The straight line (20) can be considered similarly.

3. The solution $\mathbf{x}(\mathbf{x}^0, t)$ of the system (2)-(3), determined as above, is also a generalized solution of this system in the sense of the definition of A. F. Filippov [8].

(We mention that the essence of A. F. Filippov's definition is precisely the definition of the discontinuous right-hand sides, including infinite valued functions, continuous in the same sense as the characteristic (2)). The fact that the solution $\mathbf{x}(\mathbf{x}^0, t)$ can be continued for $t \to +\infty$, its uniqueness, and the property $\mathbf{x}((\mathbf{x}^0, t_1), t_2) = \mathbf{x}(\mathbf{x}^0, t_1 + t_2)$ for $t \geq 0$ are obvious; the continuity relative to $\mathbf{x}^0$ and t can easily be proved by using (21).

Thus, for the reflection of $\mathbf{x}(\mathbf{x}^0, t)$ for $t \geq 0$, we can apply all the results of the general theory of dynamic systems [9], and this, in turn, makes it possible to use a series of results formulated for classical systems of differential equations, but permitting obvious generalizations to abstract dynamic systems – for example, the theorems of E. A. Barbashin and N. N. Krasovskii [10] concerning stability in the large.

4. We will form the Lyapunov function $V(\mathbf{x})$ for the system (2)-(3) and (5) in the case (7) by linking on the surface $\sigma(\mathbf{x}) = 0$ the Lyapunov functions

$$V_+(\mathbf{x}) = \mathbf{x}_+\mathbf{H}\mathbf{x}_+,\ \mathbf{x}_+ = \mathbf{x} + \mathbf{A}^{-1}\mathbf{b}(1 + \varphi_0), \tag{24}$$

$$V_-(\mathbf{x}) = \mathbf{x}_-\mathbf{H}\mathbf{x}_-,\ \mathbf{x}_- = \mathbf{x} - \mathbf{A}^{-1}\mathbf{b}(1 - \varphi_0) \tag{25}$$

($\mathbf{H}$ is a symmetric matrix of the nth order) for the systems (12') and (13'), respectively:

$$V(\mathbf{x}) = \begin{cases} V_+(\mathbf{x}) - V_+(0) & \text{for } \sigma(\mathbf{x}) \geqslant 0, \\ V_-(\mathbf{x}) - V_-(0) & \text{for } \sigma(\mathbf{x}) \leqslant 0. \end{cases} \tag{26}$$

The continuity of $V(\mathbf{x})$ is ensured if $V_+(\mathbf{x})$ and $V_-(\mathbf{x})$ are chosen so that the equality

$$V_+(\mathbf{x}) - V_+(0) = V_-(\mathbf{x}) - V_-(0) \text{ for } \sigma(\mathbf{x}) = \mathbf{x}'\mathbf{k} = 0. \tag{27}$$

holds. Since the relation $V_+(\mathbf{x}) - V_+(0) = V_-(\mathbf{x}) - V_-(0)$ is equivalent to $\mathbf{x}'\mathbf{H}\mathbf{A}^{-1}\mathbf{b} = 0$, the condition (27) used in the linking can be written in the form

$$\mathbf{H}\mathbf{A}^{-1}\mathbf{b} = \mu\mathbf{k}\quad (\mu = \text{const} \neq 0). \tag{28}$$

It is evident from the relation

$$V(\mathbf{x}) = \mathbf{x}'\mathbf{H}\mathbf{x} + 2\mu\sigma\cdot\varphi(\sigma), \tag{29}$$

that follows from (26) and (28), that if the function $\mathbf{x}'\mathbf{H}\mathbf{x}$ satisfying condition (28) is positive-definite, then the function $V(\mathbf{x})$ will also be positive-definite for $\mu > 0$.

For any solution $\mathbf{x}(\mathbf{x}^0, t)$ everywhere except possibly at points of intersection of $\mathbf{x}(\mathbf{x}^0, t)$ with the surface $\sigma(\mathbf{x}) = 0$, the derivative $\dot{V}(\mathbf{x})$ with respect to time of the function (29) along $\mathbf{x}(\mathbf{x}^0, t)$ exists and satisfies the relations

$$\dot{V}(\mathbf{x}) = \begin{cases} \dot{V}_+(\mathbf{x}) & \text{for } \sigma(\mathbf{x}) > 0, \\ \dot{V}_-(\mathbf{x}) & \text{for } \sigma(\mathbf{x}) < 0, \\ \dot{V}_0(\mathbf{x}) & \text{for } \sigma(\mathbf{x}) = 0,\ \dot{\sigma}_+(\mathbf{x})\cdot\dot{\sigma}_-(\mathbf{x}) < 0, \end{cases} \tag{30}$$

where $\dot{V}_+(\mathbf{x}) = \mathbf{x}_+'(\mathbf{H}\mathbf{A} + \mathbf{A}'\mathbf{H})\mathbf{x}_+$ and $\dot{V}_-(\mathbf{x}) = \mathbf{x}_-'(\mathbf{H}\mathbf{A} + \mathbf{A}'\mathbf{H})\mathbf{x}_-$ are the derivatives with respect to time of the functions $V_+(\mathbf{x})$ and $V_-(\mathbf{x})$ according to (12') and (13'), and $\dot{V}_0(\mathbf{x}) = \mathbf{x}'(\mathbf{H}\mathbf{B} + \mathbf{B}'\mathbf{H})\mathbf{x}$ is the derivative with respect to time of the function (29) according to (18).

Let the form $-\mathbf{x}'\mathbf{G}\mathbf{x}$ ($\mathbf{G}$ is a symmetric matrix of the nth order) be positive-definite. Then when the

condition (7) is satisfied, there exists one and only one form $x'Hx$, related to $x'Gx$ by

$$G = HA + A'H, \tag{31}$$

where the form $x'Hx$ is positive-definite (A. M. Lyapunov). It is evident, in addition, that the function $V_+(x)$ or the function $V_-(x)$ will be negative everywhere, except at the point x_+^0, where $x_+ = x_+^0 + A^{-1}b(1+\varphi_0) = 0$, or the point x_-^0, where $x_- = x_-^0 - A^{-1}b(1-\varphi_0) = 0$). If

$$a_1^{-1}k_p b_1 = k'A^{-1}b > 0, \tag{32}$$

corresponding to negative feedback in the system (1)-(2), then $\sigma(x_+^0) < 0$ and $\sigma(x_-^0) > 0$, so that

$$\begin{aligned} \dot V_+(x) &< 0 \quad \text{for} \quad \sigma(x) \geqslant 0, \\ \dot V_-(x) &< 0 \quad \text{for} \quad \sigma(x) \leqslant 0. \end{aligned} \tag{33}$$

If the inequalities (33) are satisfied, the function $V(x)$ decreases along any trajectory $x(x^0, t)$, while the point representing the system (2)-(3) moves according to Equations (12) or (13). It remains to show that $V(x)$ also decreases for motion according to (18). We now turn to the proof of this fact, and formulate the final results in the following theorem:

Theorem I. If the conditions (5), (7), and (32) are satisfied for the system (2)-(3), and if there exists a negative-definite form $x'Gx = x'(HA + A'H)x$, satisfying (28), then the equilibrium solution $x = 0$ of this system is asymptotically stable in the large.

In accordance with the equality $H = GA^{-1} - A'HA^{-1}$ we write the relation

$$Hbk'A = GA^{-1}bk'A - A'HA^{-1}bk'A. \tag{34}$$

Further, according to the linking conditions (28) we have

$$A'HA^{-1}bk'A = \mu A'kk'A. \tag{35}$$

If we substitute (34) and (35) in the expression

$$\dot V_0(x) = x'(HB + B'H)x =$$
$$= x'(G - (b'k)^{-1}(Hbk'A + A'kbH))x,$$

we obtain the identity

$$\dot V_0(x) = f_0'(x) A'^{-1} G A^{-1} f_0(x),$$

from which it follows that $\dot V_0(x)$ is zero only at points x_0 of the set $f_0(x) = 0$. This set and the surface $\sigma(x) = 0$ have only one common point – the origin 0. Actually, we have [see (16)-(18)], the equality

$$f_0(x) = Ax + b\xi,$$

so that $x_0 = -\xi A^{-1}b$. According to (32) the relation $\sigma(x_0) = -\xi k'A^{-1}\cdot b = 0$ can hold only for $\xi = 0$, i.e., only at the point 0. At the remaining points of the surface $\sigma(x) = 0$, the function $\dot V_0(x)$ is negative.

Thus, when the conditions of Theorem I are satisfied, the infinitely large positive-definite function (29) strictly decreases along any nonzero trajectory $x(x^\circ, t)$. The system (2)-(3) will satisfy all the essential conditions of Theorem I of [10].

• We have reduced the investigation of the equilibrium stability of the system (2)-(3) in the case considered to a study of conditions that are sufficient for the compatibility of the equality (28) and the Sylvester criterion for the sign-definiteness of the form $x'Gx$. The limitation (5) appears [7] to be a necessary condition for this compatibility. It is now clear, that in its final part, the method of obtaining the Lyapunov functions is a modification of the method of I. G. Malkin [2] (p. 163), [11, 12], of investigating absolute stability.

5. Let it be possible to choose a negative-definite form $x'Gx = x'(HA + A'H)x$, satisfying (28). If we use the results in [5], p. 251, and consider a pencil of forms

$$x'Gx + \nu\cdot x'Hx, \tag{36}$$

we obtain the inequality

$$\nu_1 x'Hx \leqslant -x'Gx \leqslant \nu_n x'Hx, \tag{37}$$

where ν_1 and ν_n are the least and greatest roots of the characteristic equation of the pencil (36), respectively. From (37) we obtain

$$\nu_1\cdot V(x) \leqslant -\dot V(x) \leqslant \nu_n\cdot V(x) \tag{38}$$

for the rate of change $\dot V(x)$ of the function (29) for the motion by using (12) and (13).

For the forms $x'Hx$ and $\dot V_0(x) = x'(HB + B'H)x$ on the surface $k'x = 0$ the relation

$$x'Hx = V(x), \quad \nu_1^0 x'Hx \leqslant -\dot V_0(x) \leqslant \nu_{n-1}^0 x'Hx, \tag{39}$$

holds, where ν_1^0, ν_{n-1}^0 are the smallest and largest roots, respectively, of the characteristic equation of the form $x^{0\prime}(HB + B'H)^0x^0 + \nu^0 x^{0\prime}H^0x^0$. obtained from the pencil

$$x'(HB + B'H)x + \nu x'Hx$$

on applying the relation $k'x = 0$.

The time taken for the distance $r^2 = V(\mathbf{x})$, from a representative point of the system (2)-(3) to the origin, to decrease from the value r_1 to the value r_2, can be given its coarsest estimate by using the inequalities (38) and (39) in a way similar to that in [13] for a linear system. The frequency method [14] is applied to calculate the roots $\nu_1, \nu_n, \nu_1^0, \nu_{n-1}^0$.

6. We now consider the case when the equations of motion of the relay system considered above have the form (2) and (4), i.e., when they are actually given in the form (1)-(2), which is the most common in control theory. The relation (31) between the matrix $\mathbf{G} = -\|\beta_{ij}\|_1^n$ and $\mathbf{H} = \|\alpha_{ij}\|_1^n$ (the abbreviated notation is borrowed from [5]) and the condition (28) for $\mu = k_p$ can be written

$$\beta_{ij} = a_{n+1}^{-1}(\alpha_{in}a_j + \alpha_{jn}a_i) - \alpha_{ji-1} - \alpha_{ij-1}\,(b_0 = \alpha_{i0} = \alpha_{0i} = 0), \tag{40}$$

$$\alpha_{1k} = a_1 b_k \quad (k = 1, \ldots, n), \tag{41}$$

and the Lyapunov function (29) takes the form

$$V(\mathbf{x}) = \sum_{i,\,j=1}^{n} \alpha_{ij}x_i x_j + 2k_p\varphi(\sigma)\,\sigma. \tag{42}$$

The investigation of the conditions under which the form $-\mathbf{x}'\,\mathbf{G}\mathbf{x} = \sum_{i,\,j=1}^{n} \beta_{ij}x_i x_j$, which satisfies the conditions (40) and (41), can be positive-definite is more convenient if it is reduced to a consideration not of the matrix $\|\beta_{ij}\|_1^n$, but of the matrix $\|\gamma_{ij}\|_1^n$ congruent to it [5] (p. 240), for which

$$\begin{gathered}\gamma_{11} = \beta_{11},\; \gamma_{i1} = \gamma_{1i} = \beta_{i1} - \beta_{11}a_i,\\ \gamma_{ij} = \gamma_{ji} = \beta_{ij} - \beta_{i1}a_j - \beta_{1j}a_i + \beta_{11}a_i a_j,\end{gathered} \tag{43}$$

since the quantities γ_{ij} can be expressed more simply in terms of α than the quantities β_{ij}.

A relatively uncomplicated calculation shows that the components of α satisfy condition (41) if, and only if, the components of γ satisfy

$$\gamma_{11} = 2a_{n+1}^{-1}b_n,\; \gamma_{kk} = 2\Big(B_{kn} + \sum_{r=0}^{k-3}(-1)^r\,\gamma_{k+r+1,\,k-r-1}\Big) \quad (k = 2, \ldots, n), \tag{44}$$

where

$$B_{kn} = b_{k-1}a_k - \sum_{r=0}^{k-2}(-1)^r\,(b_{k+r}a_{k-r-1} + b_{k-r-2}a_{k+r+1}), \tag{45}$$

$$b_{n+l} = a_{n+l+1} = \alpha_{i,\,n+l} = \gamma_{n+l+1,\,i} = 0,$$

$$\gamma_{n+1,\,i} = -\gamma_{1i}a_{n+1} \quad (l \geqslant 1).$$

We now denote by D_n the determinant of the matrix $\|\gamma_{ij}\|_1^n$, and write Theorem I in a concrete form.

<u>Theorem II.</u> If the elements of the determinant D_n satisfy the relations (44) and (45), and if the quantities γ_{ij} $(i \neq j)$ can be chosen so that all the diagonal minors D_{kn} of the determinant D_n are positive, then the equilibrium state of the system (1)-(2) is asymptotically stable as a whole if its linear part is asymptotically stable for $b_n > 0$.

7. If the condition (8) is satisfied, then the Lyapunov function for the system (2), (4) can be taken to have the form

$$V(\mathbf{x}) = \sum_{i,\,j=2}^{n} \alpha_{ij}x_i x_j + 2k_p\varphi(\sigma)\,\sigma, \tag{46}$$

obtained from (41) and (42) for $a_1 \to 0$. By calculations and reasoning similar to that in Sec. 4, and with the derivative of $V(\mathbf{x}(\mathbf{x}^0, t))$ [2, 11, 12] taken as the n-variable form,

$$x_1' = \varphi(\sigma),\; x_k' = x_k \quad (k = 2, \ldots, n),$$

it is easy to show that Theorem II is valid in this case. The construction of the Lyapunov function, however, loses its elegant geometric interpretation. The stability criteria obtained from Theorem II with the assumption (7) tend for $a_1 \to 0$ to a sufficient condition for stability of the system (2), (4) when its linear part is neutral for one coordinate. We note here that these criteria, for $b_n \to 0$, tend to the stability criteria obtained in [7] for $b_n = 0$.

The method of obtaining Lyapunov functions proposed in 4 and 6 thus becomes, for $a_1 \to 0$, an extension of the method of I. G. Malkin to relay systems.

Below (in 8 and 9 we obtain the minimum limitations for n = 3, 4 stability criteria which can be obtained by using Theorem II. These criteria are equivalent to the stability conditions that follow from the theorem of A. I. Lur'e as given by A. M. Letov in [2] (p. 124). Consequently, the assumption [11] that I. G. Malkin's method will give more precise stability conditions than the Lur'e algorithm for control systems with one nonlinear link is, generally speaking, not justified. This method is, however, [2, 12] suitable for obtaining simplified criteria for the stability of systems of high order ($n \geq 5$). In Sec. 10 of the present paper, an effective algorithm is proposed for obtaining such criteria.

8. For n = 3 the determinant D_n has the form

$$D_3 = \begin{vmatrix} 2a_4^{-1}b_3 & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & 2B_{23} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & 2(B_{33} - a_4\gamma_{12}) \end{vmatrix} \tag{47}$$

The simultaneous validity of the inequalities

$$B_{23} > 0, \quad B_{33} - a_4\gamma_{12} > 0,$$
$$D_{23} = 4a_4^{-1}b_3B_{23} - \gamma_{12}^2 > 0 \tag{48}$$

is a necessary condition that all the principal minors of the determinant (47) be positive. For $\gamma_{13} = \gamma_{23} = 0$ this condition is also sufficient.

From (48) we obtain the relation

$$\gamma_{12} < a_4^{-1} B_{33},$$
$$-2\sqrt{a_4^{-1} b_3 B_{23}} < \gamma_{12} < -2\sqrt{a_4^{-1} b_3 B_{23}}$$

and this can be used to obtain necessary and sufficient conditions for the solvability of (48) for γ_{12}:

$$B_{23} = b_1a_2 - b_2a_1 > 0,$$
$$B_{33} = b_2a_3 - b_3a_2 - b_1a_4 > -2\sqrt{b_3a_4B_{23}}. \tag{49}$$

According to Theorem II, the conditions (49) are sufficient for the stability in the large of the system (2), (4) that we are considering.

9. For n = 4

$$D_4 = \begin{vmatrix} 2a_5^{-1}b_4 & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & 2B_{24} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & 2(B_{34}+\gamma_{24}) & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & 2(B_{44}-a_5\gamma_{13}) \end{vmatrix} \tag{50}$$

The simultaneous validity of the inequalities

$$B_{24} > 0, \quad 4a_5^{-1} b_4 (B_{34} + \gamma_{24}) - \gamma_{13}^2 > 0,$$
$$4B_{24}(B_{44} - a_5\gamma_{13}) - \gamma_{24}^2 > 0 \tag{51}$$

is a necessary condition that all the principal minors of the determinant (50) be positive. For $\gamma_{12} = \gamma_{14} = \gamma_{23} = \gamma_{34} = 0$, if

$$D_{24} = 4a_5^{-1} b_4B_{24}, \quad D_{34} = 2B_{24}(4a_5^{-1} b_4 (B_{34} + \gamma_{24}) - \gamma_{13}^2),$$
$$D_{44} = [4B_{24}(B_{44} - a_4\gamma_{13}) - \gamma_{24}^2]\,[4a_5^{-1} \cdot b_4 (B_{34} + \gamma_{24}) - \gamma_{13}^2],$$

this condition also appears to be sufficient. If we use the results of [15] (p. 95), we arrive at the following necessary and sufficient conditions for the solvability of the inequality (51) for γ_{13} and γ_{24}:

$$B_{24} > 0, \tag{52}$$

$$M^3 + N^3 - M^2N^2 - \frac{9}{8}MN + \frac{27}{956} > 0, \tag{53}$$

where

$$M = 0.25\, B_{44}B_{24}^{-\frac{1}{3}} b_4^{-\frac{2}{3}} a_5^{\frac{2}{3}}, \tag{54}$$

$$N = 0.24\, B_{34}B_{24}^{-\frac{2}{3}} b_4^{-\frac{1}{3}} a_5^{\frac{1}{3}}.$$

According to Theorem II, the conditions (52)-(54) are sufficient for stability in the large for the system (2), (4).

10. For $n \geq 5$, the most precise criteria that can be obtained from Theorem II involve calculations that make them practically inapplicable in their general form. In order to obtain simplified stability criteria, the method can be recommended of insisting that only one or two elements γ_{ij} $(i \neq j)$ be exactly equal to zero, and of reducing the condition that the main minors of the determinant D_n be positive to inequalities similar to the relations (46) and (51).

Thus, for example, the simple inequalities

$$B_{55} + \gamma_{53} + a_6\gamma_{12} > 0, \quad 4a_6^{-1} b_5B_{25} - \gamma_{12}^2 > 0,$$
$$4B_{35}B_{45} - \gamma_{53}^2 > 0 \tag{55}$$

are obtained for n = 5, when the values of the elements γ_{12} and γ_{35} of the determinant D_5 are left as variable quantities. Calculations similar to those in 8, applied to the system (55), yield the criteria

$$B_{25}, \quad B_{35}, \quad B_{45} > 0,$$
$$B_{55} > -2\left(\sqrt{B_{34}B_{45}} + \sqrt{a_6b_5B_{25}}\right)$$

for the stability of the relay system considered. If we do not assume that the elements γ_{24} and γ_{53} are equal to zero, we obtain

$$D_5 = 2a_6^{-1}b_5 \begin{vmatrix} 2B_{25} & 0 & \gamma_{24} & 0 \\ 0 & 2(B_{35}+\gamma_{24}) & 0 & \gamma_{35} \\ \gamma_{24} & 0 & 2(B_{45}+\gamma_{53}) & 0 \\ 0 & \gamma_{53} & 0 & 2B_{55} \end{vmatrix}.$$

The reasoning applied in 9 leads to stability conditions given by the relations $B_{25} > 0$, $B_{55} > 0$, and (53), where $M = 0.25\, B_{25}^{-1/3} B_{35}B_{55}^{-2/3}$, $N=0.25\, B_{25}^{-2/3}B_{45}B_{55}^{-1/3}$.

In conclusion, we note that the very simple stability criterion

$$B_{kn} > 0$$

can be obtained from Theorem II for any $n \geq 2$, if we set $\gamma_{ij} = 0$, for $i \neq j$, in the determinant D_n.

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THE APPLICATION OF THE METHOD OF SMALL PARAMETERS IN THE INVESTIGATION OF SYSTEMS OF AUTOMATIC CONTROL WITH DELAY

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A method of investigation of systems of automatic regulation (SAR) with retardation is presented, based on the consideration of retardation as a small parameter and the application of the Lyapunov-Poincaré method of small parameters. The case is considered of a SAR expressed as a linear equation with constant coefficients and with a single constant retardation.

In many problems of automatic control, the delay is small and plays the role of a type of interference causing a distortion in that "undisturbed" process which would occur in the absence of any delay. It is therefore completely natural in such problems to consider the delay as a small parameter, and to attempt to apply the Poincaré-Lyapunov method of small parameters. One method of investigation of automatic control systems with delay is to seek the solution of the relevant differential equations by using successive approximations, with the first approximation taken to be the solution of these equations with a zero delay. We will consider here linear equations with constant coefficients and a single constant delay, although this method can also be used with known restrictions in the case of equations with variable coefficients with delay, and also in the case of quasi-linear equations.

1. In the general case, the equations we will consider have the form

$$\dot{x}_s(t) = \sum_{j=1}^{n} [a_{sj} x_j(t) + b_{sj} x_j(t-\mu)] \quad (s = 1, \ldots, n), \tag{1}$$

where a_{sj} and b_{sj} are constant coefficients and μ is the constant delay.

Let the functions $x_1^{(0)}, \ldots, x_n^{(0)}$ form a solution of the system (1) for $\mu = 0$ (we will call this solution undisturbed) and for the initial conditions $x_s(0) = x_{s0}$ ($t = 0$ being the initial instant of time). If we rewrite (1) in the form

$$\dot{x}_s(t) = \sum_{j=1}^{n} \bar{a}_{sj} x_j(t) + \sum_{j=1}^{n} b_{sj} [x_j(t-\mu) - x_j(t)], \quad \bar{a}_{sj} = a_{sj} + b_{sj}, \tag{2}$$

then we can consider the difference $x_j(t-\mu) - x_j(t)$ in these equations as a type of nonlinearity depending on the small parameter μ and becoming zero for $\mu = 0$. Then for obtaining the solution of the system (2), it is natural to consider the method that is applied in the theory of differential equations with small parameters (see, for example, [1]). If we take $x_s^{(0)}$ for the zeroth approximation, then, for the first approximation, we use $x_s^{(1)}$, $s = 1, \ldots, n$, which is the solution of the system

$$\dot{x}_s(t) = \sum_{j=1}^{n} \bar{a}_{sj} x_j^{(1)}(t) + \sum_{j=1}^{n} b_{sj} [x_j^{(0)}(t-\mu) - x_j^{0}(t)], \tag{3}$$

for the same initial conditions $x_s^{(1)}(0) = x_{s0}$. Since the functions $x_j^{(0)}(t)$ are already known, then the differences $x_j^{(0)}(t-\mu) - x_j^{(0)}(t)$ are known functions of $\underline{t}$ and the delay μ, where μ plays the part of a parameter. The equations (3) are ordinary linear inhomogeneous equations. Their solutions, defined uniquely for all $-\infty < t < \infty$, are obtained in the form $x_s^{(1)} = x_s + \bar{x}_s^{(1)}$, where the functions $\bar{x}_s^{(1)} = \bar{x}_s^{(1)}(t, \mu)$ become zero for $t = 0$ or $\mu = 0$. The second approximation $x_s^{(2)}$ is obtained in a similar way as the solution of the inhomogeneous linear system

$$\dot{x}_s^{(2)}(t) = \sum_{j=1}^{n} \bar{a}_{sj} x_j^{(2)}(t) + \sum_{j=1}^{n} b_{sj} [x_j^{(1)}(t-\mu) - x_j^{(1)}(t)] \tag{4}$$

with the initial conditions $x_s^{(2)}(0) = x_{s0}$. As a result we obtain $x_s^{(2)} = x_s^{(0)} + \bar{x}_s^{(1)} + \bar{x}_s^{(2)}$, where the functions $\bar{x}_s^{(2)} = \bar{x}_s^{(2)}(t, \mu)$ are of the second order of smallness relative to μ, (and so on. For the kth approximation we obtain $x_s^{(k)} = x_s^{(0)} + \bar{x}_s^{(1)} + \ldots + \bar{x}_s^{k}$. In the limit (for $k \to \infty$) the solution of the system (1) can be written as a series:

$$x_s = x_s^{(0)} + \sum_{k=1}^{\infty} \bar{x}_s^{(k)} \quad (s = 1, \ldots, n). \tag{5}$$

where the functions $\bar{x}_s^{(m)}$, $m = 1, 2, \ldots$ are of order $\underline{m}$ relative to μ. Each term of this series is a function

that is analytic in t and μ for $-\infty < t < \infty$ and any finite μ. It can be shown, that, if μ does not exceed a certain bound μ_0 which depends on the coefficients of the system (1), then the series (5) is convergent and is uniformly convergent relative to t in any finite interval of t. The sum of this series for $\mu < \mu_0$ is consequently an analytic function of t, and the above iteration process, in all cases when μ is sufficiently small, can be used to obtain a solution of the equations (1) in $-\infty < t < \infty$ having continuous derivatives of all orders at every point and satisfying the given initial conditions $x_s(0) = x_{s0}$.

2. It follows from the general theory of differential equations with delay that the solution of the system (1) depends not only on the initial values x_{s0} of the functions sought at the initial instant of time $t = 0$, but also on the so-called initial functions $\varphi_1, \ldots, \varphi_n$. The functions $x_1, \ldots, x_n$ are set equal to the respective initial functions in the interval $-\mu \le t < 0$. These initial functions express the "previous history" or the "hereditary properties" of the dynamic system being considered that is described by the equations (1). The solution obtained by our proposed method is uniquely determined by a single set of initial values x_{s0}. This solution does not exhibit the so-called "hereditary properties," and is only one particular solution among the infinite number of solutions determined by the same initial conditions but with different initial functions. The question arises of the position of this solution of the system (1) among the other solutions.

We will consider as an example the equation

$$\dot{x}(t) = ax(t-\mu), \tag{6}$$

where a is a positive number, and we denote by $\psi(t)$ the solution of this equation corresponding to some initial function $\varphi(t)$ that is defined, continuous, and bounded for $-\mu \le t < 0$. By using the "step" method we obtain successively the values of $\psi(t)$ in each of the intervals $[t_{k-1}, t_k]$, $t_k = k\mu$, $k = 1, 2, \ldots$ As is known (see [2], for example), we can be sure that the function $\psi(t)$ will possess in the interval $t_{k-1} \le t < t_k$ continuous derivatives up to and including the $(k-1)$th order, and a continuous kth derivative in the interval $t_{k-1} \le t < t_k$. It is therefore possible to express this function in the interval $t_{k-1} \le t \le t_k$ in the form of a Taylor series with a remainder as

$$\psi(t) = \psi_k + \bar{t}\dot{\psi}_k + \ldots + \frac{\bar{t}^{k-1}}{(k-1)!}\psi_k^{(k-1)} + \frac{\bar{t}^k}{k!}\psi^{(k)}(\theta_k), \tag{7}$$

where $\bar{t} = t - t_k$, $\psi_k = \psi(t_k)$, $\psi_k^{(\sigma)} = \psi^{(\sigma)}(t_k)$, $\sigma = 1, 2, \ldots, k-1$, $t_{k-1} < \theta_k < t_k$, where $\psi^{(k)}(\theta_k) = a^k\varphi(\bar{\theta})$, $\mu < \bar{\theta} < 0$. Moreover, by using the condition of continuity of the derivatives of the function $\psi(t)$ at $t = t_k$ up to and including the $(k-1)$th order (derivatives from the left, of course), we arrive at the following relations between the values of the function $\psi(t)$ at the points $t = t_k$, $t = t_{k-1}$ and its derivatives at the point $t = t_{k-1}$:

$$\begin{aligned} u_0 &= \psi_k - \mu a u_0 + \frac{\mu^2}{2} a u_1 - \ldots + (-1)^{k-1}\frac{\mu^{k-1}}{(k-1)!} a u_{k-1} + \delta_k, \\ u_1 &= a u_0 - \mu a u_1 + \ldots + (-1)^{k-2}\frac{\mu^{k-2}}{(k-2)!} a u_{k-2} + \delta_{k-1}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ u_{k-1} &= a u_{k-2} + \delta_1, \end{aligned} \tag{8}$$

where $u_j = \psi^{(j)}(t_{k-1})$, $j = 1, \ldots, k-1$, $u_0 = \psi(t_{k-1})$, $\delta_\sigma = (-1)^\sigma a^k \mu^\sigma/\sigma!\, \varphi(\theta_\sigma)$, $-\mu < \theta_\sigma < 0$.

These equations are linear algebraic equations in $u_0, \ldots, u_{k-1}$. If we solve them for $u_0, \ldots, u_{k-1}$, and then express $\dot{\psi}_k, \ddot{\psi}_k, \ldots$ by the formulas $\dot{\psi}_k = au_0$, $\ddot{\psi}_k = au_1, \ldots$ and substitute these expressions in (7), we obtain $\psi(t)$ in the interval $t_{k-1} \le t \le t_k$ in the form $\psi(t) = \tilde{\psi}_k(t) + \Delta\psi_k(t)$, where the function $\tilde{\psi}_k(t)$ corresponds to the solution of Equations (8) for $\delta_1 = \ldots = \delta_k = 0$, and the function $\Delta\psi_k(t)$ depends linearly on $\delta_1, \ldots, \delta_k$ and is of the order k relative to μ. Thus,

$$\Delta\psi_2(t) = \bar{t}\,\frac{\delta_2}{A} + \frac{\bar{t}^2}{2} a^2\varphi(\bar{\theta}_2),\ \Delta\psi_3(t) =$$

$$= \bar{t}\left(\frac{\mu^2}{2}\delta_2 + A\delta_3\right)\frac{1}{\Delta_2} + \frac{\bar{t}^2}{2}(A\delta_2 + \delta_3)\frac{1}{\Delta_2} + \frac{\bar{t}^3}{3!}a^3\varphi(\bar{\theta}_3),$$

where $A = 1 + \mu a/a$, $\Delta_2 = A^2 - \mu^2/2$, and the arguments $\bar{\theta}_2$, $\bar{\theta}_3$ of the function φ are in the interval $(-\mu, 0)$. If $|\varphi(t)| \le \gamma$, then we can obtain the inequality

$$|\Delta\psi_2(t)| \leqslant \frac{|a\mu|^2}{2}\left(1 + \frac{\mu}{|A|}\right)\gamma,\ |\Delta\psi_3(t)| \leqslant$$

$$\leqslant \frac{|a\mu|^3}{3!}\left[1 + \left(2\mu^2 + \frac{5}{2}|A|\mu\right)\frac{1}{\Delta_2}\right]\gamma$$

$$t_1 \leqslant t < t_2,\quad t_2 \leqslant t < t_3$$

and similar formulas for $\Delta\psi_4, \Delta\psi_5, \ldots$. These inequalities show that for small values of μ, or, more correctly, for small values of the product $|a|\mu$, the functions $\Delta\psi_k$ decrease very rapidly in absolute value for increasing k. For all values of k we have $|\Delta\psi_k(t)| \leq Q_k(\mu)\gamma$, where $Q_k(\mu)$ is of order k relative to μ.

We will now consider the solution $\bar{x}(t, t_k)$ of Eq. (6) obtained by the method of iterations for the initial condition $x(t_k) = \psi_k$. Since this solution has continuous derivatives of any order at $t = t_k$, a relation similar to (7) and (8) can be obtained for it. As a result, we find that, in the interval $t_{k-1} \leq t < t_k$,

$$\bar{x}(t, t_k) = \bar{\varphi}_k(t) + \Delta\bar{x}_k(t),$$

where the function $\Delta\bar{x}_k(t)$ consists of terms of order k and higher relative to μ for $t_{k-1} \leq t < t_k$. For the difference between the functions $\bar{x}(t, t_k)$ and $\psi_k(t)$ in this interval, we obtain the inequality

$$|\psi(t) - \bar{x}(t, t_k)| \leqslant |\Delta\psi_k(t)| + |\Delta\bar{x}_k(t)|.$$

For values of μ for which the solution $\bar{x}(t, t_k)$ exists and for all values of t, we have $\lim_{k\to\infty} \Delta\bar{x}_k = 0$. This is clear from the fact that if we expand $\bar{x}(t, t_k)$ in a double series of powers of $t - t_k$ and μ, then Δx_k is part of the remainder term of this series. It can be shown that for these values of μ we have $\lim_{k\to\infty} Q_k(\mu) = 0$, so that $\Delta\psi_k \to 0$, and also $|\psi(t) - \bar{x}(t, t_k)| \to 0$ for $k \to \infty$.

If we denote by $\varphi_k(t)$ and $\bar{\varphi}_k(t)$ functions given in the interval $t_{k-1} \leq t < t_k$, and coinciding in this interval with $\psi(t)$ and $\bar{x}(t, t_k)$, respectively, then these functions can be considered as initial functions determining the solution $\psi(t)$ and $\bar{x}(t, t_k)$ for $t > t_k$. Since the difference $\varphi_k(t) - \bar{\varphi}_k(t)$ tends to zero for $k \to \infty$, and in view of the continuous dependence of the solutions of differential equations with delay on the initial functions, the solution $\psi(t)$ will be a more accurate approximation to the solution $\bar{x}(t, t_k)$ for $t > t_k$, for larger t_k. The following theorem can be proved rigorously.

Let two arbitrary positive numbers ϵ and L be given, where ϵ can be as small as we wish and L can be as large as we wish. Then, in every case, if $\mu < 1/(|a| e)$, there is an instant of time T such that if $t_* > T$, then in the interval $[t_*, t_* + L]$, where $t_* > T$, the inequality $|\psi(t) - \bar{x}(t, t_k)| \leq \epsilon$, will hold where $\psi(t)$ is the solution of Equation (6) determined by the initial function $\varphi(t)$, and $x(t, t_k)$ is the solution of this equation obtained by the method described above for the initial condition $x|_{t=t_k} = \psi(t_*)$. The time T depends on ϵ, a, μ, and on the upper limit of the absolute value of $\varphi(t)$.

This theorem can easily be extended to cover the case of the equation

$$\dot{x}(t) = ax(t) + bx(t - \mu),$$

where a, b, μ are constants, and also the case of the system

$$\dot{x}_s(t) = a_{s1}x_1(t - \mu) + \ldots + a_{sn}x_n(t - \mu) \quad (s = 1, \ldots, n), \tag{8*}$$

where $a_{s\sigma}$ and μ are constant. In the latter case the theorem holds if $\mu < 1/(|\bar{\lambda}| e)$, where $\bar{\lambda}$ is the root of the characteristic equation with the largest absolute value for $\mu = 0$. In the general case of a system of type (1), we have not proved this theorem rigorously, but it is evident that it is valid.

Thus, in all cases of equations with a delay of a certain type, when the delay is sufficiently small, a phenomenon takes place that can be called the relaxation of "heredity," i.e., the weakening of the influence of the "heredity" with time.

In fact, let us consider a dynamic system described by the equations (1). Because of the dependence of the solution of these equations on the initial functions, the state of the system at any instant of time $t > 0$ will depend not only on its state of the system at any instant $t = 0$, but also, on its state in the preceding interval of time from $t = -\mu$ to $t = 0$. We will consider a certain instant t_*, that is sufficiently far from the origin, and will assume that the state of the system at this instant is fixed. Then, in view of the above-stated theorem, the state of the system for a reasonably large interval of time beginning with t_* will depend mainly on its state at the instant of time $t = t_*$, and can be adequately predicted. The influence of the state of the system in the interval of time preceding t_* becomes smaller as the time t_* becomes larger. The behavior of the system in this case corresponds to a certain extent to a so-called Markoff process.

The question is very intersting as to whether this phenomenon also occurs for systems described by more complex equations with delay—for example in the case of variable coefficients with delay, and in the case of nonlinear systems.

On the basis of what has been said, the solutions obtained by the proposed method can be called "limiting" solutions. For linear systems of the type (1) when the above-stated theorem holds, it can be proven that certain characteristics of the asymptotic behavior are the same (for $t \to \infty$) for the limiting solutions and for any solution of the system corresponding to arbitrary initial functions. Thus, it can be shown that if the functions $x_1, \ldots, x_n$ in the "limiting" solution constructed for any initial conditions $x_s(0) = x_{s0}$ tend to zero for $t \to \infty$, then any solu-

tion of the system (1) has the same property. Also, if the initial conditions are such that the functions $x_1, \ldots, x_n$ in the "limiting" solution increase without limit in absolute value, then, for arbitrary initial functions (if they are not specially chosen — a case that will be considered below), the functions $x_1, \ldots, x_n$ in the corresponding solution also increase without limit (in absolute value). If all the roots of the characteristic equation for the system (1) for $\mu = 0$ have negative real parts, and if μ does not exceed a certain bound $\bar{\mu}_0$, then the functions $x_1, \ldots, x_n$ in any "limiting" solution tend to zero as $t \to \infty$, and so do the solutions for arbitrary initial functions. In other words, the zero solution of the system (1) possesses asymptotic stability both for $\mu = 0$ and for $\mu < \bar{\mu}$, and this property does not depend either on the initial conditions or on the initial functions. If, among the roots $\lambda_1, \ldots, \lambda_n$ of the characteristic equation, there are roots with positive real parts, then the zero solution ceases to be stable both for $\mu = 0$ and for any other value of μ. This conclusion concerning the equivalence in a certain sense of the solutions of a system of the type (1) for $\mu = 0$ and $\mu \neq 0$ agrees with the results of [3]. Nothing that has been said in this section contradicts the results of A. D. Myshkis ([4], §§ 11, 23) on the classification of solutions of equations with delay. Let us assume, for example, that all the roots $\lambda_1, \ldots, \lambda_n$ of the characteristic equation for the system (1) for $\mu = 0$ have negative real parts. Then, according to this classification, all the solutions can be separated for sufficiently small μ into two categories: 1) the weakly damped solutions, tending to zero not more rapidly than $e^{\rho t}$, where ρ is the least of the absolute values of the real parts of the roots $\lambda_1, \ldots \lambda_n$; 2) the strongly damped solutions, tending to zero as rapidly as we please according to the law $e^{\alpha t}$, where $\alpha \to -\infty$ for $\mu \to 0$. If the solution of the system (1) is sought by using Euler's method in the form of exponential functions $x_s = c_s e^{\lambda t}$, then the solutions of the first type correspond to those roots of the characteristic equation

$$\| a + be^{-\lambda\mu} - E\lambda \| = 0, \tag{9}$$

of the system (1) that, for $\mu \to 0$, tend to the corresponding roots $\lambda_1, \ldots, \lambda_n$ of the characteristic equation $\| a + b - E\lambda \| = 0$. The solutions of the second type correspond to those complex roots $\alpha \pm \beta_i$ of Equation (9) for which $\alpha \to -\infty$ for $\mu \to 0$. For arbitrary initial functions we obtain a solution that can be represented as some combination of the two types of solution. The terms corresponding to solutions of the second type decrease very rapidly, and, after a relatively short time interval starting at the initial instant, they cease to influence the further behavior of the solution, which is thereafter determined by the main terms corresponding to a combination of solutions of the first type. The "limiting" solution corresponds to this latter combination of solutions of the first type.

If among the roots $\lambda_1, \ldots, \lambda_n$ there are roots with positive real parts, then for arbitrary initial functions we obtain a solution in which the functions $x_1, \ldots, x_n$ increase in absolute value without limit. It is, of course, possible to choose the initial functions in such a way that they correspond to solutions of the form $x_s = c_s e^{\alpha t} \cos \beta t$ or to a combination of such solutions, where $\alpha + \beta i$ is a root of Equation (9) that is discontinuous for $\mu = 0$, i.e., $\alpha \to -\infty$ for $\mu \to 0$. In other words, for the zero solution of the system (1) for sufficiently small μ, as for $\mu = 0$, the so-called stability condition is satisfied.

3. We will consider a system of equations arising in a problem on the stabilization of a rocket relative to its yaw angle γ by an automatic pilot with rigid inverse coupling, and a delay μ in the argument of the controlling parameter of the rudder mechanism:

$$\ddot{\gamma}(t) + b_1\dot{\gamma}(t) + b_3\delta(t) = 0, \quad \dot{\delta}(t) + \delta(t) = \sigma(t-\mu), \quad \sigma(t) = p_1\gamma(t) + p_2\dot{\gamma}(t). \tag{10}$$

Here b_1 and b_3 are positive constant coefficients characterizing the object being controlled, and p_1 and p_2 are positive constant parameters of the regulator. Since the regulator does not act until $\gamma = \dot{\gamma} = 0$, and then $\delta = 0$, the initial functions must be assumed to be zero, i.e., we set $\sigma = 0$ for $-\mu \leq t < 0$, and the initial values are $\gamma(0) = \gamma_0$, $\dot{\gamma}(0) = \dot{\gamma}_0$, $\delta(0) = 0$.

We will give the formulas that can be used in the iteration method to construct the "limiting" solution $\bar{\gamma}(t, t_0)$ of this system for the arbitrary initial conditions $\gamma(t_0) = \gamma_0$, $\dot{\gamma}(t_0) = \dot{\gamma}_0$, $\delta(t_0) = \delta_0$. Here we are interested only in the regulated coordinate γ, and so we transform the system (10) into a single equation for γ:

$$\dddot{\gamma}(t) + (1 + b_1)\ddot{\gamma}(t) + b_1\dot{\gamma}(t) + b_3[p_1\gamma(t-\mu) + p_2\dot{\gamma}(t-\mu)] = 0. \tag{10*}$$

If we set $\mu = 0$ and assume for simplicity that $t_0 = 0$, then we obtain the "undisturbed" solution $\gamma^{(0)}(t)$ of this equation in the form

$$\gamma^{(0)} = A_1 e^{\lambda t} + (A_2 \cos \omega t + A_3 \sin \omega t) e^{at}, \tag{11}$$

where

$$A_1 = \frac{1}{s}\,[\ddot{\gamma}_0 - 2a\dot{\gamma}_0 + (a^2 + \omega^2)\gamma_0],$$

$$A_2 = \frac{1}{s}\,[-\ddot{\gamma}_0 + 2a\dot{\gamma}_0 + (\lambda^2 - 2a\lambda)\gamma_0],$$

$$A_3 = \frac{1}{\omega^2 s}\,[(a-\lambda)\ddot{\gamma}_0 + (\lambda^2 + \omega^2 - a^2)\dot{\gamma}_0 + \lambda(a^2 - \omega^2 - a\lambda)\gamma_0], \quad s = (a-\lambda)^2 + \omega^2.$$

Here λ, $a \pm i\omega$ are the roots of the characteristic equation for the system (10) for $\mu = 0$:

$$k^3 + (1 + b_1)\,k^2 + (b_1 + b_3 p_2)\,k + b_3 p_1 = 0. \quad (12)$$

This equation has one real root and two complex (or purely imaginary) roots. Our main interest, of course, is to find the region of values of the coefficients of the system, for which λ and $\underline{a}$ are negative, i.e., the region of asymptotic stability of the zero solution of the system (10) for $\mu = 0$.

The first approximation of the "limiting" solution is obtained in the form $\gamma^{(1)} = \gamma^{(0)} + \bar{\gamma}^{(1)}$, where $\bar{\gamma}^{(1)}$ is the solution of the linear homogeneous equation

$$\dddot{\bar{\gamma}}^{(1)} + (1 + b_1)\,\ddot{\bar{\gamma}}^{(2)} + (b_1 + b_3 p_2)\,\dot{\bar{\gamma}}^{(1)} + b_3 p_1 \bar{\gamma}^{(1)} = \\ = -\,b_3 p_1\,[\gamma^{(0)}(t-\mu) - \gamma^{(0)}(t)] - b_3 p_2\,[\dot{\gamma}^{(0)}(t-\mu) - \\ -\ \dot{\gamma}^{(0)}(t)]$$

for zero initial conditions. This solution is found from the formula

$$\bar{\gamma}^{(1)} = \frac{b_3}{s}\int_0^t f(t-\tau)\,[p_1 \Delta\gamma^{(0)}(\tau) + p_2 \Delta\dot{\gamma}^{(0)}(\tau)]\,d\tau, \quad (13)$$

where

$$\Delta\gamma^{(0)}(t) = \gamma^{(0)}(t-\mu) - \gamma^{(0)}(t),$$

$$\Delta\dot{\gamma}^{(0)}(t) = \frac{d}{dt}\Delta\gamma^{(0)}(t),$$

$$f(t) = -\,e^{\lambda t} + \left(\cos\omega t + \frac{\lambda - a}{\omega}\sin\omega t\right).$$

The function $\Delta\gamma^{(0)}$ can be written according to (11) in the form

$$\Delta\gamma^{(0)}(t) = A_1\chi_1 e^{\lambda t} + [(A_2\chi_2 - A_3\chi_3)\cos\omega t + \\ + (A_2\chi_3 + A_3\chi_2)\sin\omega t]\,e^{at}, \quad (14)$$

where the quantities $\chi_1 = e^{-\lambda\mu} - 1$, $\chi_2 = e^{-a\mu}\cos\omega\mu - 1$, $\chi_3 = e^{-a\mu}\sin\omega\mu$ are of the first order relative to μ. If we differentiate this equation with respect to $\underline{t}$, we obtain $\Delta\dot{\gamma}^{(0)}(t)$.

A calculation leads to the formula

$$\bar{\gamma}^{(1)} = (M_1^{(1)} + M_2^{(1)}t)\,e^{\lambda t} + \\ + [(N_1^{(1)} + N_2^{(1)}t)\cos\omega t + \\ + (L_1^{(1)} + L_2^{(1)}t)\sin\omega t]\,e^{at}, \quad (15)$$

where

$$M_2^{(1)} = -\,\chi_1\frac{A_1 b_3}{s}(p_1 + p_2\lambda),\quad N_2^{(1)} = \frac{1}{2}(l_2A_2 + l_3A_3),$$

$$L_2^{(1)} = \frac{1}{2}(-l_3A_2 + l_2A_3),$$

$$M_1^{(1)} = \frac{2(a-\lambda)}{s}M_2^{(1)} - \frac{2\omega}{s}L_2^{(1)},$$

$$L_1^{(1)} = \frac{(\lambda - a)^2 - \omega^2}{\omega s}M_2^{(1)} - \frac{1}{\omega}N_2^{(1)} + \frac{2(\lambda - a)}{s}L_2^{(1)},$$

$$l_2 = \frac{b_3}{s}\,[(p_1 + p_2 a)\,\bar{\chi}_2 + p_2\omega\bar{\chi}_3],$$

$$l_3 = \frac{b_3}{s}\,[p_2\omega\bar{\chi}_3 + (p_1 + p_2 a)\,\bar{\chi}_2],$$

$$\bar{\chi}_2 = \chi_2 - \frac{\lambda - a}{\omega}\chi_3,\quad \bar{\chi}_3 = \chi_3 - \frac{\lambda + a}{\omega}\chi_2.$$

All the approximations can be obtained similarly in the form

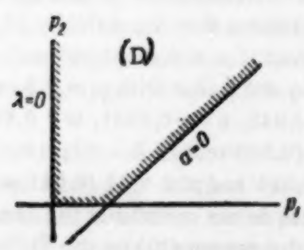
$$\gamma^{(k)} = \gamma^{(0)} + \bar{\gamma}^{(1)} + \ldots + \bar{\gamma}^{(k)},$$

where

$$\bar{\gamma}^{(m)} = M_m(t)\,e^{\lambda t} + [N_m(t)\cos\omega t + \\ + L_m(t)\sin\omega t]\,e^{at} \quad (16)$$

and M_m, N_m, and L_m are polynomials in $\underline{t}$ of degree $\underline{m}$ with constant coefficients of order m relative to μ. It can be shown that when $\lambda < 0$, $a < 0$, and $|\lambda| > |a|$, the sequence $\{\gamma^{(k)}\}$ converges for any finite value of $\underline{t}$, if $\mu^2 e^{-\lambda\mu} < 1/e \approx 0.368$.

An investigation of this "limiting" solution shows directly that if the "undisturbed" solution $\gamma^{(0)}$ tends to zero for $t \to \infty$, then the function $\bar{\gamma}(t, t_0)$ will have this property for all values of μ that do not exceed a certain value. From what has been said in Section 2, it follows that in this case all solutions of the system (10) also tend to zero for $t \to \infty$. We thus arrive at an estimate of the limiting value of μ for which the delay does not destroy the asymptotic stability of the control process that occurs for $\mu = 0$.



The region of stability D of the zero solution of the system (10) for $\mu = 0$ in the plane of the control parameters p_1 and p_2 has the form shown in the sketch. On the left boundary of this region (taking $p_1 = 0$) we have $\lambda = 0$ and $a = -0.525$, while on the right boundary [where $p_2 = 1/(1 + b_1)\,p_1 - b_1/b_3$] we have $\lambda = -(1 + b_1)$ and $a = 0$. As ω increases, the frequency increases like $\sqrt{p_2}$. It is of interest to investigate the stability near the right boundary of the region D where $\underline{a}$ is small in absolute

value in comparison with λ and where the delay can serve as a disturbance in a system of undamped oscillations. We will not give the derivation, but will give an approximate estimate which is easily obtained in the case $a < 0$, $\lambda < 0$ for the difference $\Delta\bar{\gamma}(t) = \bar{\gamma}(t, t_0) - \gamma^{(0)}(t) = \sum_{k=1}^{\infty} \bar{\gamma}^{(k)}(t)$. For "undisturbed" systems depending on the delay μ, this estimate has the form

$$|\Delta\bar{\gamma}(t)| \leqslant \sqrt{A_2^2 + A_3^2}\,[e^{(q+a)t} - e^{at}], \tag{17}$$

where A_2 and A_3 are the coefficients in the "undisturbed" solution $\gamma^{(0)}$; and the value of q, with precision up to terms of the first order relative to μ, is equal to $q = \mu\bar{q}$, where

$$\bar{q} = \frac{b_3}{2\omega}\sqrt{\frac{a^2+\omega^2}{(a-\lambda)^2+\omega^2}\,[p_1^2 + 2p_1p_2a + p_2^2(a^2+\omega^2)]}. \tag{18}$$

Although the inequality (17) is not exact, it is, however, completely satisfactory for all practical purposes, and calculations show that it is actually valid for all t. It follows from this inequality that if μ is small enough, then $q + a < 0$, i.e.,

$$\mu < \frac{|a|}{\bar{q}}, \tag{19}$$

$\Delta\bar{\gamma}(t) \to 0$ for $t \to \infty$, and the stability of the control process is ensured.

The inequality (19) is convenient because of the fact that it directly gives an approximate limit of the delay for which the control process retains its stability.

Let us set, for example, $b_1 = 0.05$, $b_3 = 0.15$, $p_2 = 3$ and $p_2 = 2$, where the unit of time is 0.1 sec. These values correspond to a system that can be realized in practice, and yield the following values of the parameters: $\lambda = -0.871$, $a = -0.0893$, $\omega = 0.580$, $\bar{q} = 0.192$. The inequality (19) shows that the stability of the control process is ensured if $\mu < 0.46$ (0.046 sec). For the same values of b_1, b_3 and p_2, but with $p_1 = 2.5$ or $p_1 = 3$, we obtain: $\lambda = -0.942$, $a = -0.0541$, $\omega = 0.629$, $\bar{q} = 0.208$ and $\mu < 0.26$ (0.026 sec) or $\lambda = -1$, $a = -0.025$, $\omega = 0.670$, $\bar{q} = 0.221$ and $\mu < 0.11$ (0.011 sec), respectively. These estimates do not contradict the results obtained by modelling the system (10) on the ÉMU-8 modelling apparatus.

If μ has a value such that the inequality (19) is not satisfied, then the control process cannot be stable, and, as calculations show, $\gamma(t)$ increases with t. Thus, if μ is given, it then follows from (18) that for a fixed p_2 the values of p_1 for which the process is stable are bounded on the right by some number p_1^* lying inside the region D, and for a fixed p_1 the values of p_2 are bounded above by some number p_2^*. These results do not contradict those of other authors who have considered systems of the type (10) (for example, see [5]), but the condition (19) is in our opinion more convenient in practical applications.

In the investigation of the system (10) it is essential to have an estimate of the rate of damping of the controlled coordinate $\gamma(t)$, i.e., to solve the problem of the quality of the regulation. If this problem is solved for the system (10) for $\mu = 0$, then, by using the inequality (17), we can estimate the dependence on the time of the magnitude of the disturbances caused by a delay, and also solve the problem of finding the quality of control in the case $\mu \neq 0$. It should be mentioned that the factor $\sqrt{A_2^2 + A_3^2}$ in formula (18) is comparable in value to the initial value γ_0. The function $f(t) = e^{(q+a)t} - e^{at}$ thus determines approximately the amount of "disturbance" due to γ_0. For example, if $b_1 = 0.05$, $b_3 = 0.15$, $p_1 = 2.5$, $p_2 = 3$ and $\mu = 0.1$ (0.01 sec), the maximum of the function $f(t)$ is attained for $t = 23.6$ (2.36 sec), and the value of this maximum is approximately 0.18, and we can conclude that this "disturbance" is not greater than about 20% of γ_0. In this case the delay has a relatively small effect on the quality of control. For the same coefficients, however, but with $\mu = 0.2$, we find that $\max f(t) \approx 0.51$ (for $t = 3.6$ sec) and that for $t = 10$ sec we have $f \approx 0.30$. Consequently, the "disturbances" are so large, that even though the values of the coefficients lie inside the region of stability of the process for $\mu = 0.2$, the quality of the control is unsatisfactory: the damping of the oscillations of the coordinate $\gamma(t)$ is too slow, due to the influence of the delay.

If it is necessary to investigate the character of the regulation process and the character of the "disturbances" in more detail, then we must obtain an approximate solution by using the method described in Section 2. For example, let $b_1 = 0.05$, $b_3 = 0.15$, $p_1 = 2.5$, $p_2 = 3$, $\gamma_0 = 1$, $\dot{\gamma}_0 = 0$, $\delta_0 = 0$, and $\mu = 0.1$, and let the unit of time be 0.1 sec. If we use the "step" method, we find that in the interval $0 \leq t \leq t_1 = \mu$ the function $\gamma(t)$ can be obtained from the equation $\dddot{\gamma} + 1.05\ddot{\gamma} + 0.05\dot{\gamma} = 0$ with the initial conditions $\gamma_0 = 1$, $\dot{\gamma}_0 = \ddot{\gamma}_0 = 0$. Therefore, $\gamma(t) \equiv 1$.

In the interval $t_1 \leq t \leq t_2 = 2\mu$, the function $\gamma(t)$ satisfies the equation

$$\dddot{\gamma} + 1.05\ddot{\gamma} + 0.05\dot{\gamma} + 0.375 = 0$$

and the initial conditions $\gamma_0 = 1$, $\dot{\gamma}_0 = \ddot{\gamma}_0 = 0$, so that

$$\gamma(t) = 1 - \frac{15}{38}(1 - e^{-(t-t_1)}) + \frac{3000}{19}(1 - e^{-0.05(t-t_1)}) - 7.5(t - t_1). \tag{20}$$

We now obtain the "limiting" solution $\bar{\gamma}(t, t_2)$ of the equation (10) and choose the instant of time $t = t_2$ as the initial time with the initial conditions defined by (20). With an error of less than 0.0001 we obtain $\gamma_0 = 0.9999$, $\dot{\gamma}_0 = -0.0018$, $\ddot{\gamma}_0 = -0.0356$.

The calculation of the functions $\gamma^{(0)}$, $\bar{\gamma}^{(1)}$, and $\bar{\gamma}^{(2)}$ yields

$$\gamma^{(0)}(t) = 0.306\, e^{\lambda \bar{t}} + (0.694 \cos \omega \bar{t} + 0.516 \sin \omega \bar{t})\, e^{a \bar{t}}.$$

$$\bar{\gamma}^{(1)}(t) = (-0.00773 + 0.00125\, \bar{t})\, e^{\lambda \bar{t}} + \\ + [(0.00773 + 0.0156\, \bar{t}) \cos \omega \bar{t} + \\ + (0.0193 + 0.00904\, \bar{t}) \sin \omega \bar{t}]\, e^{a \bar{t}},$$

$$\bar{\gamma}^{(2)}(t) = (-0.000263 - 0.0000760\, \bar{t} + 0.00000254\, \bar{t}^2)\, e^{\lambda \bar{t}} + \\ [(0.000263 - 0.000070\, \bar{t} + 0.000172\, \bar{t}^2) \cos \omega \bar{t} + (0.000173 + \\ + 0.000278\, \bar{t} + 0.000075\, \bar{t}^2) \sin \omega \bar{t}]\, e^{a \bar{t}},$$

where $\bar{t} = t - t_2$, $\lambda = -0.9419$, $a = -0.05407$, $\omega = 0.6287$, and the coefficients are given with an accuracy of one-half the value of the last figure. With an error of less than 0.005 for all values of t, we can write the second approximation to $\bar{\gamma}(t, t_2)$ in the form

$$\bar{\gamma}(t, t_2) = \gamma^{(0)} - 0.008\, e^{\lambda \bar{t}} + [(0.008 + 0.0155\, \bar{t} + \\ + 0.00017\, \bar{t}^2) \cos \omega \bar{t} + \\ + (-0.019 + 0.0093\, \bar{t} + 0.00008\, \bar{t}^2) \sin \omega \bar{t}]\, e^{a \bar{t}}. \tag{21}$$

It is not difficult to see that the predominant part in each function is played, as in the case of the functions $\bar{\gamma}^{(1)}$ and $\bar{\gamma}^{(2)}$, by the terms of the highest degree in $\bar{t}$ in the coefficients of $e^{a\bar{t}} \cos \omega \bar{t}$ and $e^{a\bar{t}} \sin \omega \bar{t}$ (in the function $\bar{\gamma}^{(1)}$ these terms attain a value of approximately 0.12 in absolute value, while the remaining terms do not exceed 0.03 in absolute value; in the function $\bar{\gamma}^{(2)}$ these terms reach 0.035 in absolute value, while all the remaining terms are not larger than 0.0026). In the function $\bar{\gamma}^{(3)}$ these terms are

$$(0.000\,00125 \cos \omega \bar{t} + 0.000\,00037 \cdot \sin \omega \bar{t})\, \bar{t}^3 e^{a \bar{t}}$$

and do not exceed 0.001 for any t. In $\bar{\gamma}^{(4)}$ these terms are only one-fifth as large, etc., so that if we want to calculate $\bar{\gamma}(t, t_2)$ with an accuracy of 1%, we can limit our calculation to the second approximation.

We now estimate the error committed in using the "limiting" solution $\bar{\gamma}(t, t_2)$ to represent the exact solution $\gamma(t)$ for $t \geq t_2$. In order to do this, we compare the function $\gamma(t)$ in the interval $t_1 \leq t < t_2$, represented by the formula (20), with the function $\bar{\gamma}(t, t_2)$ in the same interval. It is convenient to write both these functions in the form of a Taylor's series of powers of $t - t_2$ with a remainder. For $\gamma(t)$ we obtain

$$\gamma(t) = \gamma_0 + \dot{\gamma}_0 \bar{t} + \frac{\ddot{\gamma}_0}{2} \bar{t}^2 - 0.05627\, \bar{t}^3 + 0.01476\, \bar{t}^4 - \\ - 0.00328\, \bar{t}^5 + \Delta_1, \tag{22}$$

where $|\Delta_1(t)| < 0.00055\, (\bar{t} + \mu)$, $\bar{t} = t - t_2$ and γ_0, $\dot{\gamma}_0$, $\ddot{\gamma}_0$ are the values of the function $\gamma(t)$ and its derivatives at the instant $t = t_2$, calculated by using the expression (20). For the second approximation (21) of the "limiting" solution, we obtain

$$\bar{\gamma}(t, t_2) = \gamma_0 + \dot{\gamma}_0 \bar{t} + \frac{\ddot{\gamma}_0}{2} \bar{t} - 0.05626\, \bar{t}^3 + \\ + 0.01484\, \bar{t}^4 - 0.00127\, \bar{t}^5 + \Delta_2, \tag{22*}$$

where $|\Delta_2(t)| < 0.00045\, \bar{t}^6$. The difference between the expressions for the function is

$$\gamma(t) - \bar{\gamma}(t, t_2) = -0.00001\, \bar{t}^3 - 0.00008\, \bar{t}^4 - 0.00201\, \bar{t}^5 + \\ + \Delta_3,$$

where $|\Delta_3(t)| < 0.00055\, (\bar{t} + \mu)^6 + 0.00045\, \bar{t}^6$. The difference between the corresponding expressions for the function $\sigma = p_1 \gamma + p_2 \dot{\gamma}$ is

$$\Delta\sigma(t) = \sigma(t) - \bar{\sigma}(t, t_2) = -0.00009\, \bar{t}^2 - 0.00098\, \bar{t}^3 - \\ - 0.03\, \bar{t}^4 - 0.005\, \bar{t}^5 + \varepsilon, \tag{23}$$

where $|\epsilon| < 2 \cdot 10^{-7}$. It follows from this last formula that $|\Delta\sigma(t)| < 5.1 \cdot 10^{-6}$. We should mention that the coefficients in (22*), written with an accuracy of 0.00001, do not vary if the function $\bar{\gamma}^{(2)}$, is left out, and this is, of course, also true for the functions $\bar{\gamma}^{(3)}$, $\bar{\gamma}^{(4)}$, etc. We can therefore assume that formula (23) is valid not only for the second approximation, but also for the "limiting" solution itself.

We must therefore compare two solutions of the equation (10*) for $t \geq t_2$ for which the functions $\sigma(t)$ and $\bar{\sigma}(t, t_2)$ (which play the part of initial functions and are given in the interval $t_2 - \mu \leq t < t_2$) differ from one another by not more than $5.1 \cdot 10^{-6}$. It is not difficult to establish that, in all cases for any $t \geq t_2$, these two solutions will not differ from one another by more than this amount. The solution $\bar{\gamma}(t, t_2)$ has an accuracy that is more than that needed in practice. The influence of "heredity" on the solution $\gamma(t)$ as early as for $t = t_2 = 0.02$ sec has practically vanished.

We mention that the accuracy would be completely satisfactory if the "limiting" solution were calculated directly for the initial conditions of the problem $\gamma_0 = 1$, $\dot{\gamma}_0 = \ddot{\gamma}_0 = 0$. In other words, the control process described by the system of equations (10) is approximated sufficiently well by the solution obtained by the iteration method, if only the initial values of the problem $\gamma_0 = 1$, $\gamma_0 = \delta_0 = 0$ are used, and without taking into account the initial functions.

It can be established from formula (21) that the "disturbance" related to μ does not exceed 0.15, i.e., 15% of the initial value γ_0. It should be mentioned that a more detailed investigation of the "disturbances" leads to

approximately the same estimate as that obtained by using the approximate formula (18).

I feel that I must express my thanks to A. M. Letov, who suggested to me the problem of calculating the role of the initial functions, or the "heredity," in systems of automatic control, and whose advice, in the course of several discussions of the problem, was very helpful.

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THE APPLICATION OF LYAPUNOV'S METHOD TO PROBLEMS IN THE STABILITY OF SYSTEMS WITH DELAY

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We consider the application of the direct Lyapunov method to problems in the stability of systems with delay. Certain criteria are given for the stability and asymptotic stability of such systems. The possibility is shown of applying quadratic forms in stability problems in a first approximation, and estimates are proposed for the perturbations.

One of the important problems attracting the attention of many scientific workers is the stability problem for systems with delay.

Any automatic control system is in some measure a system with delay, and the influence of this delay on the stability must be ascertained. The influence of even small delays can be very significant, for example, in systems with transitional high-frequency vibration processes. The scientific development of the last three decades has shown that the most effective and universal method of investigating stability is the direct Lyapunov method. There is therefore a natural tendency to derive methods for investigating the stability of systems with delay based on the ideas of the Lyapunov theory of the stability of motion.

1. Definitions and Statement of Problem

We consider the system of differential equations

$$\frac{dx_i}{dt} = X_i\,[t;\; x_1(t-\tau), \ldots,\; x_n(t-\tau)] \qquad (i = 1, \ldots, n), \tag{1}$$

where $X_i[t; x_1(t-\tau), \ldots, x_n(t-\tau)]$ is a functional that is continuous and bounded in the region $t \geq t_0$, $|x_i| < H$ $(j = 1, \ldots, n)$ and satisfies the condition $X_i(t; 0, \ldots, 0) = 0$. For a given t, the value of the functional X_i is determined by the values of the function $x_1(t-\tau), \ldots, x_n(t-\tau)$ of τ in the intervals $0 \leq \tau \leq h_1(t), \ldots, 0 \leq \tau \leq h_n(t)$. The functions $h_1(t), \ldots, h_n(t)$ are assumed to be positive and bounded, i.e., $0 \leq h_j(t) \leq h$ $(j = 1, \ldots, n)$ for $0 \leq t < \infty$.

Definition 1. The functional $X_i[t; x_1(t-\tau), \ldots, x_n(t-\tau)]$ is called continuous, if, for any arbitrarily small $\epsilon > 0$, there exists a positive number $\delta(\epsilon, t_0) > 0$ such that

$$|X_i\,[t;\; x_1^{(1)}(t-\tau), \ldots,\; x_n^{(1)}(t-\tau)] - X_i\,[t;\; x_1^{(2)}(t-\tau), \ldots,\; x_n^{(2)}(t-\tau)]\,| < \varepsilon \quad \text{for} \quad \| x^{(1)} - x^{(2)} \| \leqslant \delta,$$

where

$$\| x \| = \sup |\, x_j(t-\tau)\,| \quad \text{for} \quad 0 \leqslant \tau \leqslant h_j(t) \quad (j = 1, \ldots, n).$$

If δ can be chosen independently of t_0, the functional is called uniformly continuous.

Definition 2. The solution $x_1(t) = 0, \ldots, x_n(t) = 0$ of the system (1) is stable, if, for every $t_0 > 0$ and $\epsilon > 0$, we can find a number $\delta(\epsilon, t_0)$ such that from the condition $\| x(t_0-\tau) \| \leq (\epsilon, t_0)$ follows $|\overline{x}(t)| < \epsilon$ for any $t > t_0$. Here, and in what follows, $\| x(t_0-\tau)\| = \sup \{ |x_j(t_0-\tau)| \}$ for $0 \leq \tau \leq h_j(t_0)$, $|\overline{x}_j(t)| = \sup \{ |x_j(t)| \}$ for $t > t_0$.

If the number δ is independent of t_0, then the stability is called uniform.

Definition 3. If, for the conditions stated in definition 2, there exists a positive number δ_1 such that from the condition $\|x(t_0-\tau)\| < \delta_1$ it follows that $\lim\limits_{t\to\infty} |\overline{x}(t)| = 0$, then the undisturbed motion $x_1 = \ldots = x_n = 0$ is called asymptotically stable.

Definition 4. The functional $U[t; x_1(t-\tau), \ldots, x_n(t-\tau)]$ is called sign-definite, if there exists a continuous function $\varphi(r)$, satisfying the conditions $\varphi(0) = 0$, $\varphi(r) > 0$ for $r \neq 0$, and such that in the region where the inequalities $t \geq T$, $|x_i(t)| < H$ are satisfied where T is a sufficiently large, and H a sufficiently small, positive number, then either $U[t; x_1(t-\tau), \ldots, x_n(t-\tau)] \geq \varphi(|x(t)|)$, or $U[t; x_1(t-\tau), \ldots, x_n(t-\tau)] \leq -\varphi(|x(t)|)$. In the first case the functional is positive-definite; in the second, negative-definite.

A pecularity of the Lyapunov method in its application to stability problems for systems of the type (1) is that either the Lyapunov function is replaced by a functional *, the derivative of which, in view of (1), is also a functional, or the functional is the derivative of an ordinary Lyapunov function in view of (1). The minimal class of curves, on which these functionals must satisfy the conditions of the Lyapunov theorem, are the set of solutions of the system of equations for the disturbed motion which correspond to the chosen system of initial functions. In stability problems a knowledge of the solutions is not assumed, and therefore a basic problem is the selection of those classes of curves on which the fulfilling of the conditions of Lyapunov's theorem ensures the stability of the undisturbed motion.

The present work has the object of showing that Lyapunov's V-function method can be effectively applied in stability problems for systems with delay. Here the class of functions mentioned above is determined by the Lyapunov functions themselves.

2. Stability Theorems

We will consider functions $V(t; x_1, \ldots, x_n)$ that are continuous, that have bounded continuous, partial derivatives, and that are sign-definite in the region $t \geq t_0, |x_j| < H$ $(j = 1, \ldots, n)$. Because of (1) the derivative of such a function represents the continuous functional

$$U[t; x_1(t-\tau), \ldots, x_n(t-\tau)] =$$

$$= \frac{\partial V}{\partial t} + \sum_{k=1}^{n} \frac{\partial V}{\partial x_k} X_k [t; x_1(t-\tau), \ldots, x_n(t-\tau)]. \quad (2)$$

The functional (2) evidently also has a meaning when it is expressed on a set of curves which are not integral curves of the system (1). In what follows we will consider a functional formally defined by the relation (2) on a set of continuous curves $\{y_1(t), \ldots, y_n(t)\}$ that contains not only integral curves of the system (1). A sufficient condition for the stability of undisturbed motion of the system considered is given by the following theorem.

Theorem 1. If for the disturbed-motion equations (1) there exists a positive-definite function $V(t; x_1, \ldots, x_n)$, whose derivative $dV/dt = U[t; x_1(t-\tau)]$, by the above equations, is such that for every $t \geq t_0$ the functional $U[t; y_1(t-\tau), \ldots, y_n(t-\tau)]$ is negative or identically zero on the set of continuous functions $\{y_1(\sigma), \ldots, y_n(\sigma)\}$, satisfying the condition $V[\sigma; y_1(\sigma), \ldots, y_n(\sigma)] \leq V[t; y_1(t), \ldots, y_n(t)]$ for $\sigma < t$, then the undisturbed motion is stable.

In the proof of this theorem we use a theorem on the continuous dependence of the solutions of the system (1) on the initial functions, according to which, for any number $c > 0$ as small as we please and any number $T > t_0 + h$, a positive number $\delta(c, T)$ can be found such that the condition $\| x(t_0-\tau)\| \leq \delta(c, T)$ (see definition 2) for all t in the interval (t_0, T) ensures the validity of the inequality $V[t; x_1(t), \ldots, x_n(t)] \leq c$. The fact that the inequality $V[t; x_1(t), \ldots, x_n(t)] \leq c$ holds for all $t \geq t_0$ follows from the conditions of the theorem.

The criterion for asymptotic stability is obtained from the following theorem.

Theorem 2. If, for the conditions of Theorem 1, the function $V(t; x_1, \ldots, x_n)$ admits an infinitely small upper limit, and the function $U[t; y_1(t-\tau), \ldots, y_n(t-\tau)]$ is negative-definite, then the undisturbed motion is asymptotically stable.

The proof of this theorem is based on the estimate obtained for the functional U along all the solutions of the system (1) satisfying the conditions $V[t; x_1(t), \ldots, x_n(t)] = c$, i.e., on the relation

$$V(\sigma, x_1(\sigma), \ldots, x_n(\sigma)) \leqslant c \quad \text{for } \sigma \leqslant t,$$

where c is any arbitrarily small positive number.

Theorems 1 and 2 give sufficient conditions for stability and asymptotic stability, regardless of the size of the delay. These conditions are close to being necessary for large values of the delay. It is a more important fact that Theorems 1 and 2 determine the boundary set of curves on which the functional dV/dt must be of a constant sign or sign-definite. These conditions are expressed by the inequality

$$V[\sigma, x_1(\sigma), \ldots, x_n(\sigma)] \leqslant$$

$$\leqslant V[t; x_1(t), \ldots, x_n(t)] \quad \text{for} \quad t-h \leqslant \sigma \leqslant t \quad (3)$$

and are needed for obtaining other stability criteria.

Less stringent stability conditions can be obtained by excluding from the set of continuous lines satisfying condition (3) those which in view of the system (1) do not yield solutions. In this case, it is possible to obtain stability criteria as the criteria for the sign-definiteness of the functional U on the curves obtained from one or other of the estimates of solutions satisfying condition (3).

We will consider one of the possible methods of obtaining such stability criteria. Let

* See the results of N. N. Krasovskii in [1].

$$S_i^{(1)}(t; c) = \sup |X_i[\sigma; y_1(\sigma - \tau), \ldots, y_n(\sigma - \tau)]| \quad (i = 1, \ldots, n)$$

on the set of continuous curves satisfying the conditions

$$V(t; y_1(t), \ldots, y_n(t)) = c, \quad V(\sigma; y_1(\sigma), \ldots, y_n(\sigma)) \leqslant c \quad \text{for} \quad t - h \leqslant \sigma \leqslant t$$

It follows from the system (1) that the inequalities

$$|x_i(t') - x_i(t'')| \leqslant S_i^{(1)}(t; c)\,|t' - t''|. \quad (i = 1, \ldots, n) \tag{4}$$
$$\text{for} \quad t - h \leqslant t' \leqslant t, \ t - h \leqslant t'' \leqslant t.$$

must hold.

It is not difficult, in addition to this, to prove the validity of the following criterion.

Theorem 3. If, for the equations of the disturbed motion (1), there exists a positive-definite function $V(t; x_1, \ldots, x_n)$ having an infinitely small upper limit, and a positive number $\overline{c}$ such that the derivative of the function V determines the functional $dV/dt = U$ by using the system (1) where this functional is negative-definite along every curve $(y_1(t), \ldots, y_n(t))$ satisfying the conditions $V[t; y_1(t), \ldots, y_n(t)] = c$, $V[\sigma; y_1(\sigma), \ldots, y_n(\sigma)] \le c$ for $\sigma \le t \ge t_0$, $|x_i(t'')| \le S_i^{(1)}(t; c)|t' - t''|$ $(i = 1, \ldots, n)$ for $t - h \le t' \le t \ge t_0$, $t - h \le t'' \le t \ge t_0$, then the undisturbed motion is asymptotically stable.

This criterion involves the delay, and yields the possibility of estimating its influence on the stability. Better precision in the stability criterion can be obtained by replacing the functions $S_i^{(1)}(t; c)$ by the functions $S_i^{(2)}(t; c)$ defined by the condition $S_i^{(2)}(t; c) = \sup| X_i[\sigma_i; y_1(\sigma - \tau), \ldots, y_n \times (\sigma - \tau)]|$ on the set of curves $\{y_1(\sigma), \ldots, y_n(\sigma)\}$ satisfying the conditions of Theorem 3. Further methods of improving the criteria are obvious. The effectiveness of the method proposed can be shown by many examples considered in [2]. We will give some simple examples:

1. The region of stability of the trivial solution of the equation

$$\frac{dx}{dt} = -ax(t) - bx(t - \tau),$$

determined by the Lyapunov method, is shown in Fig. 1.

The broken line shows the limit of the region of stability, obtained from the accurate solution.

2. The trivial solution of the equation

$$\frac{dx}{dt} = -\int_{t-h}^{t} x(\zeta)\,d\zeta$$

is stable if $h \le \pi/\sqrt{2} \approx 2.24$. If we apply the Lyapunov V-function theorem, we obtain the condition $h \le 2$.

Thus, in the case of a system with delay, the Lyapunov method is an effective and universal method of investigating stability.

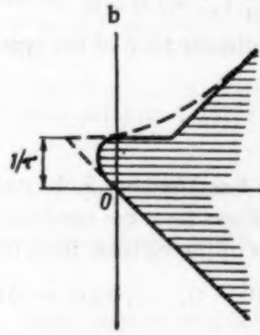

Fig. 1.

3. Stability According to the First Approximation to Systems With Delay

In stability problems relative to the first approximation for general systems, quadratic forms have found a wide application as Lyapunov functions. Quadratic forms have just as wide an application in stability problems of systems with delay.

We will consider the system of differential equations for disturbed motion,

$$\frac{dx_i}{dt} = F_i[t; x_1(t), \ldots, x_n(t), x_1(t - \tau), \ldots, x_n(t - \tau)], \tag{5}$$

where the F_i are holomorphic functions of the variables $x_1(t), \ldots, x_n(t), x_1(t-\tau), \ldots, x_n(t-\tau)$, satisfying the condition $F_i(t; 0, \ldots, 0) = 0$ $(i = 1, \ldots, n)$, with τ a constant.

If we use the notation $p_{ij}(t) = \partial F_i/\partial x_j$, $q_{ij} = \partial F_i/\partial x_j(t-\tau)$, then, for $x_1(t) = \ldots = x_n(t) = x_1(t-\tau) = \ldots = x_n(t-\tau) = 0$, we obtain the system of equations

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} [p_{ij}(t)\,x_j(t) + q_{ij}(t)\,x_j(t - \tau)] \tag{6}$$
$$(i = 1, \ldots, n)$$

For a system of the type (6) we can look for a Lyapunov function in the form of a sign-definite quadratic

form $V = \sum_{i,\, j=1}^{n} \alpha_{ij}(t) x_i x_j$. As a Lyapunov function for (6), we can take, in particular, the quadratic form that is a Lyapunov function for the usual system:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} [p_{ij}(t) + q_{ij}(t)] x_j \quad (i = 1, \ldots, n), \qquad (7)$$

which is obtained from (6) for $\tau = 0$ and is obtained by some one of the known methods.

The derivative $\frac{d}{dt}\left(\sum \alpha_{ij} x_i x_j\right)$, in view of the system (6), will be a bilinear form of the type

$$\frac{dV}{dt} \sum_{i,\, j=1}^{n} [b_{ij}(t) x_i(t) x_j(t) + c_{ij}(t) x_i(t) x_j(t-\tau)]. \qquad (8)$$

By using Theorem 1 or Theorem 2, the stability or asymptotic stability follows from the constancy of sign or the sign-definiteness of the bilinear form (8) if

$$V[t-\tau;\ x_1(t-\tau), \ldots, x_n(t-\tau)] \leqslant \leqslant V[t;\ x_1(t), \ldots, x_n(t)]. \qquad (9)$$

For these conditions the bilinear form (8) can be majorized by a quadratic form, and the stability conditions can be obtained from a known inequality of Sylvester. The proposed method can also be used to obtain an estimate for the disturbances, to determine the degree of stability (quality criteria), and to solve the stability problem for a finite interval of time.

The majorant of the bilinear form (8) can be obtained in the following ways:

First method. The quadratic form majorizing the bilinear form dV/dt can be obtained as the solution of the problem of maximizing the function (8) under the condition (9). This problem can be solved by applying the Lagrange method of undetermined parameters. Then, if we use a known inequality for quadratic forms, obtained by using their canonical representation, we obtain the following inequality for the bilinear form (8):

$$\sum_{i,\, j=1}^{n} [\beta_{ij}(t) + \delta_{ij}\sqrt{h'(t)}]\zeta_i(t)\zeta_j(t) \geqslant \frac{dV}{dt} \geqslant \geqslant \sum_{i,\, j=1}^{n} [\beta_{ij}(t) + \delta_{ij}\sqrt{h''(t)}]\zeta_i(t)\zeta_j(t), \qquad (10)$$

where ζ_i is the canonical variable, $\beta_{ij}(t)$, $h'(t)$ and $h''(t)$ are linear combinations of the coefficients of the system of equations for the disturbed motion and the coefficients of the quadratic form V are $\delta_{ij} = 1$ for $i = j$, $\delta_{ij} = 0$ for $i \neq j$.

On the basis of the inequality (10), the asymptotic stability of the trivial solution of system (6) for any value of the delay $\tau > 0$ follows from the positive-definite quadratic form V and the negative-definite quadratic form

$$\sum_{i,\, j=1}^{n} [\beta_{ij}(t) + \delta_{ij}\sqrt{h'(t)}]\zeta_i(t)\zeta_j(t). \qquad (11)$$

Second method. This method of obtaining a majorant for the bilinear form dV/dt can be used to estimate the influence of the size of the delay on the stability of the undisturbed motion.

It is obvious that the expression (8) for dV/dt can be written in the form

$$\frac{dV}{dt} = \sum_{i,\, j=1}^{n} \{[b_{ij}(t) + c_{ij}(t)] x_i(t) x_j(t) + c_{ij}(t) x_i(t)[x_j(t-\tau) - x_j(t)]\}. \qquad (12)$$

A known formula due to Lagrange then yields

$$x_j(t-\tau) - x_j(t) = -\tau \dot{x}_j(\sigma_j), \qquad (13)$$

where $\sigma_j = t - \theta_j\tau$, $0 < \theta_j < 1$.

If we substitute (13) in (12) and replace $\dot{x}_j$ by the right-hand side of the corresponding equation from the system (6), we obtain

$$\frac{dV}{dt} = \sum_{i,\, j=1}^{n} \Big\{[b_{ij}(t) + c_{ij}(t)] x_i(t) x_j(t) - \qquad (14)$$

$$-\tau\Big[x_i(t) c_{ij}(t) \sum_{s=1}^{n} \big(p_{js}(\sigma_j) x_s(\sigma_j) + q_{js}(\sigma_j) x_s(\sigma_j - \tau)\big)\Big]\Big\}.$$

A multilinear form is obtained for dV/dt, and the condition for asymptotic stability will be the negative-definiteness of this form for the condition (3). If we transform to canonical variables $y_1, \ldots, y_n$, in which $V = \sum_{i=1}^{n} y_i^2$, we obtain conditions that are equivalent, in the case considered, to the condition (3):

$$\sum_{i=1}^{n} y_i^2(\sigma_j) \leqslant \sum_{i=1}^{n} y_i^2(t), \qquad (15)$$

$$\sum_{i=1}^{n} y_i^2(\sigma_j - \tau) \leqslant \sum_{i=1}^{n} y_i^2(t) \quad (j = 1, \ldots, n).$$

The majorant for dV/dt can be obtained here by replacing the conditions (15) by the less stringent conditions

$$|y_i(\sigma_j)| \leqslant \sqrt{\sum_{i=1}^{n} y_i^2(t)},$$
$$|y_i(\sigma_j - \tau)| \leqslant \sqrt{\sum_{i=1}^{n} y_i^2(t)}. \quad (16)$$

If we substitute (16) in (14), we obtain, after some simple transformations, the inequality

$$\frac{dV}{dt} < \sum_{i,\, j=1}^{n} [a_{ij}(t) + \delta_{ij}\, \tau\omega(t;\, \tau)]\, y_i y_j, \quad (17)$$

where δ_{ij} is the Kroneker delta, and $\omega(t;\, \tau)$ is a known function of the coefficients of the system (6), of the coefficients of the quadratic form V, and of the magnitude of the delay τ.

The asymptotic stability criterion is obtained from the Sylvester conditions for the right-hand side of the inequality (17) to be negative.

4. Estimates of the Disturbances

The desired estimate cannot be directly obtained from the inequalities (10) or (17), since these inequalities were obtained by using condition (3) which is not satisfied by the estimate obtained in this way. We therefore start from the assumption that the desired estimate has the form

$$V \leqslant V_0 \exp \int_{t_0}^{t} \lambda(t)\, dt. \quad (18)$$

From this inequality follows the condition

$$V[t-\tau;\, x_1(t-\tau), \ldots, x_n(t-\tau)] \leqslant$$
$$\leqslant V[t;\, x_1(t), \ldots, x_n(t)] \exp \int_{t-\tau}^{t} -\lambda(t)\, dt, \quad (19)$$

which must replace condition (3) in the reasoning in Sec. 3.

If we use the notation $\varphi(t) = \exp \int_{t-\tau}^{t} -\lambda(t)\, dt$ and repeat the calculation in Sec. 3, we obtain for dV/dt the inequality, to replace (10),

$$\sum_{i,\, j=1}^{n} [\beta_{ij}(t) + \delta_{ij} \sqrt{\varphi(t)\, h'(t)}]\, \zeta_i \zeta_j \geqslant \frac{dV}{dt} \geqslant \sum_{i,\, j=1}^{n} [\beta_{ij}(t) + \delta_{ij} \sqrt{\varphi(t)\, h''(t)}]\, \zeta_i \zeta_j \quad (20)$$

or the inequality, to replace (17),

$$\frac{dV}{dt} < \sum_{i,\, j=1}^{n} \{a_{ij}(t) +$$
$$+ \delta_{ij} [\omega_1(t;\, \tau) \sqrt{\varphi(t)} + \omega_2(t,\, \tau) \sqrt{\psi(t)}]\, \tau\}\, y_i y_j, \quad (21)$$

where $\omega_1(t;\, \tau)$ and $\omega_2(t;\, \tau)$ are known functions of the same arguments occurring in the functions $\omega_1(t;\, \tau)$ [(17)] and

$$\psi(t) = \exp \int_{t-2\tau}^{t} -\lambda(t)\, dt.$$

In addition, in the case of the inequality (20), the function $\lambda(t)$ must not be less than the largest root of the equation

$$\det \| \beta_{ij}(t) + \delta_{ij} [\sqrt{\varphi(t)\, h'(t)} - \lambda] \| = 0. \quad (22)$$

In the case of the inequality (21), the function $\lambda(t)$ must be not less than the largest of the roots of the equation

$$\det \| a_{ij}(t) + \delta_{ij} \{\tau [\omega_1(t;\, \tau) \sqrt{\varphi(t)} +$$
$$+ \omega_2(t;\, \tau) \sqrt{\psi(t)}] - \lambda\} \| = 0. \quad (23)$$

We now obtain the inequality for determining the function $\lambda(t)$:

in the first case —

$$\lambda(t) \geqslant \bar{\mu}(t) - \sqrt{h'(t)} \exp\left[-\frac{1}{2} \int_{t-\tau}^{t} \lambda(t)\, dt\right], \quad (24)$$

in the second case —

$$\lambda(t) \geqslant \bar{\mu}(t) - \tau\left[\omega_1(t;\, \tau) \exp \frac{1}{2} \int_{t-\tau}^{t} -\right.$$
$$\left. -\lambda(t)\, dt + \omega_2(t;\, \tau)\ \exp \frac{1}{2} \int_{t-2\tau}^{t} -\lambda(t) dt\right]. \quad (25)$$

In both cases $\bar{\mu}(t)$ is the largest root of the equation

$$\det \| a_{ij}(t) - \delta_{ij}\, \mu \| = 0. \quad (26)$$

For determining the function $\lambda(t)$, we can now apply the known method of successive approximations.

EXAMPLES

The value of Lyapunov's direct method lies in the fact that at the present time it is difficult to find stability problems where the application of this method is doubtful. The results given below of the solution of particular problems are of an illustrative character, since these problems can be solved by other methods, and a comparison can be made of the results of the various methods of solution.

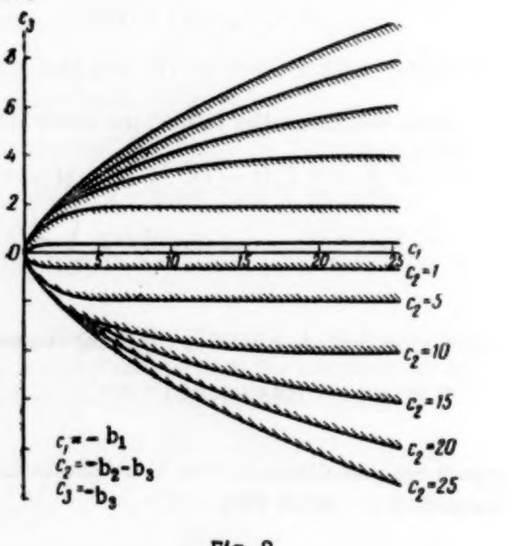

Fig. 2.

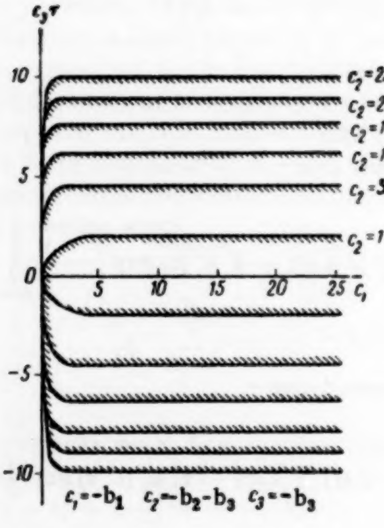

Fig. 3.

1. In Fig. 2 and Fig. 3 are shown the stability regions in the coefficient space for the equations of disturbed motion for the stability problem of a control system described by the differential equations

$$\frac{dx_1}{dt} = b_1x_1(t) + b_2x_2(t) + b_3x_2(t-\tau),$$
$$\frac{dx_2}{dt} = x_1(t). \tag{27}$$

The Lyapunov function for the given problem is taken to be the quadratic form $V = \alpha_{11}x_1^2 + 2\alpha_{12}x_1x_2 + \alpha_{22}x_2^2$, which satisfies the equation

$$\frac{dV}{dx_1}[b_1x_1 + (b_2+b_3)x_2] + \frac{\partial V}{\partial x_2}\cdot x_1 = -x_1^2 - x_2^2. \tag{28}$$

The corresponding stability regions, shown in Fig. 2 and Fig. 3, were obtained by using Sylvester's conditions for the sign-definiteness of the majorant of the bilinear form dV/dt obtained by the first and second methods given in Sec. 3.

2. In Fig. 4 we give the results of calculating the permissible delay in an automatic pilot, if the airplane is to be longitudinally stable. The equations for the disturbances in the motion of a plane with an automatic pilot in the case considered have the form

$$\frac{dx_1}{dt} = p_{11}x_1(t) + p_{12}x_2(t) + p_{13}x_3(t-\tau),$$
$$\frac{dx_2}{dt} = x_1(t), \tag{29}$$
$$\frac{dx_3}{dt} = p_{31}x_1(t) + p_{32}x_2(t).$$

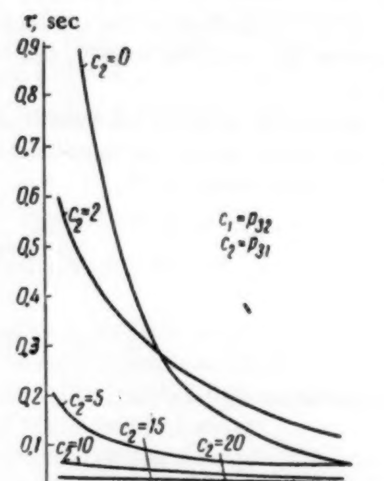

Fig. 4.

The Lyapunov function in this example is also taken to be the quadratic form $V = \alpha_{11}x_1^2 + 2\alpha_{12}x_1x_2 + \alpha_{22}x_2^2 + 2\alpha_{23}x_2x_3 + \alpha_{33}x_3^2$, which satisfies the equation

$$\frac{\partial V}{\partial x_1}(p_{11}x_1 + p_{12}x_2 + p_{13}x_3) + \frac{\partial V}{\partial x_2}x_1 + \frac{\partial V}{\partial x_3}(p_{31}x_1 + p_{32}x_2) = -(x_1^2 + x_2^2 + x_3^2). \tag{30}$$

The majorant for $\partial V/\partial t$ is obtained by using the second method in Sec. 3.

The results of the calculation show that the fact that the system is highly stable for $\tau = 0$ does not guarantee its stability in the presence of delay.

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SOME QUESTIONS CONCERNING COORDINATED CONTROL SYSTEMS

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A theorem concerning inverse transfer matrices is considered, which can be used to formulate the conditions of static independence of the control of several variables in a short form. Also considered are the conditions for dynamic independence of several nonlinear coordinated control systems.

1. The Theorem of Control Inverse Transfer Matrices

In the works of L. N. Voznesenskii [1, 2], there appears a theory of autonomous or independent control of several coordinated variables. In these works a general method is given for the determination of the transfer functions of control couplings. In designing control systems with several controlled variables, in particular, control systems for central power station turbines, we can apply the theorem proven below, which states that the matrix of machine couplings is the inverse of the matrix of control couplings. This theorem can be used not only for the simple determination of the transfer functions of the control couplings, but also for an easy verification of the correctness of the results obtained. We will prove this theorem as applied to an example of the control of three coordinated variables.

The equations for the control system will be assumed to be:

the equations for the machine (relative to the original equilibrium position)

$$\begin{aligned} T_{a1}\dot{\psi}_1 + c_{11}\mu_1 + c_{12}\mu_2 + c_{13}\mu_3 &= \lambda_1, \\ T_{a2}\dot{\psi}_2 + c_{21}\mu_1 + c_{22}\mu_2 + c_{23}\mu_3 &= \lambda_2, \\ T_{a3}\dot{\psi}_3 + c_{31}\mu_1 + c_{32}\mu_2 + c_{33}\mu_3 &= \lambda_3. \end{aligned} \tag{1}$$

the equations for the servomotors in the case of one amplification stage

$$\begin{aligned} T_{s1}\dot{\mu}_1 + \mu_1 - (d_{11}\eta_1 + d_{12}\eta_2 + d_{13}\eta_3) &= 0, \\ T_{s2}\dot{\mu}_2 + \mu_2 - (d_{21}\eta_1 + d_{22}\eta_2 + d_{23}\eta_3) &= 0, \\ T_{s3}\dot{\mu}_3 + \mu_3 - (d_{31}\eta_1 + d_{32}\mu_2 + d_{33}\eta_3) &= 0. \end{aligned} \tag{2}$$

In the equations (1) we have used the relative variables $\psi_n = \Delta q_n/\delta_n q_{n0} = \varphi_n/\delta_n$, for the machine, where φ_n is the usual relative machine variable. This is done in order to eliminate the nonuniformities δ_1, δ_2, and δ_3.

If we use the fact that for ideal regulators $\eta_1 = \psi_1$, $\mu_2 = \psi_2$, and $\eta_3 = \psi_3$, we can rewrite (2) in the form

$$\begin{aligned} T_{s1}\dot{\mu}_1 + \mu_1 - (d_{11}\psi_1 + d_{12}\psi_2 + d_{13}\psi_3) &= 0, \\ T_{s2}\dot{\mu}_2 + \mu_2 - (d_{21}\psi_1 + d_{22}\psi_2 + d_{13}\psi_3) &= 0, \\ T_{s3}\dot{\mu}_3 + \mu_3 - (d_{31}\psi_1 + d_{32}\psi_2 + d_{33}\psi_3) &= 0. \end{aligned} \tag{3}$$

The coupling coefficients c_{ik} in the equations (1) are for the internal machine coupling, while d_{ik} in Eq. (3) are for the external control coupling.

The problem of realizing statically independent control reduces to that of finding the values of the regulator-to-servomotor transfer functions d_{ik} for which the external control couplings disconnect the internal couplings of the machine. This problem has often been solved for special cases, and it has been solved in general form by L. N. Voznesenskii. In the most concise form this problem can be stated as the briefest theorem of inverse transfer matrices: the matrix $\|d_{ik}\|$ of the control couplings transfer functions that ensure static independence is the inverse of the matrix $\|c_{ik}\|$ of the internal machine couplings.

For the proof, we apply to the three machine variables in turn the following three systems of unit loads:

$$\begin{aligned} &1)\ \lambda_1 = 1,\ \lambda_2 = \lambda_3 = 0; \\ &2)\ \lambda_2 = 1,\ \lambda_1 = \lambda_3 = 0; \\ &3)\ \lambda_3 = 1,\ \lambda_1 = \lambda_2 = 0. \end{aligned} \tag{4}$$

For statically independent control, the static conditions for the first case must be

$$\begin{aligned} &1)\ \eta_1 = \psi_1 = \lambda_1 = 1, \\ &\eta_2 = \psi_2 = \lambda_2 = 0,\ \eta_3 = \psi_3 = \lambda_3 = 0. \end{aligned} \tag{5}$$

Similarly, in the second and third cases,

$$2)\ \eta_2 = 1,\ \eta_1 = \eta_3 = 0;$$

$$3)\ \eta_3 = 1,\ \eta_1 = \eta_2 = 0.$$

Static displacements of the servomotors are determined by the substitution of (5) in (3):

$$\begin{aligned} &1)\ \mu_1 = d_{11},\quad \mu_2 = d_{21},\quad \mu_3 = d_{31}; \\ &2)\ \mu_1 = d_{12};\quad \mu_2 = d_{22},\quad \mu_3 = d_{32}; \\ &3)\ \mu_1 = \mu_{13},\quad \mu_2 = d_{23},\quad \mu_3 = d_{33}. \end{aligned} \tag{6}$$

If we substitute (4) and (6) in (1), we obtain the static conditions for the first case:

$$1)\ c_{11}d_{11}+c_{12}d_{21}+c_{13}d_{31}=1,\quad c_{21}d_{11}+c_{22}d_{21}+c_{23}d_{31}=0,\quad c_{31}d_{11}+c_{32}d_{21}+c_{33}d_{31}=0. \tag{7}$$

Thus,

$$d_{11}=\frac{C_{11}}{\triangle_c},\quad d_{21}=\frac{C_{12}}{\triangle_c},\quad d_{31}=\frac{C_{13}}{\triangle_c}, \tag{8}$$

where Δ_c is the matrix determinant of $\|c\|$, and C_{ik} is its cofactor. Similarly, for the second and the third case, we obtain:

$$2)\ d_{12}=\frac{C_{21}}{\triangle_c},\quad d_{22}=\frac{C_{22}}{\triangle_c},\quad d_{23}=\frac{C_{32}}{\triangle_c};$$

$$3)\ d_{13}=\frac{C_{31}}{\triangle_c},\quad d_{23}=\frac{C_{32}}{\triangle_c},\quad d_{33}=\frac{C_{33}}{\triangle_c}.$$

We now note that, in the system (7), the subscripts of each term $c_{ik}d_{ki}$ are such that the subscripts ki of the unknown quantities are in the opposite order to the subscripts ik of the coefficients c_{ik}. Thus, the general expression for the transfer function d_{ik} can be written in the form $d_{ik}=C_{ik}/\Delta_c$. The matrix $\|c_{ik}\|$ is thus adjoint to the matrix $\|d_{ik}\|$. It follows from this by a known theorem from matrix theory that $\|d_{ik}\|=\|c_{ik}\|^{-1}$. The theorem is thus proven.

As a consequence of this theorem we have $|d_{ik}|=|c_{ik}|^{-1}$, or, in other notation, $\Delta d=\Delta c^{-1}$. This result yields the shortest method of checking whether the control transfer functions correspond to the integral machine couplings.

We will demonstrate the application of the theorem to the classic example for this problem, i.e., the problem of regulating a two-point extraction power station turbine by the application of the theory of independent control [1, 2].

We introduce the following notation: ω, p_1, and p_2 are the controlled variables; ω_0, p_{10}, and p_{20} are the nominal values of these variables; N_1, N_2, and N_3 are the powers of the three sections of the turbine; N_{10}, N_{20}, and N_{30} are the maximum powers of the three sections; D_1, D_2, and D_3 are the values of steam flow through the three sections; D_{10}, D_{20}, and D_{30} are the maximum steam flow through the sections; D_I and D_{II} are steam flow through separators; N_0, D_{I0}, and D_{II0} are the nominal electric and heat loads of the turbine; V_1 and V_2 are the volumes of the separators; γ_1 and γ_2 are the specific weights of the steam in the separators.

We write the equations for the machine in terms of absolute variables:

$$\begin{gathered} I\omega\frac{d\omega}{dt}=N_1+N_2+N_3-N_{\text{gen}} \\ V_1\frac{d\gamma_1}{dt}=D_1-D_2-D_{\text{I}}, \\ V_2\frac{d\gamma_2}{dt}=D_2-D_3-D_{\text{II}}. \end{gathered} \tag{9}$$

We shall rewrite these equations in terms of relative variables:

$$\begin{aligned} T_{a1}\dot{\psi}_1+c_{11}\mu_1+c_{12}\mu_2+c_{13}\mu_3&=0,\\ T_{a2}\dot{\psi}_2+c_{21}\mu_1+c_{22}\mu_2\qquad&=0,\\ T_{a3}\dot{\psi}_3\qquad+c_{32}\mu_2+c_{33}\mu_3&=0, \end{aligned} \tag{10}$$

where $\psi_1=\Delta\omega/\delta_1\omega_0$, $\psi_2=\Delta p_1/\delta_2 p_{10}$, $\psi_3=\Delta p_2/\delta_3 p_{20}$.

The coefficients c_{ik} are given by

$$\|c_{ik}\|=\begin{Vmatrix} c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & 0\\ 0 & c_{32} & c_{33}\end{Vmatrix}=\begin{Vmatrix} \frac{N_{10}}{N_0} & \frac{N_{20}}{N_0} & \frac{N_{30}}{N_0}\\ \frac{D_{10}}{D_{I0}} & -\frac{D_{20}}{D_{I0}} & 0\\ 0 & \frac{D_{20}}{D_{I0}} & -\frac{D_{30}}{D_{II0}}\end{Vmatrix} \tag{11}$$

The matrix $\|d_{ik}\|$ of the control transfer functions is uniquely defined by the inverse matrix theorem $\|d_{ik}\|=\|c_{ik}\|^{-1}$, and

$$\triangle_c=\frac{N_{10}}{N_0}\frac{D_{20}}{D_{I0}}\frac{D_{30}}{D_{II0}}+\frac{D_{10}}{D_{I0}}\frac{D_{20}}{D_{II0}}\frac{N_3}{N_0}+\frac{D_{10}}{D_{I0}}\frac{N_{20}}{N_0}\frac{D_{30}}{D_{II0}},$$

$$d_{11}=\frac{C_{11}}{\triangle_c}=\frac{\frac{D_{20}}{D_{I0}}\frac{D_{30}}{D_{II0}}}{\triangle_c},$$

$$d_{12}=\frac{C_{21}}{\triangle_c}=\frac{\frac{N_{20}}{N_0}\frac{D_{30}}{D_{II0}}+\frac{N_{30}}{N_0}\frac{D_{20}}{D_{II0}}}{\triangle_c},$$

$$d_{13}=\frac{C_{13}}{\triangle_c}=\frac{\frac{N_{30}}{N_0}\frac{D_{20}}{D_{I0}}}{\triangle_c}, \tag{12}$$

$$d_{21}=\frac{C_{12}}{\triangle_c}=\frac{\frac{D_{10}}{D_{I0}}\frac{D_{30}}{D_{II0}}}{\triangle_c},$$

$$d_{22}=\frac{C_{22}}{\triangle_c}=-\frac{\frac{N_{10}}{N_0}\frac{D_{30}}{D_{II0}}}{\triangle_c},\quad d_{23}=\frac{C_{32}}{\triangle_c}=-\frac{\frac{N_{30}}{N_0}\frac{D_{10}}{D_{I0}}}{\triangle_c}.$$

As is to be expected, these more simply expressed values of the transfer functions coincide with those obtained by I. N. Voznesenskii [2, 3]. What is new here is the compact form of the entire derivation in the form of the inverse matrix theorem, and also the consequent possibility of thoroughly verifying the final result by using the relation $\Delta d\times\Delta c=1$.

It is obvious from the proof of the theorem that only after the transformation to the relative coordinates ψ

are the matrices $\|c\|$ and $\|d\|$ related by the inverse matrix theorem. The matrices $\|i\|$ and $\|k\|$ [2, 3], reduced in [2], with respect to nonrelative variables, do not satisfy the inverse matrix theorem. The reason for the transformation to relative variables is thus made clear.

For the APT-12, one of the two-point extraction turbines produced in this country, with parameters $t_0 = 435°C$, $p_0 = 35$ atm, $p_1 = 10$ atm, $p_2 = 1.2$ atm, $p_k = 0.05$ atm, the flows through the sections are $D_{10} = 130{,}000$ kg/hr, $D_{20} = 80{,}000$ kg/hr, and $D_{30} = 40{,}000$ kg/hr.

The heat drops used are:

$H_1\eta_1 = 55$ kcal/kg, $H_2\eta_2 = 93$ kcal/kg, $H_3\eta_3 = 81$ kcal/kg.

The maximum powers of the sections are

$$N_{10} = \frac{D_{10}H_1\eta_1}{860} = 8320 \text{ kw} \qquad N_{20} = \frac{D_{20}H_2\eta_2}{860} = 8650 \text{ kw}$$

$$N_{30} = \frac{D_{30}H_3\eta_3}{860} = 3770 \text{ kw}$$

The nominal loads are

$$N_0 = 12{,}000 \text{ kw}, \; D_{10} = 50{,}000 \text{ kg/hr}, \; D_{II_0} = 40{,}000 \text{ kg/hr}$$

The coefficients $\underline{c}$, obtained by using the formulas (11), are

$$\begin{aligned} &c_{11} = 0.69, \quad c_{12} = 0.72, \quad c_{13} = 0.31, \\ &c_{21} = 2.6, \quad c_{22} = -1.6, \quad c_{23} = 0.0, \\ &c_{31} = 0.0, \quad c_{32} = 2.0, \quad c_{33} = -1.0 \end{aligned}$$
and $\triangle_c = 4.581$.

The coefficients $\underline{d}$, calculated from formula (12), are

$$\begin{aligned} &d_{11} = 0.35, \quad d_{12} = 0.293, \quad d_{13} = 0.108, \\ &d_{21} = 0.568, \quad d_{22} = -0.151, \quad d_{23} = 0.176, \\ &d_{31} = 1.135, \quad d_{32} = -0.31, \quad d_{33} = -0.6\text{[illegible]} \end{aligned}$$
and $\triangle_d = 0.218$.

Thus, $\Delta d \times \Delta c = 1$.

The fact that the product $\Delta d \Delta c$ is equal to one verifies the results of the calculation.

2. The Dynamic Independence of Certain Nonlinear Coordinated Control Systems

A. Possible methods for the control of central power station turbines

The central problem of contemporary turbine control is to find methods of increasing the rapidity of action of control systems without a corresponding increase in oil-pump capacities and a further expansion of the oil-systems. The importance of this problem is shown on the one hand by the continual decrease in the time of acceleration of the turbine rotors and the growth in the control forces with the consequent increase in the operating volumes of the servomotors and, on the other hand, by the impossibility of any further development in the oil-system, which has already grown in some cases to abnormal proportions.

The importance of the problem of increasing the rapidity of action is greatest for the turbines in central power stations, where oil pumps with limited capacity are called on to ensure the supply for a complex control system which, for a two-point extraction turbine, contains three main servomotors, and where all the difficulties are consequently increased. The conditions of dynamic independence demand the maintenance of equality of the time for the three servomotors [2]. If the operating volume for each servomotor is 10 dm^3, and the time for each servomotor is 0.25 sec (which is usual for condensing turbines), then the oil-pump capacity needed to supply only the main servomotors amounts to 430 m^3/hr, and this is practically unattainable.

In uncoordinated control of two-point extraction turbines, the problem is made easier by the fact that the control of the pressures, in the separators—in particular, of the pressure in the second separator—can be rather slow, and the second and third servomotor times can be rather large (Fig. 1a$_1$). In coupled independent autonomous control, the times of all three servomotors must be equal (Fig. 1a$_3$). This is the condition for dynamic independence. As was shown in detail by P. E. Boloban in [4], the working of statically independent systems is not significantly worsened by using servomotors with essentially different times, which are used in the case of uncoordinated control (Fig. 1a$_2$). It is shown [4] that such control is sufficiently stable. The mutual disturbances of the variables also reach significant values. The best results are obtained with a solution due to A. V. Shcheglyaev (Fig. 1b$_2$). Here servomotors with different times are also used, but the supply of oil to the servomotors is limited, not by the opening in the valves, but by different diaphragms in the supply lines, the servomotors being identical. This leads to nonlinearity of the servomotors. We will compare the possible systems of coordinated control using the same over-all oil supply $Q_{x\,max}$. In systems with three different servomotors, this quantity is

$$Q_{x\,max} = \frac{V_1}{T_{s1}} + \frac{V_2}{T_{s2}} + \frac{V_3}{T_{s3}}.$$

In systems with identical servomotors (Fig. 1a$_3$, b$_3$), we have $Q'_{x\,max} = 1/T_s\,(V_1 + V_2 + V_3)$. In order to satisfy the condition $Q_{x\,max} = Q'_{x\,max}$, we must have

$$\frac{V_1}{T_{s1}} + \frac{V_2}{T_{s2}} + \frac{V_3}{T_{s3}} = \frac{1}{T_s}(V_1 + V_2 + V_3).$$

Assuming all the servomotors identical, i.e., $V_1 + V_2 = V_3$, we obtain

$$\frac{1}{T_{s1}} + \frac{1}{T_{s2}} + \frac{1}{T_{s3}} = \frac{3}{T_s}.$$

In Fig. 1a$_2$ we have taken the following relation between the servomotor times: $T_{s1} : T_{s2} : T_{s3} = 1 : 3 : 9$.

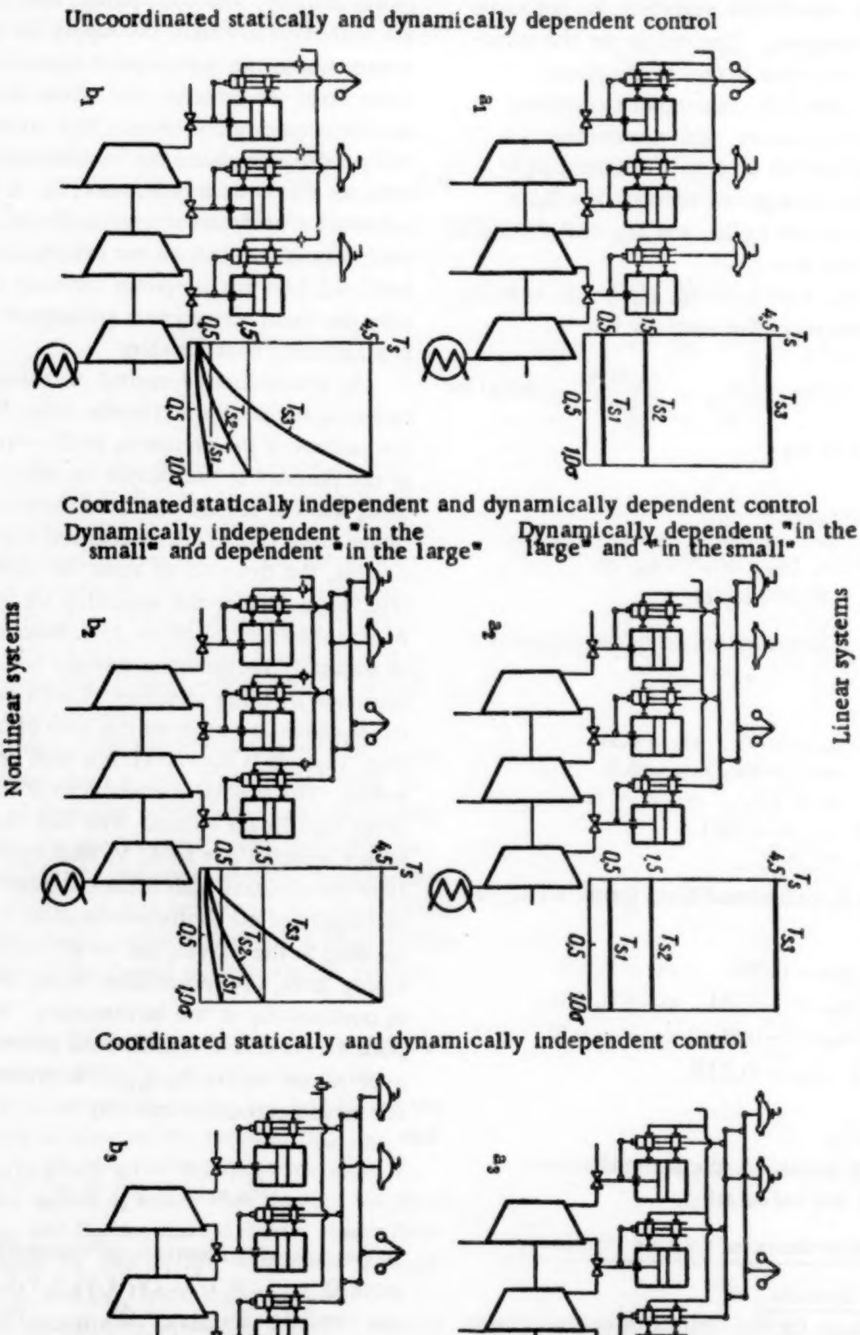

Fig. 1. Possible method of control of central power station turbines.

In addition $1/T_{s1} + 1/3T_{s1} + 1/9T_{s1} = 3/T_{s1}$ or $13/9\,T_{s1} = 3 \cdot 1/T_s$; $T_{s1} = 13/27\,T_s$. If we take the times T_s for the identical servomotors in the system in Fig. 1 to be 1 sec, then $T_{s1} = 13/27$ sec ≈ 0.5 sec, $T_{s2} = 3T_{s1} \approx 1.5$ sec, $T_{s3} \approx 4.5$ sec. These relations are shown in Fig. 1.

Thus, even a very large difference in the servomotor times – in the ratio of 1 : 3 : 9 – with the same maximum oil supply, leads to a decrease in the time of the first servomotor of only 50% (0.5 sec instead of 1.0 sec). However, the time for the third servomotor increases to 4.5 times its value. These results are very unsatisfactory, and we are forced to look for other solutions to the problem.

In A. V. Shcheglyaev's method (Fig. 1b$_2$) a higher oil-pump output was used than in the first method (Fig 1a$_2$). Moreover, A. V. Shcheglyaev's systems, from the point of view of stability, are dynamically independent, since for small oscillations (small σ) their servomotor times are equal. Deviations from dynamic independence occur only for large disturbances, and in this case they are significantly large.

Dynamic independence "in the small" and "in the large" in a nonlinear system in which the rapidity of action approximates that for a system with instantaneous servomotor action, is attained in the method of control proposed by the author and described below. In this method, a common diaphragm in the supply lines of the main servomotors is used.

B. Coordinated control with a common diaphragm in the supply lines of the main servomotors

Let all three main servomotors of a central power station turbine have identical performances (i.e., identical times, identical operating volumes, and identical valves), and let the total calculated operating sections of all three valves be connected to the common main line supplying all three servomotors, in the shape of a diaphragm (Fig. 1b$_3$), the valve cross sections being as great as possible within the permissible limits of construction. Since in this case the pressures ahead of all three servomotors (the pressure beyond the diaphragm) are the same, the times of all three servomotors are the same at any given instant of time. These times will vary, however, as functions of the total opening of all three valves $\Sigma\sigma + \sigma_1 + \sigma_2 + \sigma_3$, and the system is consequently dynamically nonlinear.

The necessary and sufficient condition for dynamical independence in a nonlinear system, as given by I. N. Voznesenskii, is the equality of the servomotor times. The conditions for dynamic independence of nonlinear systems have not yet been considered in general. It can be proven, however, that in the case of such nonlinear systems, where the servomotor times are functions of μ, η, and $\sigma = \eta - \mu$, the equality of the times of the corresponding servomotors at every instant of time will also be the condition of dynamic independence. The mode of variation of the times as functions of $\Sigma\sigma_x = \sigma_{1x} + \sigma_{2x} + \sigma_{3x}$ is shown in Fig. 2a for various values of the ratio $\alpha = R_3/R_0$, where R_3 is the total resistance of the fully open valves ($\Sigma\sigma = 3$) of all three servomotors ($R_3 = 1/9\ R_1$), R_0 is the resistance of the diaphragm, T_{s0} is the time of servomotor for fully open values and no diaphragm, and $T_{sx\ max}$ is the value of T_{sx} for fully open valves ($\sigma_1 = \sigma_2 = \sigma_3 = 1$). A decrease in α brings the control conditions closer to those of instantaneous operation of the servomotors. The smaller α, the greater the extent to which the diaphragm becomes the only resistance limiting the discharge, since downstream from the valve, there only remain functions of the diaphragm activation, i.e., of limiting the discharge through it.

In Fig. 2b the dependence of the speed of the servomotor and of the oil discharge on $\Sigma\sigma$ is shown for various

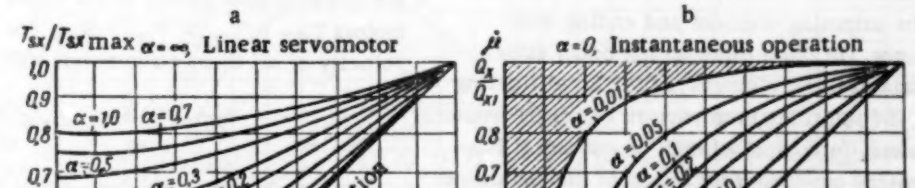

Fig. 2. Nonlinear servomotor characteristics.

$$a - T_{sx} = T_{s0}\sqrt{\frac{(\Sigma\sigma)^2 + 18\alpha}{18\alpha}},\quad T_{sx}\ \max = T_{s0}\sqrt{\frac{1+2\alpha}{2\alpha}};\ \dot{b} - \dot{\mu} = \frac{\sigma}{T_{sx}},\ Q_x = \dot{\mu}Q_{x1}$$

values of α. The shaded area represents the unused capacity of the oil pump for $\alpha = 0.01$. The calculation of the relations shown in Fig. 2 was carried out for a constant maximum oil discharge.

The advantages of nonlinear control with a common diaphragm, as found in the VTI system, are demonstrated by the investigation of the transfer process in an APT-12 turbine.

C. The dynamic independence of systems with nonlinear servomotors with a common diaphragm in the supply feed line.

It is known that the condition for static independence is the static linearity of the machine and static linearity of the coupling system, i.e., the constancy of all the transfer functions. I. N. Voznesenskii has shown that the condition for dynamic independence in a linear system is the equality of the servomotor times. This condition is obtained under the assumption not only of static, but also of dynamic linearity, i.e., the constancy of the dynamic parameters — the times of the machine and of the servomotors. In connection with the proposed nonlinear solution, in which the servomotor times, although they are equal one to the other at any instant of time, all vary as functions of the sum $\sigma_1 + \sigma_2 + \sigma_3$, the question arises as to whether dynamic independence can exist in such a dynamically nonlinear system, and under what conditions. In order to answer this question, it is necessary to consider the question of dynamic independence in its general form.

It should be noted that the conditions of dynamic independence have been obtained up to the present time [2, 3] from a consideration of the combined system of equations of a closed-loop system. This leads to unnecessary complication. All the conditions of dynamic independence are localized in series open-loop servosystems, and the appearance of these systems in control with power amplification involves these supplementary conditions. Not all of the amplification loop of each of the separate systems enters these systems, but only a part, beginning with the summing element and ending with the main servomotor. In them there do not occur either the controlled variables, the elements of the amplification loops preceding the summing elements, or the distribution elements. Therefore, in control without power amplification, no supplementary conditions for dynamic independence arise, and static independence is the condition for dynamic independence. This occurs because there are no servosystems with new degrees of freedom included between the distributing elements (the regulators in the case considered) and the valves (the summing elements here), and the valves are controlled directly by the regulators with the necessary transmission ratios. In the same way, supplementary conditions of dynamic independence would also be absent if the main servomotors were the distributing elements. This method, although theoretically ideal, is unsuitable in practice, since it leads to an excessively large weight of the lever coupling mechanism.

With the appearance of control with power amplification of servosystems, which are included between the distributing elements (not coinciding with the main servomotors) and the valves, only one supplementary condition must arise — the servosystems must be arranged so that the valves are controlled as before in accordance with the necessary transmission ratios. When this condition is satisfied, the undistorted transfer of pulses from the regulators to the servomotors is ensured for the same assigned ratios when new degrees of freedom—servomotors of the follow-up systems — are introduced. We will consider the conditions of such an undistorted transfer.

Let η_1, η_2, η_3 be the pulse-distributing elements, and μ_1, μ_2, μ_3 be the pulse-summing elements. In the consideration of the general case, we will assume that η_1, η_2, and η_3 are, respectively, the arbitrary functions $\varphi_1(t)$, $\varphi_2(t)$, and $\varphi_3(t)$ of the time.

The equations for μ_1, μ_2, and μ_3 are

$$\begin{aligned} T_{s1}\dot{\mu}_1 + \mu_1 - (d_{11}\eta_1 + d_{12}\eta_2 + d_{13}\eta_3) = 0, \\ T_{s2}\dot{\mu}_2 + \mu_2 - (d_{21}\eta_1 + d_{22}\eta_2 + d_{23}\eta_3) = 0, \\ T_{s3}\dot{\mu}_3 + \mu_3 - (d_{31}\eta_1 + d_{32}\eta_2 + d_{33}\eta_3) = 0. \end{aligned} \tag{13}$$

The times T_s are here the above-mentioned function of $\Sigma\sigma$. Let any of the regulators, for example η_1, suffer a very small displacement. This causes valve displacements

$$\begin{aligned} \triangle\sigma_1 = d_{11}\triangle\eta_1, \\ \triangle\sigma_2 = d_{21}\triangle\eta_1, \\ \triangle\sigma_3 = d_{31}\triangle\eta_1. \end{aligned} \tag{14}$$

For very small variations of $\Delta\eta_1$, and correspondingly small values of $\Delta\sigma_1$, $\Delta\sigma_2$, and $\Delta\sigma_3$, we can assume that the pressure beyond the diaphragm is constant. Since the pressure in front of all three servomotors is identical and equal to the pressure behind the common diaphragm, the instantaneous values of the times of all three servomotors $T_{s1x} = T_{s2x} = T_{s3x} = T_{sx}$, are also identical. The velocity of all three servomotors will be

$$\begin{aligned} \dot{\mu}_1 = \frac{\triangle\sigma_1}{T_{sx}} = d_{11}\frac{\triangle\eta_1}{T_{sx}}, \\ \dot{\mu}_2 = \frac{\triangle\sigma_2}{T_{sx}} = d_{21}\frac{\triangle\eta_1}{T_{sx}}, \\ \dot{\mu}_3 = \frac{\triangle\sigma_3}{T_{sx}} = d_{31}\frac{\triangle\eta_1}{T_{sx}}. \end{aligned} \tag{15}$$

The displacement of all three servomotors during the interval of time Δt will be

$$\begin{aligned} \triangle\mu_1 = \dot{\mu}_1\triangle t = d_{11}\frac{\triangle\eta_1\triangle t}{T_{sx}}, \\ \triangle\mu_2 = \dot{\mu}_2\triangle t = d_{21}\frac{\triangle\eta_1\triangle t}{T_{sx}}, \\ \triangle\mu_3 = \dot{\mu}_3\triangle t = d_{31}\frac{\triangle\eta_1\triangle t}{T_{sx}}. \end{aligned} \tag{16}$$

In each small interval of time, the displacement of the servomotors will have the ratios $\Delta\mu_2/\Delta\mu_1 = d_{21}/d_{11} = j_{21}$, $\Delta\mu_3/\Delta\mu_1 = d_{31}/d_{11} = j_{31}$, and this ensures the dynamic independence. The displacements μ_1, μ_2, and μ_3 of the servomotors for the regulator control input η_1, in a finite interval of time expressed in terms of the same ratios, are obtained by integrating the relations (16):

$$\mu_1 = \int \dot{\mu}_1 dt = d_{11} \int \frac{\eta_1}{T_{sx}} dt,$$

$$\mu_2 = \int \dot{\mu}_2 dt = d_{21} \int \frac{\eta_1}{T_{sx}} dt, \tag{17}$$

$$\mu_3 = \int \dot{\mu}_3 dt = d_{31} \int \frac{\eta_1}{T_{sx}} dt.$$

The equality of the times of all three servomotors at every instant of time is thus a sufficient condition for dynamic independence. The times do not have to be constant, but can be arbitrary functions of μ and η, i.e., the system can be dynamically nonlinear.

A system with a common diaphragm in the supply line satisfies these conditions. This system is nonlinear, and in it $T_{s1} = T_{s2} = T_{s3} = f(\Sigma\sigma)$; it is, however, dynamically independent of the equality of the servomotor times, and is the most direct and radical solution of the problem of dynamic independence. If the supply of oil to the servomotors is limited by the maximum capacity of the pump, this system permits operation with partially open servomotors with significantly smaller times than in the case of fully open servomotors. In this system, the constancy of the sum of the velocities of all three servomotors is ensured, while the separate velocities are determined by the transfer functions. This has, in general, the same advantages as operation in parallel. Thanks to the constancy of the velocity sum, the peaks in the amount of oil necessary for one servomotor are covered by the smaller amounts necessary for the others. This also guarantees the greatest possible operating speed for a given pump capacity, if the dynamic independence is strictly preserved.

D. Transient processes in central power plant turbines

If the stability conditions of a certain nonlinear system could be investigated analytically in closed form, then the transient processes occurring in it could be determined only by numerical integration at the present time. The accuracy of such an integration was verified by comparison with the analytic solution.

In Fig. 3 is shown the transient process in a system of coordinated control of a two-point extraction turbine with nonlinear servomotors. The continuous curve shows the process for the times of the main servomotors $T_{s1\,max} = T_{s2\,max} = T_{s3\,max} = 1$ sec. The curve is not drawn for the nominal electric load drop ($\lambda = 1$) ($N_0 = 12{,}000$ kw), since this load was beyond the limits of independent control, but for the maximum electric load within the limits of independence ($\lambda = 0.88$).

The equations of the machine in this case are

$$T_{a1}\dot{\varphi}_1 + c_{11}\mu_1 + c_{12}\mu_2 + c_{13}\mu_3 = 0;$$

$$T_{a2}\dot{\varphi}_2 + c_{21}\mu_1 + c_{22}\mu_2 + c_{23}\mu_3 = 0;$$

$$T_{a3}\dot{\varphi}_3 + c_{31}\mu_1 + c_{32}\mu_2 + c_{33}\mu_3 = 0.$$

The equations of the regulators (ideal) are

$$\eta_1 = \varphi_1;\ \ \eta_2 = \varphi_2;\ \ \eta_3 = \varphi_3.$$

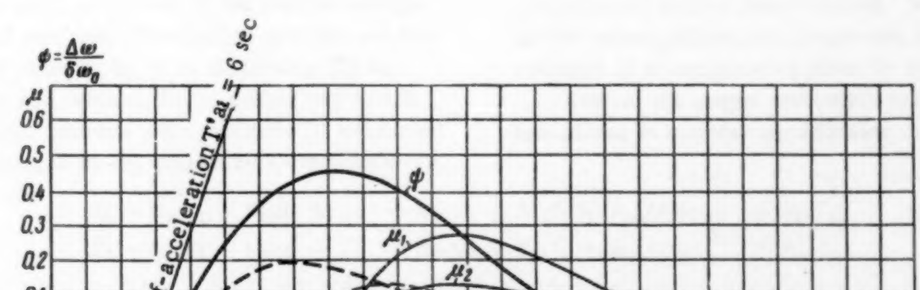

Fig. 3. The transient process in the control system of a two-point extraction turbine with nonlinear servomotors.

The servomotor equations:

$$T_{s1}\dot{\mu}_1 + \mu_1 - (d_{11}\eta_1 + d_{12}\eta_2 + d_{13}\eta_3) = 0;$$
$$T_{s2}\dot{\mu}_2 + \mu_2 - (d_{21}\eta_1 + d_{22}\eta_2 + d_{23}\eta_3) = 0;$$
$$T_{s3}\dot{\mu}_3 + \mu_3 - (d_{31}\eta_1 + d_{32}\eta_2 + d_{33}\eta_3) = 0.$$

The numerical values of the coefficients c_{ik}:

$$c_{11} = 0.69, \quad c_{12} = 0.72, \quad c_{13} = 0.31,$$
$$c_{12} = 2.6, \quad c_{22} = -1.6, \quad c_{23} = 0,$$
$$c_{13} = 0. \quad c_{32} = 2.0, \quad c_{33} = -1.0.$$

The numerical values of the coefficients d_{ik}:

$$d_{11} = 0.85, \quad d_{12} = 0.293, \quad d_{13} = 0.108,$$
$$d_{21} = 0.568, \quad d_{22} = -0.151, \quad d_{23} = 0.176,$$
$$d_{31} = 1.135, \quad d_{32} = -0.31, \quad d_{33} = -0.165.$$

The times of the machine:

$$T_{a1} = \delta_1 T'_{a1} = 0.05 \cdot 6 = 0.3 \text{ sec},$$
$$T_{a2} = \delta_2 T'_{a2} \approx 0.1 \cdot 1.95 \approx 0.2 \text{ sec},$$
$$T_{a3} = \delta_3 T'_{a3} \approx 0.2 \cdot 1.95 \approx 0.4 \text{ sec}.$$

The transient process in Fig. 3 shows a strict static and dynamic independence of control in this nonlinear system. Because of the application of servomotors with large nonlinearities ($\alpha = 0.05$) which were similar to servomotors with constant velocity, the overshoot of the number of revolutions was $\Delta\psi = 0.45$ or, in absolute values, $\Delta n = \Delta\psi\delta\omega_0 = 67.5$ rpm. For $T_{a1} = 6$ sec and $T_{s\,max} = 1.0$ sec, i.e., for the maximum pump output $Q_{x\,max} = 91\ m^3/hr$, such an overshoot is evidence of quick response.

The broken curves in Fig. 3 show the drop of the same load for $T_{s\,max} = 0.65$ sec, which corresponds to $Q_{x\,max} = 120\ m^3/hr$. These values were also taken in the VTI project. To them corresponds a dynamic overshoot of the number of revolutions less than 1%.

In the coordinated control of several variables, there are therefore nonlinear systems that use the capacity of the oil-pump reasonably fully, and which, relative to the use of this capacity, are close to systems with instantaneous servomotor actuation. Static and dynamic independence are maintained in such systems of coordinated control. It must be assumed that in the coordinated control of several variables, nonlinear systems must be used in solving the problems that arise.

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ON THE PRINCIPLES OF SYNTHESIZING AUTOMATIC SYSTEMS WITH MANY CONTROLLED QUANTITIES

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The paper suggests principles for synthesizing systems with many controlled variables. The basic mathematical relationships for combined-type automatic systems are derived in matrix form.

It is shown that the use of the principles of physical realizability, invariance, and optimality permit one to select a controller's operator coefficients (elements of matrices B and C) for multiloop systems.

Solution of the problem of the complex automation of production is inseparably related to the development of methods for synthesizing systems with many controlled quantities.

The widespread use of complicated systems for the simultaneous control of several physical quantities is a result of the tendency to get the greatest energy or most economical output from the employment of complicated objects (plants).

It is convenient, in analyzing and synthesizing systems with many variables, to use matrix and operational calculi. The use of matrix calculus allows one to give the solution of the synthesis and analysis problems compactly and elegantly.

The importance of the use of the matrix calculus for analysis and synthesis of multiloop systems was first asserted, in our country, by V. S. Kulebakin [1] and N. N. Luzin [2] and, abroad, by A. Boksenbom and R. Hood [3]. Matrix methods were successfully employed in automatic control theory abroad by Ts'ien Süeh-sen [4], M. Colomb and E. Usdin [5], R. L. Kavanagh [6], and, in our country, by M. V. Meerov [7], P. V. Bromberg [8], E. I. Baranchuk [9], and others.

We now formulate the synthesis problem for systems with many controlled quantities. Let there be a complicated plant, i.e., a plant in which there flow many processes which are subject to control. We denote by $x_1, x_2, \ldots, x_n$ the plant's output coordinates. These physical quantities characterize the plant's state. In the general case, two types of stimuli act on the plant. On the one hand, there are the external, uncontrolled, stimuli $f_1, f_2, \ldots f_n$ which disturb the desirable state of the plant and, on the other, the so-called controlling (regulated) stimuli $y_1, y_2, \ldots, y_n$ by means of which the disturbed state is reconstituted. With account taken of the nomenclature just introduced, we can present the structure of a complicated plant by the scheme of Fig. 1.

Each of the output controlled coordinates x_i can be considered as a linear combination of the input quantities:

$$\begin{aligned} x_1 &= a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n + h_{11}f_1 + h_{12}f_2 + \ldots + h_{1n}f_n, \\ x_2 &= a_{21}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n + h_{21}f_1 + h_{22}f_2 + \ldots + h_{2n}f_n, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_n &= a_{n1}y_1 + a_{n2}y_2 + \ldots + a_{nn}y_n + h_{n1}f_1 + h_{n2}f_2 + \ldots + h_{nn}f_n. \end{aligned} \quad (1)$$

The quantities a_{ij} and h_{rk} are polynomials in the operator $D = d/dt$ of degree not higher than the second. They reflect the effect of the input quantities y_i and f_i on the corresponding output quantities.

By using matrix notation, we can write (1) in the form

$$X = AY_{ob} + HF,$$

where

$$X = \begin{Vmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Vmatrix}, \quad Y = \begin{Vmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Vmatrix}, F = \begin{Vmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{Vmatrix}, \quad A = \begin{Vmatrix} a_{11}a_{12} \ldots a_{1n} \\ a_{21}a_{22} \ldots a_{2n} \\ \ldots\ldots \\ a_{n1}a_{n2} \ldots a_{nn} \end{Vmatrix}, \quad H = \begin{Vmatrix} h_{11}h_{12} \ldots h_{1n} \\ h_{21}h_{22} \ldots h_{2n} \\ \ldots\ldots \\ h_{n1}h_{n2} \ldots h_{nn} \end{Vmatrix}$$

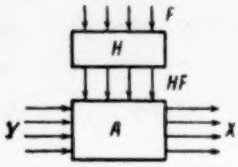

Fig. 1. Block schematic of an object with many parameters. A is an operator matrix whose elements characterize the object's natural motions; H is an operator matrix whose elements characterize the transformation of the external stimuli F to the form X with account taken of A; X, Y, F are column matrices whose elements are the coordinates, respectively, of the controlled (x_i), controlling (y_i) and disturbing (f_i) quantities.

Since the quantities f_i vary arbitrarily, one should then expect that, for a definite initial state of the plant and unchanged values of y_i (for an unchanged position of the controlling organs), the x_i will vary arbitrarily.

The attempt to obtain a desirable character of the variations of quantities x_i leads either to an auxiliary transformation of matrix A or to such a preliminary variation of the inputs y_i that, with the previous matrix A, a desirable variation of the x_i will be obtained. Intervention in the structure of matrix A is undesirable, since the plant ordinarily presents itself as an invariable system element.

Consequently, there remains one possibility, namely, to introduce controlling stimuli (the coordinates y_i) by such laws that the resulting x_i will be desirable. In other words, instead of the plant matrix A, one must find a system matrix W which includes A ($A \subset W$) such that the previous inputs y_i and f_i will result in a desirable temporal variation of the quantities x_i.

We denote the controller matrices by B and C. Then, the controlling action of the controller on the plant can be written in the form

$$Y_{reg} = BF + CX, \tag{2}$$

where

$$B = \begin{Vmatrix} b_{11} b_{12} \dots b_{1n} \\ b_{21} b_{22} \dots b_{2n} \\ \dots\dots \\ b_{n1} b_{n2} \dots b_{nn} \end{Vmatrix}, \quad C = \begin{Vmatrix} c_{11} c_{12} \dots c_{1n} \\ c_{21} c_{22} \dots c_{2n} \\ \dots\dots \\ c_{n1} c_{n2} \dots c_{nn} \end{Vmatrix}$$

The comparison condition for the required controlling stimulus of the object (plant) Y_{ob} and the intended Y_{reg} is

$$Y = Y_{ob} = -Y_{reg} \tag{3}$$

which is simultaneously the condition for a closed system.

Therefore, the definitive equations for the complicated plant can be written in the form

$$X = AY + HF, \quad Y = -BF - CX. \tag{4}$$

By multiplying the second equation on the left by matrix A and then substituting into the first equation, we obtain

$$X = -ABF - ACX + HF$$

or

$$(E + AC)X = (H - AB)F,$$

where

$$E = \begin{Vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 1 \end{Vmatrix}$$

Equation (4) is the generalization of an ordinary differential equation for a single-loop automatic system to the case of a multiloop system of the combined type (Fig. 2).

For a multiloop servosystem (Fig. 3), Eq. (4) will have the form

$$(E + AC)X = ACG \tag{5}$$

or

$$(E + AC)X = ACG + HF.$$

For a multiloop stabilization system ($B \equiv O$) (Fig. 4):

$$(E + AC)X = HF. \tag{6}$$

All our further discussion will have to do with systems of the combined type.

If the inverse matrix $(E + AC)^{-1}$ exists, then

$$X = (E + AC)^{-1}(H - AB)F$$

or

$$X = WF,$$

where

$$W = (E + AC)^{-1}(H - AB). \tag{7}$$

From the synthesis point of view, the matrices A, H, F, and X are given, and the matrices B and C are unknown.

Since there are two unknowns and only one equation, it is necessary to find an additional condition relating the coefficients of matrices A, B, C, H, F, and X. Such conditions can be found on the basis of the following principles: the principle of realizability, the principle of invariance, and the principle of optimality.

We first explain the essential points of these principles, and then go on to show how it is practical to use them for obtaining the required additional relationships between B, C, A, H, F, and X.

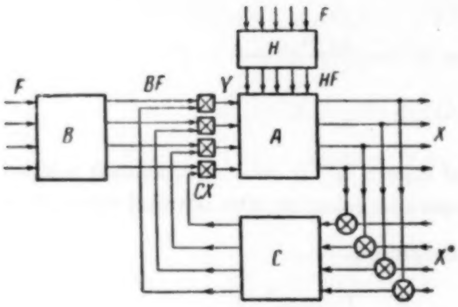

Fig. 2. Block schematic of a multiloop combined system. A, H, F, X, Y are the same as in Fig. 1; B is an operator matrix whose elements implement the transformation of F to Y (the controller matrix for disturbances); C is an operator matrix whose elements implement the transformation of the deviations X to CX (controller matrix for deviations); X^0 is a column matrix whose elements are the nominal output coordinates CX. ⊠ is the symbol for summation of the controlling stimuli; ⊗ is the symbol for summation of the output coordinates and their nominal values.

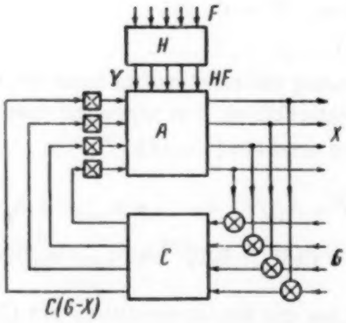

Fig. 3. Block schematic of a multiloop servosystem. A, H, C, X, Y, F are the same as in Figs. 1 and 2; G is a column matrix whose elements are the coordinates of the controlling actions on the system.

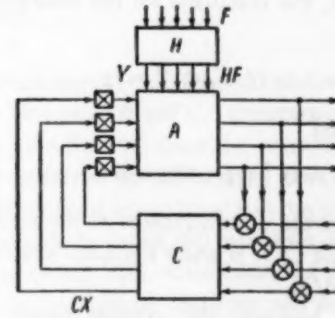

Fig. 4. Block schematic of a multiloop stabilization system. A, H, C, X, Y, X^0 are the same as on Fig. 2.

We turn first to the principle of realizability. In creating a controller, the designer must be certain that it is possible to realize such a controller. First, there must be certainty that the controller is capable of functioning in general and, second, there must be certainty that the controller can operate within given limits under the conditions of physical realizability (conditions of stability, speed limitations, overload limitations).

From the mathematical point of view, the realizability condition is equivalent to the existence of the inverse matrix $(E + AC)^{-1}$. To find the inverse matrix, we can use the efficient methods of matrix theory [10, 11]. Thus, for example, the bordering method [12] is very convenient. According to this method, the nth-order inverse matrix $M_n^{-1} = (E + AC)^{-1}$ is found in terms of the (n-1)th-order inverse matrix M_{n-1}^{-1} and the coefficients of the original matrix M_n:

$$M_n^{-1} = \left\| \begin{matrix} M_{n-1}^{-1} + \dfrac{M_{n-1}^{-1} u_{n-1} v_{n-1} M_{n-1}^{-1}}{\alpha_n}\,; & -\dfrac{M_{n-1}^{-1} u_{n-1}}{\alpha_n} \\ -\dfrac{v_{n-1} M_{n-1}^{-1}}{\alpha_n}\,, & \dfrac{1}{\alpha_n} \end{matrix} \right\|. \tag{8}$$

Here, u_{n-1} and v_{n-1} are (n-1)th-order columns and rows in the original matrix:

$$M_n = (E + AC) = \left\| \begin{matrix} m_{11} m_{12} \ldots m_{1n} \\ m_{21} m_{22} \ldots m_{2n} \\ \ldots\ldots\ldots\ldots \\ m_{n1} m_{n2} \ldots m_{nn} \end{matrix} \right\|;$$

$$u_{n-1} = \begin{Vmatrix} m_{1n} \\ m_{2n} \\ \vdots \\ m_{n-1,\,n} \end{Vmatrix}, \quad v_{n-1} = [m_{n1} m_{n2} \ldots m_{n,\,n-1}], \quad \alpha_n = m_{nn} - v_{n-1} M_{n-1}^{-1} u_{n-1}. \tag{9}$$

The synthesis problem is easily solved if one has prior knowledge of the optimal matrix – the transformer of system W* or the corresponding matrix W* of the operator equation with optimal matrices – with the coefficients

$$(A_0^{\bullet} D^n + A_1^{\bullet} D^{n-1} + A_2^{\bullet} D^{n-2} + \ldots + A_{n-1}^{\bullet} D + A_n^{\bullet}) X =$$
$$= (B_0^{\bullet} D^n + B_1^{\bullet} D^{n-1} + \ldots + B_n^{\bullet}) F,$$
$$(A_0^{\bullet},\ A_1^{\bullet}, \ldots,\ A_n^{\bullet},\ B_0^{\bullet},\ B_1^{\bullet}, \ldots,\ B_n^{\bullet} \text{ do not depend on } \quad D = d/dt).$$

In this case, the condition for the existence of the solution X

$$(E + AC)(E + AC)^{-1} = l\,E \tag{10}$$

and the condition

$$(E + AC)^{-1}(H - AB) = W^{*}$$

are sufficient for the determination of the coefficients of matrices B and C; l is some constant which does not depend on D = d/dt.

Frequently, however, the optimal matrix W* is not known. Besides, the mathematical solution X may not lie within the same limits as those given by technological conditions (the conditions of physical realizability).

For finding the relationship from the condition of physical realizability, it is suggested that one use the equivalence theorem [13, 14]. If

$$(A_0 D^n + A_1 D^{n-1} + \ldots + A_{n-1} D + A_n) X = $$
$$= (B_0 D^n + B_1 D^{n-1} + \ldots + B_n) F \tag{10'}$$

and F ≡ E, but the initial conditions are $(D^i X)_0 \equiv 0$, then it is equivalent to seek the solution for the equation

$$(A_0 D^n + A_1 D^{n-1} + \ldots + A_{n-1} D + A_n) X = 0$$

with the initial conditions which satisfy the following system of inhomogeneous algebraic equations:

$$\begin{aligned} A_0 X_0 + 0\,(DX)_0 + \ldots + 0\,(D^{n-1}X)_0 &= -B_0, \\ A_1 X_0 + A_0 (DX)_0 + \ldots + 0\,(D^{n-1}X)_0 &= -B_1, \\ \ldots\ldots\ldots\ldots\ldots \quad & \ldots\ldots\ldots\ldots \\ A_{n-1} X_0 + A_{n-2}(DX)_0 + \ldots + A_0 (D^{n-1}X)_0 &= -B_{n-1}. \end{aligned} \tag{11}$$

In conditions (11), $X_0 = X_0(-0)$.

For the compensation of jumps in the external disturbance F ≡ E, it is necessary, in addition, to give the initial value of the solution

$$X(+0) = -A_n^{-1} B_n. \tag{12}$$

Conditions (11) can be extended to the more general law of variation f [15], specifically,

$$f_i(t) = \sum_{k=0}^{m} [f_{ki}(t) - f_{k-1,\,i}(t)]\, E(t - k\tau_i) \quad (i = 1, 2, \ldots, n), \tag{13}$$

where E(t) is a unit step function, m is the number of segments of the linear approximation of f(t), and τ is the length of these segments.

The representation of any analytic function in the form of (13) is given in [16].

If, instead of $f_i(t)$ and $E(t - k\tau_i)$, we consider the matrices, then the expression

$$F = \sum_{k=0}^{m} [f_{ki}(t) - f_{k-1,\,i}(t)]\, E(t - k\tau) \tag{14}$$

is an approximate expansion of the arbitrary vector function F(t) in terms of unit step disturbances. Then, the equivalent initial conditions will be found as the

solution of the following system of inhomogeneous equations:

$$
\begin{aligned}
&A_0X_0 + 0(DX)_0 + \ldots + 0(D^{n-1}X)_0 = \\
&= -B_0\sigma_0 + B_1\sigma_1 - B_2\sigma_2 + \ldots + (-1)^{s+1}B_s\sigma_s, \\
&A_1X_0 + A_0(DX_0) + \ldots + 0(D^{n-1}X)_0 = \\
&= -B_1\sigma_0 + B_3\sigma_2 - B_3\sigma_2 + \ldots + (-1)^{s+1}B_{s+1}\sigma_s, \\
&A_2X_0 + A_1(DX)_0 + A_0(D^2X)_0 + \ldots + 0(D^{n-1}X)_0 = \\
&= -B_2\sigma_0 + B_3\sigma_1 - \ldots + (-1)^{s+1}B_{s+2}\sigma_s, \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&A_{n-2}X_0 + A_{n-3}(DX)_0 + \ldots + A_0(D^{n-2}X)_0 + 0(D^{n-1}X)_0 = \\
&= -B_{n-2}\sigma_0 + B_{n-1}\sigma_1 - B_n\sigma_2, \\
&A_{n-1}X_0 + A_{n-3}(DX)_0 + \ldots + A_0(D^{n-1}X)_0 = -B_{n-1}\sigma_0 + B_n\sigma_1, \\
&X(+0) = -A_n^{-1}B_n\sigma_0,
\end{aligned}
\tag{15}
$$

where

$$\sigma_s = \frac{\tau^s}{s!}\sum_{k=0}^{m} k^s(f_k - f_{k-1}) \quad (s = 0, 1, 2, \ldots, r).$$

It ordinarily suffices to limit oneself to the case when $r = 2$. In addition to conditions (15), the following conditions, defining the accuracy ϵ of the equivalent substitution, must be observed:

$$
\begin{aligned}
&-B_0\sigma_1 + B_1\sigma_2 + \ldots + (-1)^sB_{s-1}\sigma_s \leqslant \varepsilon_1, \\
&B_0\sigma_2 - B_1\sigma_3 + \ldots + (-1)^sB_{s-2}\sigma_s \leqslant \varepsilon_2, \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&(-1)^{s-1}B_0\sigma_{s-1} + (-1)^sB_1\sigma_s \leqslant \varepsilon_{s-1}, \\
&(-1)^sB_0\sigma_s \leqslant \varepsilon_s.
\end{aligned}
\tag{16}
$$

Each of the quantities $\epsilon_1, \epsilon_2, \ldots, \epsilon_s$ must be chosen less than the quantity ϵ, which depends on the initial equation and on the form of $f(t)$.

Thus, thanks to (15), one can, with the condition that $|E + AC| \neq 0$, establish the relationships between the initial conditions of the solution $(D^{(i)}X)_0$ ($i = 0, 1, \ldots, n-1$) and the coefficient matrices $A_0, A_1, \ldots, A_{n-1}$ and $B_0, B_1, \ldots, B_{n-1}$ or, what amounts to the same thing, between the elements of the known and unknown matrices, A, B, C, and H.

For brevity, we write the equivalence conditions (15) in the form

$$(D^{(i)}X)_0 = \Phi_i(A, B, C, H, \sigma) \quad (i = 0, 1, 2, \ldots, n-1). \tag{17}$$

By the conditions of physical realizability, the initial values X_0 and their derivatives $(D^{(i)}X)_0$ ($i = 1, 2, \ldots, n-1$) are bounded (for example, the accelerations are bounded by conditions of reliability, etc.):

$$\Phi_i(A, B, C, H, \sigma) \leq N_i. \tag{18}$$

Conditions (18) can be taken as auxiliary limitations on the choice of B and C. Limitations on the coefficients of matrices B and C from the condition of required quality can be obtained on the basis of the two other principles: the principle of invariance and the principle of optimality. We consider in detail the principle of invariance [17].

Following V. S. Kulebakin, we shall distinguish two forms of invariance: absolute and selective (autonomous) [18].

In the first case, the controlled quantity X will be unchanged under any arbitrary stimuli $F(f_1, f_2, \ldots, f_n)$. This means that

$$H - AB \equiv 0. \tag{19}$$

Equation (19) can hold if

1) $H \equiv 0$, $AB \equiv 0$,
2) $(H - AB) \equiv 0$.

The condition $H \equiv 0$ means that F is not applied directly to the plant.

The condition that $AB \equiv 0$ holds if

$$
\begin{aligned}
&a_{11}b_{1p} + a_{12}b_{2p} + \cdots + a_{1n}b_{np} = 0, \\
&a_{21}b_{1p} + a_{22}b_{2p} + \cdots + a_{2n}b_{np} = 0, \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&a_{n1}b_{1p} + a_{n2}b_{2p} + \cdots + a_{nn}b_{np} = 0
\end{aligned}
$$

for all $p = 1, 2, \ldots, n$.

For $|A| \neq 0$, all the $b_{ij} = 0$, i.e., the controller is also insulated from the external disturbances (the trivial case).

The case when $|A| = 0$ gives the possibility of obtaining a system which is invariant with respect to F for $b_{ij} \neq 0$.

If the determinant $|A|$ of the plant matrix equals zero, this means that there is a linear relationship between the rows (or columns). But, since a relationship of the

rows (or columns) corresponds to a relationship between the processes in the plant, then this latter fact means that there is the possibility of an internal compensation of the external disturbances (by virtue of the properties of the plant itself).

It is possible to create a complicated plant in which each natural process is exactly simulated. Then, the rows of the determinant $|A|$ will always be proportional. The model (simulated processes) can be so constructed that the processes in it will flow significantly faster than in the natural plant. Therefore, before the plant reacts to changes of the external conditions, there will be developed, thanks to the speed of the model, the necessary reconstitution of the stimuli to the plant to compensate the applied disturbances, so that the plant's state, characterized by the coördinates $x_i(t)$, can remain unchanged. However, with this, the stimuli do not act directly on the plant ($H \equiv 0$).

Case 2 refers to the generally known case of external disturbance compensation [17].

A controller with matrix B must be so chosen that the following conditions are met:

$$\begin{aligned} &a_{11}b_{1p} + a_{12}b_{2p} + \cdots + a_{1n}b_{np} = h_{1p}, \\ &a_{21}b_{1p} + a_{22}b_{2p} + \cdots + a_{2n}b_{np} = h_{2p}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &a_{n1}b_{1p} + a_{n2}b_{2p} + \cdots + a_{nn}b_{np} = h_{np} \quad (p = 1, 2, \ldots n). \end{aligned} \tag{20}$$

From this, with the condition $|A| \neq 0$, there are determined all the elements of the projected controller B which measures the external disturbances. The elements of controller C (which measures the deviations of controlled quantity X) are found from condition (10). The case of invariance considered above refers to the case when there is no a priori information as to F. It is obtained by means of controllers B and C, both by direct measurements (by controller B) and by indirect (by controller C) (by means of a preliminary deviation of the controlled quantity).

The choice of the controller parameters on the basis of the invariance principle can be significantly eased if there is information as to F. For example, if it is known that F is the solution of some homogeneous equation $K(D)F = 0$, where $K(D)$ – the Kulebakin operator [13] – enters into operator Eq. (10).

From this there follows the practical approach to setting up auxiliary limitations on the coefficients of matrices A, B, C, and H.

From the known F there is found the K(D) representation (or the corresponding matrix K). The right member of Eq. (4) or Eq. (10) takes the form

$$(H - AB) = KL \tag{21}$$

or

$$\begin{aligned} &B_0D^n + B_1D^{n-1} + \ldots + B_{n-1}D + B_n = \\ &= K(D)(L_0D^m + L_1D^{m-1} + \ldots + L_m). \end{aligned} \tag{22}$$

The matrix L or the operator L(D) are so chosen that the order of matrix KL will equal the order of the original matrix $(H - AB)$. If the operator K(D) has order $\underline{n}$ then, obviously, L = constant, and the matrix L can be taken as a diagonal matrix with constant coefficients.

Finally, one may use the criterion of absolute invariance, presented in [18].

As applied to system (4), this takes the form: the necessary and sufficient condition that the system's first unknown function x_1 be independent of any analytic function f_1 consists in the identical vanishing of the minor Δ_{11}.

Consequently, each of the unknown x_i will be invariant with respect to f_i if the corresponding minors Δ_{ii} equal zero identically. By equating these minors Δ_{ii} to zero, we obtain the additional limitations on the coefficients of matrices B and C.

The condition of absolute invariance is implemented with difficulty, so that one frequently limits oneself to the condition of selective invariance. In [4] and [19] this was formulated as the criterion of autonomy. In almost all systems with many variables, there is a reciprocal effect on the control loops via the plant. These so-called natural connections, caused by the interactions between the physical processes, can have an adverse effect on the dyanmic properties of the system as a whole.

Selective invariance supposes that each system output x_i reacts only to "its" input f_i. Mathematically, this means that the system's matrix operator W must have the diagonal form. As was shown in works [2, 10, 11], any nonsingular matrix operator W can be transformed to the Hermite triangular form by being multiplied left or right by some matrix S. The matrix S possesses the property that its determinant is not identically zero and does not depend on $D = d/dt$. By successively carrying out elementary transformations on the rows, we can take the matrix W to the form where all the elements below the diagonal are zero, and if we apply, to the already transformed matrix, a succession of elementary operations on the columns, we shall finally obtain a matrix with only its diagonal elements different from zero.

Thus, we present the original matrix W in the form

$$W = SE(D)T. \tag{23}$$

Matrices S and T are called transforming matrices. They are triangular matrices whose determinants do not equal zero and do not depend on D.

The diagonal elements of the matrix E(D) are defined by the following formulas:

$$e_{11} = \frac{\Delta_n(D)}{\Delta_{n-1}(D)}, \quad e_{22} = \frac{\Delta_{n-1}(D)}{\Delta_{n-2}(D)}, \ldots, e_{nn} = \frac{\Delta_1(D)}{\Delta_0} = \Delta_1(D). \tag{24}$$

where Δ_r is the greatest common divisor of the rth-order minors.

If we impose the requirement that all nondiagonal elements of matrices S and T be zero, then this will mean the holding of the criterion of selective invariance. The criterion of autonomy [4] flows from this as a particular case.

By writing matrix Eq. (23) in expanded form:

$$\begin{Vmatrix} w_{11}w_{12}\dots w_{1n} \\ w_{21}w_{22}\dots w_{2n} \\ \dots\dots\dots \\ \dots\dots\dots \\ w_{n1}w_{n2}\dots w_{nn} \end{Vmatrix} = \begin{Vmatrix} s_{11}s_{12}\dots s_{1n} \\ 0\ s_{22}\dots s_{2n} \\ 0\ 0\ \dots\dots \\ \dots\dots\dots \\ 0\ \ 0\dots s_{nn} \end{Vmatrix} \begin{Vmatrix} e_{11}\ 0\dots 0 \\ 0\ e_{22}\dots 0 \\ \dots\dots\dots \\ \dots\dots\dots \\ 0\ 0\dots e_{nn} \end{Vmatrix} \begin{Vmatrix} \tau_{11}\ \ 0\dots 0 \\ \tau_{21}\ \tau_{22}\dots 0 \\ \dots\dots\dots \\ \dots\dots\dots \\ \tau_{n1}\tau_{n2}\dots\tau_{nn} \end{Vmatrix}, \quad (25)$$

and by carrying out the multiplications on the right side, and then setting up the equations $w_{ij} = s_{ij}e_{ij}\tau_{ji}$ of the matrix elements, we obtain the additional necessary conditions for determining B and C.

If, further, the requirements are also imposed that all the s_{ij} $(j \neq i)$ and τ_{kr} $(k \neq r)$ equal zero, then this will also mean the possibility of autonomous control.

Finally, we consider the third principle — the principle of optimality.

The fact of the matter is that one does not succeed in establishing an explicit relationship between the desirable variations of the x_i and the coefficients of matrices A_i and B_i $(i = 0, 1, \dots, n)$. Therefore, the optimality condition denotes the best approximation of the solution X being sought to some optimal X^*, chosen from the condition of least dynamic error and least duration of the transient response in the integral sense. As a criterion of optimality, one can take the minimum mean square error for the multiloop system $\overline{\epsilon^2} = \min$. This case was considered in detail in [5, 9]. This last method requires that $F(f_1, \dots, f_n)$ be considered as stationary random functions of time.

We consider optimality in the integral sense. We place on X the additional requirement of 2n-fold differentiability. We write the system of n homogeneous equations for X:

$$\begin{gathered} XA_0D^{(n)}X + XA_1D^{(n-1)}X + \dots + XA_{n-1}DX + XA_nX = 0, \\ DXA_0D^{(n)}X + DXA_1D^{(n-1)}X + \dots + DXA_{n-1}DX + DXA_nX = 0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ D^{(n-1)}XA_0D^{(n)}X + D^{(n-1)}XA_1D^{(n-1)}X + \dots + D^{(n-1)}XA_{n-1}DXA_nX = 0. \end{gathered} \quad (26)$$

We consider the homogeneous equation $A_0D^{(n)}X + A_1D^{(n-1)}X + \dots + A_{n-1}DX + A_n = 0$ instead of the original Eq. (10'), so that, on the basis of the equivalence theorem, we can obtain a close solution from the left member of (10') if the initial conditions are taken to be nonzero (equivalent) [cf. (15)].

By integrating this system by parts between the limits of zero and infinity, we obtain a new inhomogeneous system of scalar equations where, as the variables, we consider the integral estimates [20] of the form

$$I_0 = \int_0^\infty (X)^2 dt, \quad I_1 = \int_0^\infty (DX)^2 dt, \dots, \quad I_{n-1} = \int_0^\infty (D^{(n-1)}X)^2 dt.$$

This system establishes the connections I_i $(i = 0, 1, \dots, n-1)$ among the equivalent initial conditions $(D^iX)_0$ $(i = 0, 1, \dots, n-1)$, the boundary conditions (at infinity) $(D^{(i)}X)_\infty$ $(i = 0, 1, \dots, n-1)$, and the matrix coefficient A_i, B_i $(i = 0, 1, \dots, n-1)$ or, what amounts to the same thing, the coefficients of matrices A, B, C, H and σ.

Since $(D^{(i)}X)_0$, in its turn, is also expressed in terms of A, B, C, H, and σ (17), and all the $(D^{(i)}X)_\infty = 0$ $(i = 1, 2, \dots, n-1)$ (condition for a stable process) and $(X)_\infty$ is given as the new steady-state condition of the system from the static calculations of the system, then Eqs. (26), after integration, can be put in the form

$$\begin{Vmatrix} A_n & -A_{n-2}\dots 0 \\ 0 & A_{n-1}\dots 0 \\ 0 & -A_n\dots 0 \\ \dots & \dots\dots -A_0 \\ 0 & 0 \quad A_1 \end{Vmatrix} \times \begin{Vmatrix} I_0 \\ I_1 \\ \cdot \\ \cdot \\ I_{n-1} \end{Vmatrix} = \begin{Vmatrix} G(A, B, C, H, \sigma) \end{Vmatrix} \quad (27)$$

The determinant of the characteristic matrix is $|E + AC| \neq 0$ (by the Hurwitz condition), so that system (27) is solvable for the I_i. If we now impose on the I_i or on their combinations the requirement that they have minimal value [20-22], we then obtain the relationship sought for the choice of the coefficients for the controllers of the multiloop system.

Let $I_i = I_i (A, B, C, H, \sigma)$. We set up the linear combinations of the integral estimates:

$$\int_0^\infty [\alpha_{0l}(X)^2 + \alpha_{1l}(T_1DX)^2 + \ldots + \alpha_{r-1,\,l}(T_{r-1}^{r-1}D^{(r-1)}X)^2]\,dt = \\ = \psi_l(A, B, C, H, \sigma) \quad (l = 0, 1, 2, \ldots, r). \tag{28}$$

Then the minimum of the functionals will be found by solving the Euler-Lagrange equations.

The parameters A, B^*, C^*, H, σ which satisfy the optimality condition

$$\min \psi_l \ (A, B, C, H, \sigma), \tag{29}$$

will also be sought.

Thus, based on the use of the three principles presented for the synthesis of automatic systems with many variables, we obtain relationships from which we find the optimal coefficients of the controller matrices B and C:

the realizability condition

$$(E + AC)(E + AC)^{-1} = lE,$$

the condition of physical realizability

$$(D^{(i)}X)_0 = \Phi_i (A, B, C, H, \sigma) \leq N_i,$$

the Kulebakin invariance condition

$$(H - AB) = KL,$$

the absolute invariance condition

$$\Delta_{ii} = 0, \ H \equiv 0, \ AB \equiv 0, \ (H - AB) = 0, \tag{30}$$

the selective invariance condition (autonomy criterion)

$$w = SE(D)T, \ s_{ij} = r_{ij} = 0 \ (i \neq j),$$

the optimality condition

$$(E + AC)^{-1} (H - AB) = W^*,$$

the optimality condition by the mean square estimate

$$\bar{\epsilon}^2 = \min,$$

the optimality condition in the integral sense

$$\psi_l \ (A, B, C, H, \sigma) = \min.$$

For systems with many variables, satisfying these relationships presents large computational difficulties. However, if they are reduced to programs for high-speed computers, which permit a large number of computations to be made, then the problem of synthesizing systems with many variables will be solved without a great deal of work.

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A METHOD FOR OBTAINING TRANSFER FUNCTIONS OF AUTOMATIC CONTROL SYSTEMS OPERATING ON ALTERNATING CURRENT

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A general method is suggested for obtaining transfer functions of automatic control systems operating on ac, where the systems are described by linear differential equations with periodic coefficients. The limitations which are thereby imposed on the systems are established. The transfer functions are obtained for several devices, and methods for simplifying them are cited.

In systems operating on ac, the amplitude-modulated oscillations obtained in the modulator pass along the control loop and are applied to a phase detector which separates out the modulating function.

Thus far, the literature has contained no method that would apply without limitation to all systems operating on ac which are described by linear differential equations with periodic coefficients. Below, we present a method which allows the problem posed to be solved.

1. Transfer Function of a Linear Passive Quadripole Connected between an Ideal Modulator and an Ideal Phase Detector

We consider a segment of circuitry (Fig. 1) consisting of an ideal modulator M, a linear passive quadripole and an ideal phase detector D.

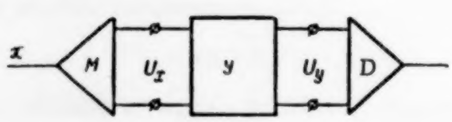

Fig. 1.

Let x(t) be the function at the input of the ideal modulator, which multiplies x(t) by $\cos(\omega_0 t + \varphi)$. At the modulator output we obtain

$$u_x(t) = x(t)\cos(\omega_0 t + \varphi). \tag{1}$$

Let $u_y(t)$ be the function at the linear quadripole's output and the ideal phase detector's input. The ideal phase detector multiplies the function $u_y(t)$ by $\cos(\omega_0 t + \psi)$. At the phase detector's output we shall have

$$u_g(t) = u_y(t)\cos(\omega_0 t + \psi). \tag{2}$$

Equations (1) and (2) can be considered as linear equations with periodic coefficients. The quadripole in the circuit is linear and, consequently, is described by linear differential equations. Thus, the entire system under consideration is described by linear differential equations with periodic coefficients.

The Laplace transform of the quadripole's equation has the form

$$U_y(p) = Y(p)\,U_x(p). \tag{3}$$

Here, Y(p) is the quadripole's transfer function.

The transforms of Eqs. (1) and (2) are

$$U_x(p) = \tfrac{1}{2}[X(p - j\omega_0)e^{j\varphi} + X(p + j\omega_0)e^{-j\varphi}], \tag{4}$$

$$U_g(p) = \tfrac{1}{2}\,[U_y(p - j\omega_0)e^{j\psi} + U_y(p + j\omega_0)e^{-j\psi}]. \tag{5}$$

By substituting $U_x(p)$ from Eq. (4) in Eq. (3), and then substituting $U_y(p)$ from Eq. (3) in Eq. (5), we obtain the following expression for the Laplace transform $U_g(p)$:

$$U_g(p) = \frac{1}{4}\{Y(p - j\omega_0)e^{j\psi}[X(p - j2\omega_0)e^{j\varphi} + X(p)e^{-j\varphi}] + Y(p + j\omega_0)e^{-j\omega}[X(p)e^{j\varphi} + X(p + j2\omega_0)e^{-j\varphi}]\}. \tag{6}$$

The terms $X(p - j2\omega_0)$ and $X(p + j2\omega_0)$ are the transforms of the function x(t), modulated at twice the carrier frequency. We shall assume that the phase detector contains low-frequency filters which do not pass oscillations modulated at twice the carrier frequency. We therefore discard these terms, and rewrite Eq. (6) in the form

$$U_g(p) = \frac{1}{4}[Y(p - j\omega_0)e^{j(\psi-\varphi)} + Y(p + j\omega_0)e^{-j(\psi-\varphi)}]\,X(p). \tag{7}$$

We now formulate the conditions whose holding permits the terms $X(p - j2\omega_0)$ and $X(p + j2\omega_0)$ to be discarded.

Let the function Y(p) be represented as the quotient of polynomials

$$Y(p) = \frac{F_1(p)}{F_2(p)}.$$

We shall assume that the polynomials $F_1(p)$ and $F_2(p)$ have no common factors, and that the degree of poly-

nomial $F_2(p)$ is not lower than the degree of polynomial $F_1(p)$. Then, if none of the complex conjugate roots of polynomial $F_2(p)$ has a frequency close, or equal, to the carrier frequency, the aforementioned terms may be discarded.

For the transfer function of the segment of circuitry under consideration, we shall have

$$Y_g(p) = \frac{U_g(p)}{X(p)} = \frac{1}{4}[Y(p - j\omega_0)e^{j(\psi-\varphi)} + Y(p + j\omega_0)e^{-j(\psi-\varphi)}]. \tag{8}$$

We note that $Y_g(p)$ is a function of the operator $\underline{p}$ with real coefficients, since it is the sum of two complex conjugate expressions.

We now change the argument of the function $Y_g(p)$ to $S = p/\omega_0$, and we set $\psi - \varphi - \delta$. Then,

$$Y_g(s) = \frac{1}{4}[Y(s - j)e^{j\delta} + Y(s + j)e^{-j\delta}]. \tag{9}$$

The transfer function defined by Eqs. (8) and (9) depends, in the general case, on the frequency of the generator which supplies the modulator and the phase detector. Let the quadripole be designed to operate for some definite carrier frequency ω_0, while the generator supplying the modulator and the phase detector has the frequency ω_g. In this case, the transfer function of the series of links of Fig. 1, instead of (8), is given by

$$Y_g(p) = \frac{1}{4}[Y(p - j\omega_g)e^{j(\psi-\varphi)} + Y(p + j\omega_g)e^{-j(\psi-\varphi)}]. \tag{10}$$

We introduce the parameter $\lambda = \omega_g/\omega_0$.

By making the transition to the operator $\underline{s}$ in Eq. (10), we obtain, instead of (9), the transfer function

$$Y_g(s, \lambda) = \frac{1}{4}[Y(s - j\lambda)e^{j\delta} + Y(s + j\lambda)e^{-j\delta}]. \tag{11}$$

Formula (11) allows one to establish how the transfer function changes as the carrier and generator frequencies move apart.

2. Application of Formulas (8)-(11) to a Differentiating Quadripole, and Some Deductions from the Theory

In ac servosystems, the quadripole whose circuit is shown on Fig. 2 is used as a differentiating loop. The

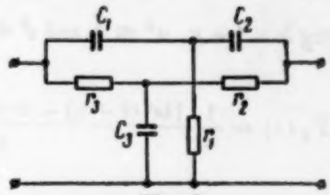

Fig. 2.

transfer function of such a quadripole can be written in the following way (bearing in mind that the filters on Fig. 2 are symmetric):

$$Y(p) = \frac{p^2 + k_1\omega_0 p + \omega_0^2}{p^2 + k_2\omega_0 p + \omega_0^2} \tag{12}$$

In order to obtain from Y(p) the transfer function for the modulating oscillation, we use the formulas for the approximate transformation of (12):

$$\frac{1}{2p}(p^2 + \omega_0^2) = p'. \tag{13}$$

By using formula (13), we obtain the following expression for the transfer function of the differentiating quadripole's modulating oscillation:

$$Y' = \frac{2p' + k_1\omega_0}{2p' + k_2\omega_0} = \frac{2s' + k_1}{2s' + k_1}. \tag{14}$$

For such a quadripole, we find the transfer function $Y_g(s)$ by means of formula (9):

$$Y_g(s) = \frac{[s^2(s^2+4) + s(k_1+k_2)(s^2+2) + k_1k_2(s^2+1)]\cos\delta + s^2(k_2-k_1)\sin\delta}{2[s^2(s^2+4) + 2sk_2(s^2+2) + k_2^2(s^2+1)]}. \tag{15}$$

We set $s^2 \ll 1$. The meaning of this inequality is very simple. It is known, from Laplace transform theory, that there exists a certain region of values of the parameter $\underline{p}$ on the complex variable plane for which there is a one-to-one correspondence between the original time domain function and its transform. The inequality $s^2 \ll 1$ isolates, in this region, some subregion in which the approximate function $Y_g(s)$ can be used. For $\delta = 0$, this approximate function has the form

$$Y_g(s) = \frac{1}{2}\frac{(2s + k_1)}{(2s + k_2)}. \tag{16}$$

The function $Y_g(s)$ coincides to within a factor of 1/2 with (14). The factor 1/2 is the transfer factor of an ideal modulator and an ideal phase detector.

We now compute the differentiating quadripole's transfer function for deviations of the carrier from the generator frequency.

By using formula (11), we obtain

$$Y_g(s) = \frac{1}{2}\frac{[c + s(k_1 + k_2)(s^2 + \lambda^2 + 1) + k_1 k_2 (s^2 + \lambda^2)] \cos\delta + (k_2 - k_1)\lambda(s^2 + \lambda^2 - 1)\sin\delta}{[c + 2sk_2(s^2 + \lambda^2 + 1) + k_2^2(s^2 + \lambda^2)]}, \tag{17}$$

where

$$c = s^2(s^2 + 2\lambda^2 + 2) + (\lambda^2 - 1)^2.$$

By setting $\lambda = 1 + \mu$, $\mu^2 \ll 1$ and $s^2 \ll 1$, we find the approximate transfer function

$$Y_g(s) = \frac{1}{2}\frac{[4s^2(1 + \mu) + 2s(k_1 + k_2)(1 + \mu) + k_1 k_2 (1 + 2\mu)]\cos\delta + 2(k_2 - k_1)\mu \sin\delta}{[4s^2(1 + \mu) + 4sk_2(1 + \mu) + k_2^2(1 + 2\mu)]}. \tag{18}$$

Computations showed that, for $s \leq 0.3$, the transfer functions computed by formulas (15) and (16) were sufficiently close.

Analogous results were provided by calculations with formulas (17) and (18) for $\mu = \pm 0.1$. The curves constructed from formulas (17) and (18) are analogous to those given in [1].

With certain assumptions [3], the following function is found at the Selsyn modulator's output:

$$u_x(t) = U_m \left[x(t)\cos(\omega_0 t + \psi) + \frac{1}{\omega_0}\frac{dx}{dt}\sin(\omega_0 t + \varphi)\right]. \tag{19}$$

For the transfer function of the circuit segment consisting of a modulator, a linear quadripole and a phase detector, we obtain

$$Y_g(s) = \frac{U_m}{4} j\left[(s - j)Y(s - j)e^{j\delta} - (s + j)Y(s + j)e^{-j\delta}\right]. \tag{20}$$

Let the quadripole in the circuit under consideration be inertialess with gain $Y(s) = k_0$. We may then write

$$Y_g(s) = \frac{U_m}{2} k_0 (\cos\delta + s\sin\delta). \tag{21}$$

In this case, for the proper choice of phase angles φ and ψ, the circuit under consideration is equivalent to a differentiating link [3].

We derive still one more conclusion from the results obtained.

In the circuit segment between the modulator and the phase detector, let there be connected in series two directed links, with transfer functions $Y_1(p)$ and $Y_2(p)$. By using the method presented, we obtain the transfer function for the modulating oscillation in the form

$$Y_g(s) = \frac{1}{4}\left[Y_1(s - j)Y_2(s - j)e^{j\delta} + Y_1(s + j)Y_2(s + j)e^{-j\delta}\right]. \tag{22}$$

Suppose the transfer functions Y'_1 and Y'_2 for both links can be found by a transformation using operation (13). If one forms the function $Y' = Y'_1 Y'_2$, this function will obviously not be equivalent to function (22). Since it is necessary to have modulators and phase detectors in servo and automatic control systems operating on ac, it is then obviously impossible to seek the circuit's transfer function for the modulating oscillation by transforming each link separately in accordance with formula (13); the circuit as a whole must be transformed.

3. Servosystem with a Two-Phase Asynchronous Motor

We now consider a servosystem in which the phase detector is a two-phase asynchronous motor. We find the system's transfer function, and answer the question as to whether it is possible to transform the circuit links con-

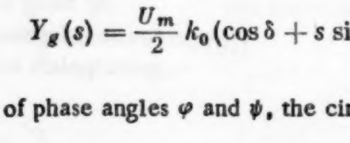

Fig. 3.

nected in the control loop separately from the phase detector, the asynchronous motor. This is frequently the method employed in investigating this system. The servosystem's schematic is shown in Fig. 3. We now write the system's equations:

the equation of the differential

$$\theta_1 - \theta_2 = x(t), \tag{23}$$

the equation of the ideal modulator

$$u_x(t) = x(t)U_m \cos(\omega_0 t + \varphi), \tag{24}$$

the transformed equation of the quadripole

$$U_y(p) = Y(p)U_x(p), \tag{25}$$

the equation of motor rotation

$$J\frac{d^2\theta_2}{dt^2} = M_e(t) - M_r(t), \tag{26}$$

where

$$M_e(t) = \frac{1}{r}\left[\psi_a u_2 - \psi_b u_y - \frac{d\theta_2}{dt}(\psi_b^2 + \psi_a^2)\right]. \tag{27}$$

Here, ψ_a and ψ_b are the stator winding flux linkages, $\underline{r}$ is the rotor resistance, $u_y = d\psi_a/dt$, $u_2 = d\psi_b/dt$;

$$u_2 = U_{2m}\sin(\omega_0 t + \psi), \tag{28}$$

is the voltage on the stator's second winding.

We assume that

$$M_r(t) = k\, d\theta_2/dt. \tag{29}$$

We also assume, as is frequently done, that

$$\psi_a^2 \ll \psi_b^2$$

and discard the ψ_a^2 in the right member of Eq. (27). Then, Eqs. (23)-(26) which describe the scheme will be linear equations with periodic coefficients. Equation (25) is written for the Laplace transformed equation. We apply the Laplace transform to the other equations just enumerated. By solving the transformed equations for $\Theta_2(p)$ and then discarding the terms for twice the carrier frequency, we obtain the following expression for the transfer function, written in terms of the operator $\underline{s}$:

$$\frac{\Theta_2(s)}{X(s)} = \frac{1}{T_2 s(T_1 s + 1)}\left[\frac{s - j2}{s - j}Y(s - j)e^{j\delta} + \frac{s + j2}{s + j}Y(s + j)e^{-j\delta}\right]. \tag{30}$$

Here, T_1 and T_2 are dimensionless constants.

Let the quadripole in the circuit be inertialess, with gain k_0. Then, instead of (30), we shall have

$$\frac{\Theta_2(s)}{X(s)} = \frac{2k_0[(s^2+2)\cos\delta + s\sin\delta]}{T_2 s(T_1 s + 1)(s^2 + 1)}. \tag{31}$$

In the system considered, the asynchronous motor is the sole link with an operator transfer factor. A formula analogous to (31) was obtained by L. M. Sadovskii [4].

The transfer function is simplified still further if we set $\delta = 0$ and $s^2 \ll 1$.

Then,

$$\frac{\Theta_2(s)}{X(s)} = \frac{4k_0}{T_2 s(T_1 s + 1)}. \tag{32}$$

In this form, the transfer function of the asynchronous motor is used in many works (for example, in [5]).

It follows from expressions (30) and (31) that it is impossible to present the system's transfer function as the product of the transfer functions of the quadripole and the asynchronous two-phase motor.

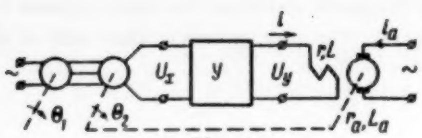

Fig. 4.

4. Servosystem with a Single-Phase Commutator Motor

1. Control is effected on the side of the excitation winding (Fig. 4). Such a system was investigated by E. I. Chernov, and the transfer function was obtained for it by means of a very cumbersome mathematical apparatus (integration by means of infinite series) with rigid limitations (the action of unit input functions and a simplified transfer function of the differentiating quadripole were considered) [6]. We now write the system's equations. As the equations for the differential, modulator and quadripole, we use Eqs. (23), (24) and (25), respectively.

The equation of the excitation winding circuit is

$$u_y(t) = ir + L\frac{di}{dt}, \tag{33}$$

the equation of the armature circuit

$$u(t) = i_a r_a + L_a \frac{di_a}{dt} + Ci\frac{d\Theta_2}{dt}, \tag{34}$$

the equation of armature rotation

$$J\frac{d^2\Theta_2}{dt^2} = Cii - M_r(t). \tag{35}$$

It was assumed in [6] that the two last terms in the right member of Eq. (34) are small. We shall make the assumption that the last term, which makes the equation nonlinear, is small. With this condition, if u(t) is a sinusoidal function of time, then $i_a(t)$ is also a sinusoidal function.

We set

$$i_a = I_m \cos(\omega_0 t + \psi) \tag{36}$$

and take Eq. (29) for $M_r(t)$.

By transferring to the transforms of these equations and then solving them for $\Theta_2(p)$, dropping the terms modulated by twice the carrier frequency, we obtain the following equation for the transfer function:

$$\frac{\Theta_2(s)}{X(s)} = \frac{1}{T_2 s(T_1 s + 1)}\left[\frac{Y(s - j)e^{j\delta}}{1 + T_a(s - j)} = \frac{Y(s + j)e^{-j\delta}}{1 + T(s + j)}\right]. \tag{37}$$

Here T_1, T_2 and T_a are dimensionless constants.

No limitations are placed on either the signal or the quadripole's transfer function in formula (37).

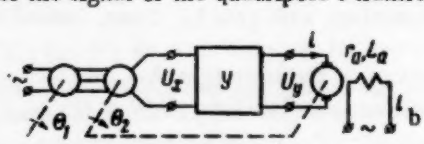

Fig. 5.

2. System with control on the armature side (Fig. 5). We again make use of Eqs. (23)-(25), and we write the motor equations as follows:

the equation of the armature circuit

$$u_y(t) = ir_a + L_a\frac{di}{dt} + Ci_b\frac{d\Theta_2}{dt}, \tag{38}$$

the equation of armature rotation

$$J\frac{d^2\Theta_2}{dt^2} = Ci_b i - M_r(t). \tag{39}$$

We take

$$i_b = I_m \cos(\omega_0 t + \psi), \tag{40}$$

and we use Eq. (29) for $M_r(t)$.

By transferring to the transforms of these equations and then solving them for $\Theta_2(p)$, dropping the terms modulated by twice the carrier frequency, we obtain the following expression for the transfer function:

$$\frac{\Theta_2(s)}{X(s)} = \frac{1}{2} \frac{Y(s-j)\,[T_a(s+j)+1]\,e^{j\delta} + Y(s+j)\,[T_a(s-j)+1]\,e^{-j\delta}}{T_k s\{k_m(T_m s+1)\,[(T_a s+1)^2+T_a^2]+T_a s+1\}}. \tag{41}$$

There is no difficulty in obtaining the transfer function for servosystems with feedback, for systems stabilized by differentiating quadripoles, etc. The necessary condition remains the linearity of the equations which describe the system.

SUMMARY

1. The method presented permits one to obtain the transfer functions for servosystems and automatic control systems operating on ac, for circuit configurations of any degree of complexity. These functions can be investigated by the methods used in the investigation of dc systems. The systems must be described by linear differential equations with periodic coefficients.

2. The method makes it possible to establish the limitations which must be imposed on the circuit configuration in order that, for the investigation, one might discard the terms modulated by twice the carrier frequency, such terms arising in systems of this nature.

3. The transfer functions for the discrepancies of the carrier and generator frequencies are obtained.

4. It was shown that the transfer functions obtained by this method can be simplified by using the inequality $s^2 \ll 1$.

5. The method can be used for investigating mixed systems in which only part of the links operate on ac.

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DETERMINATION OF DYNAMIC CHARACTERISTICS OF CHEMICAL ENGINEERING APPARATUS AS WELL AS THE RELEVANT CRITERIA OF PERSISTENCE AND CONTROLLABILITY

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Nonstationary processes in moving media are considered from the point of view of the control problem for a broad class of technological apparatus operating on the counterflow principle.

The criteria of persistence and controllability of continuous processes in moving media are defined with the aim of explaining their dependence on the apparatus parameters and mode of operation.

INTRODUCTION

Until now, in the design of technological apparatus no investigation was ordinarily made of the stability of the chosen mode of operation or of the possibilities of controlling it.

In some cases, the very simplest controller provides sufficient accuracy of process flow, while sometimes this cannot be provided by a very complicated and expensive controller. Sometimes, the cost of the controller can exceed the cost of the plant itself.

Today, the automation of chemical engineering apparatus entails controllers which, as a rule, are designed for a preexisting plant, and any changes in the plant to improve stability or controllability become either impossible or extremely difficult. In many cases, the operation of new technological systems is impossible without automatic control.

In connection with what has just been said, it becomes very important to study the plant as a basic link in the automatic control system. The problems of optimal parameters of the plant and of the controller must be solved simultaneously and compatibly. The general posing of the problem was given by V. S. Kulebakin at the Second All-Union Conference on Automatic Control [1].

It is from this point of view that the present paper solves the problem of analyzing the persistence and controllability of processes in the widely used apparatus of chemical technology.

For widely used types of engineering continuous-flow apparatus, a characteristic of the processes is the interaction of the media along the entire path of their motion by the principle of direct flow or, even more frequently, by the counterflow principle. Therefore, in considering the questions of control of continuous processes, one must solve the general nonstationary problem of the interaction processes of the moving media in these apparatus.

As apparatus of this type we have: heat exchangers, absorbers, scrubbers, extractors, fractionating columns, roasting ovens, etc. In all of these devices, the speed of interaction of the moving media is basically determined by the difference in concentration at their boundary surfaces, analogous to the circumstance that heat transmission is basically determined by the temperature difference of the interacting media.

In the present work, the problem is considered in terms of the example of implementing a process by the counterflow method, although the corresponding results can also be obtained for the case of direct flow.

I. Basic Equations of Nonstationary Processes of Moving Media Interaction in Engineering Apparatus

1. We consider heat exchangers with thin walls [2, 3] and, simultaneously, apparatus with diffusion processes (absorbers, scrubbers, extractors, etc.) without taking account of the thermal effect of the reactions in them.

Let u(x, t) and v(x, t) be the relative temperatures* (or, for other apparatus, the corresponding concentrations) of the moving media, where $\underline{x}$ is the coordinate of the apparatus cross section, measured in the direction of motion of medium $\underline{u}$.

If, in the course of time $\underline{t}$, the ideally mixed fluid portions touch one another along some boundary surface with thermal resistance or, as the case may be, with diffusion resistance, then

$$\frac{du}{dt} = k_1(v - u), \qquad \frac{dv}{dt} = k_2(u - v),$$

where k_1 and k_2 are positive constants. With these conditions, we easily convince ourselves, by the use of the concept of a function's complete differential, that the general equations for the interaction of moving media,

* For the unit of measurement of $\underline{u}$ and $\underline{v}$ we take the value u(0, 0) in the transient response and, as the zero of measurement, we take the value v(l, 0), where l is the length of the apparatus.

in the case of counterflow† have the form

$$\frac{\partial u}{\partial t} + w_1 \frac{\partial u}{\partial x} = k_1 (v - u), \qquad \frac{\partial v}{\partial t} - w_2 \frac{\partial u}{\partial x} = k_2 (u - v). \tag{1}$$

With this, $k_1 = kp/s_1c_1\gamma_1$ for heat exchangers. Here, k is the heat transmission factor, p is the perimeter of the lateral cross section of the surface separating the media, s_1 is the area of the lateral cross section of the first medium, c_1 is heat capacity, γ_1 is specific weight, w_1 and w_2 are the absolute values of the linear velocities of the moving media and k_2 is the corresponding coefficient for the second medium.

Obviously, for other apparatus, certain of these constants are replaced by others which express the analogous laws of diffusion.

The solutions of these equations will depend on the initial and the boundary conditions

$$u(x, 0) = u(x), \ v(x, 0) = v(x);$$

for direct flow

$$u(0, t) = \varphi_1(t), \ v(0, t) = \psi_1(t)$$

for counterflow

$$u(0, t) = \varphi_2(t), \ v(l, t) = \psi_2(t).$$

With the methods of operational calculus, the problem can be conveniently solved for any initial and boundary conditions for either direct flow or counterflow.

2. We now consider the more general case of heat exchangers, namely, heat exchangers with thick walls.

In the transient mode we shall take into account the thermal capacity of the walls separating the two media, ignoring, with this, the heat flux along the walls and the transient responses in the directions of the walls' lateral cross sections.

In the given case, the equations for counterflow have the form [4]

$$\begin{aligned}
&\frac{\partial T_1}{\partial t} + \frac{\partial T_1}{\partial x} w_1 = k_1(T - T_1), && T_1 = T_1(x, t),\\
&\frac{\partial T}{\partial t} = k_2 (T_1 - T) + k_3 (T_2 - T), && T = T(x, t),\\
&\frac{\partial T_2}{\partial t} - \frac{\partial T_2}{\partial x} w_2 = k_4 (T - T_2), && T_2 = T_2(x, t),
\end{aligned} \tag{2}$$

where k_1, k_2, k_3 and k_4 are constants; T, T_1, and T_2 are the temperatures of the walls and the moving media at cross section x in relative units.

3. Further, with account taken of the walls' thermal capacity and of the distribution of heat along the walls, the equations of the nonstationary heat exchange assume the form

$$\begin{aligned}
&\frac{\partial T_1}{\partial t} + \frac{\partial T_1}{\partial x} w_1 = k_1 (T - T_1),\\
&\frac{\partial T}{\partial t} - a \frac{\partial^2 T}{\partial x^2} = k_2 (T_1 - T) + k_3 (T_2 - T),\\
&\frac{\partial T_2}{\partial t} - \frac{\partial T_2}{\partial x} w_2 = k_4 (T - T_2).
\end{aligned} \tag{3}$$

With this, a is a constant which corresponds to the heat distribution along the walls.

4. If one considers general Eqs. (1) for the nonstationary processes in moving media as applied to apparatus in which chemical reactions occur, then one can obtain a generalization of these equations when account is taken of the reactions' thermal effects. With this, the question is considered only in a first approximation (the problem is linearized).

In the general case, the equations of the nonstationary processes take the form

$$\begin{aligned}
&\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} w_1 = \alpha_1 u + \beta_1 v + \gamma_1 T_1 + \delta_1 T_2,\\
&\frac{\partial v}{\partial t} - \frac{\partial v}{\partial x} w_2 = \alpha_2 u + \beta_2 v + \gamma_2 T_1 + \delta_2 T_2,\\
&\frac{\partial T_1}{\partial t} + \frac{\partial T_1}{\partial x} w_1 = \alpha_3 u + \beta_3 v + \gamma_3 T_1 + \delta_3 T_2,\\
&\frac{\partial T_2}{\partial t} - \frac{\partial T_2}{\partial x} w_2 = \alpha_4 u + \beta_4 v + \gamma_4 T_1 + \delta_4 T_2,
\end{aligned} \tag{4}$$

where u, v, T_1 and T_2 are indices of concentration and temperature in their proper relative units; α_i, β_i, γ_i, δ_i ($i = 1, 2, 3, 4$) are constants of the linear approximation.

If the reaction in the apparatus occurs in a diffusion region, then with account taken of the thermal effect in a first approximation, these equations, in the particular case, take the simpler form

$$\begin{aligned}
&\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} w_1 = k_1 (v - u) + m_1 v + m_2 T_2,\\
&\frac{\partial v}{\partial t} - \frac{\partial v}{\partial x} w_2 = k_2 (u - v) - m_3 v - m_4 T_2,\\
&\frac{\partial T_1}{\partial t} + \frac{\partial T_1}{\partial x} w_1 = a_1 (T_2 - T_1),\\
&\frac{\partial T_2}{\partial t} - \frac{\partial T_2}{\partial x} w_2 = a_2 (T_1 - T_2) + a_3 [k_2 (u - v) - m_3 v - m_4 T_2],
\end{aligned} \tag{5}$$

where k_i, a_i ($i = 1, 2$) and m_j ($j = 1, 2, 3, 4$) are the corresponding constants. Small changes in the k_i are compensated approximately by the proper choice of the m_j.

II. Equations of Stationary Processes of Moving Media Interaction in Engineering Apparatus

Under the conditions of a stationary mode of operation, the basic system of partial differential equations obtained becomes a system of ordinary differential equations. The solutions are obtained in an elementary manner in the form of linear combinations of characteristic (eigen) functions, and depend on the roots of the corresponding characteristic equations.

1. For the case corresponding to the general Eqs. (1), we get

$$w_1 = \frac{du}{dx} = k_1(v - u); \quad w_2 \frac{dv}{dx} = k_2 (v - u).$$

† In the given case, and in the sequel, the equations for direct flow processes are obtained by replacing w_2 by $-w_2$.

The characteristic equation has the form

$$\lambda^2 - \left(\frac{k_2}{w_2} - \frac{k_1}{w_1}\right)\lambda = 0.$$

2. For the second case, that of general Eqs. (2), with account taken of the walls' thermal capacity, the steady-state equations are simplified, and assume a form analogous to the foregoing:

$$w_1 \frac{dT_1}{dx} = \bar{k}_1 (T_2 - T_1), \qquad w_2 \frac{dT_2}{dx} = \bar{k}_2 (T_2 - T_1),$$

$$\lambda^2 - \left(\frac{\bar{k}_2}{w_2} - \frac{\bar{k}_1}{w_1}\right)\lambda = 0,$$

where

$$\bar{k}_1 = \frac{k_1 k_3}{k_2 + k_3}, \qquad \bar{k}_2 = \frac{k_2 k_4}{k_2 + k_3}.$$

In the conditions of the steady state, this corresponds completely to the physical meaning of the assumptions made earlier.

3. For the steady state of a heat exchanger when account is taken of heat flux along the walls, one obtains from general Eqs. (3) a system of ordinary differential equations which leads to a fourth-degree characteristic equation. With this, one of the roots equals zero, while the remaining three roots are found from the third-degree equation

$$\lambda^3 + \left(\frac{k_1}{w_1} - \frac{k_4}{w_2}\right)\lambda^2 - \left(\frac{k_3}{a} + \frac{k_1 k_4}{w_1 w_2} + \frac{k_2}{a}\right)\lambda + $$

$$+ \left(\frac{k_2 k_4}{a w_2} - \frac{k_1 k_3}{a w_1}\right) = 0.$$

4. In the remaining two cases, one obtains the corresponding fourth-degree characteristic equations.

III. Determination of the Transient Responses from the Basic Equations of Moving Media Interaction in Apparatus for the Case of Counterflow

Using the methods of operational calculus, one can easily solve the problem for direct flow and for counterflow for various methods of disturbing the steady state. In practice, these disturbances might be disturbances in concentration (or temperature), in the velocities of the moving media, and also in the volume of reagent applied to the input of the apparatus. Obtaining exact solutions for the most general equations is arduous. In those cases when such solutions can be found in general form, they are inconvenient, due to the complicated structure and the unwieldiness of the expressions.

It is convenient to carry out the analysis of quality of transient responses by operator methods, using as a basis their approximate representation by means of a series expansion of the transforms, as this was done in the monographs of A. V. Lykov [5] for the classical problem of heat conduction in nonmoving media. Such a choice of the approximate solution corresponds completely to the method of small disturbances which, in many cases, can be made basic to the investigation of transient responses from the point of view of the control problem.

However, while retaining coincidence of the approximate and exact curves of the transient response in the neighborhood of the initial point, we can also achieve their complete coincidence at the end point – in the steady state. With this, we employ an exponential law for the approximate solution‡ with the proper choice of the undefined coefficients in accordance with two conditions:

a) a common point and a common tangent with the curve of the exact solution at the beginning of the process;

b) a common asymptote as $t \to \infty$.

The solution has the form

$$u \,|\, \text{ or } \; T_1 \,| = \begin{cases} 0 & \text{for} \quad t < \tau \\ \varphi(t-\tau) & \text{for} \quad t > \tau, \; \varphi(t) = \end{cases}$$
$$= m - n e^{-rt}, \tag{6}$$

where τ is the lag time, and $\underline{m}$, $\underline{n}$ and $\underline{r}$ are determined in correspondence with conditions a) and b) for the given form of basic equations and the method of disturbing the steady state.

We consider some examples.

1. <u>The transient response of the first medium at the apparatus output for a unit step disturbance of its concentration (or temperature) at the input.</u> In the descriptions of the transient responses we shall introduce dimensionless complex parameters and, as the unit of time, we choose $(\tau_1 + \tau_2)/2$ $(\tau_1 = l/w_1, \tau_2 = l/w_2)$.

By assuming zero initial conditions, we find the expression for the curve of the transient response in the form of (6) with the corresponding relationships for $\underline{m}$, $\underline{n}$ and $\underline{r}$, while taking $\tau = 2/(1 + (w_1/w_2))$.

For general Eqs. (1), we get

$$m - n = e^{-2\alpha}, \; nr = 2\alpha\beta e^{-2\alpha}, \; m =$$

$$= \frac{(\alpha - \beta)\, e^{2(\beta-\alpha)}}{\alpha - \beta\, e^{2(\beta-\alpha)}}.$$

Here, the two dimensionless complex parameters are

$$\alpha = \frac{l k_1}{2 w_1}, \qquad \beta = \frac{l k_2}{2 w_2}.$$

For general Eqs. (2) we get

$$m - n = e^{-2\left(1 + \frac{k_2}{k_3}\right)\alpha}, \; nr =$$

$$= 2\alpha^2 \left(1 + \frac{k_2}{k_3}\right)\left(\frac{k_2}{k_1} + \frac{\beta}{\alpha}\frac{k_3}{k_4}\right) e^{-2\left(1 + \frac{k_2}{k_3}\right)\alpha},$$

$$m = \frac{(\alpha - \beta)\, e^{2(\beta-\alpha)}}{\alpha - \beta e^{2(\beta-\alpha)}},$$

where there are now four dimensionless parameters which, in the given case, have the form

‡In the first approximation, the exponential law is generally characteristic of processes flowing at the expense, as it were, of a loss of activity. This is attested by theoretical considerations and much experimental data.

$$\alpha = \frac{\bar{k}_1 l}{2w_1}, \quad \beta = \frac{\bar{k}_2 l}{2w_2}, \quad \frac{k_2}{k_3}, \quad \frac{k_3}{k_4}.$$

Note: With the given method of disturbing the steady state, the sole case of general equations of the form of (3) requires special consideration, since the curve of the transient response will have a break at time $t = 2/(1 + (w_1/w_2))$.

For general Eqs. (4), the quantity m, as in the previous cases, is simply determined from the conditions for the new steady state, while n and r are determined from the following relationships:

$$m - n = \frac{1}{2}\left(1 + \frac{1}{k}\right)\varepsilon_1 + \frac{1}{2}\left(1 - \frac{1}{k}\right)\varepsilon_2, \quad nr =$$

$$= \left(1 + \frac{1}{k}\right)\frac{l^2(\beta_1\alpha_2 + \delta_1\alpha_4)}{4w_1w_2} \times \left(\varepsilon_1 - \frac{1-k}{1+k}\varepsilon_2\right) -$$

$$- \frac{(1-k^2)}{2k^2}\frac{l}{2w_2}\left(\frac{\beta_1\alpha_2 + \delta_1\alpha_4 - \beta_3\gamma_2 - \delta_3\gamma_4}{\alpha_1 - \gamma_3} - \frac{\beta_3\alpha_2 + \delta_3\alpha_4}{2\alpha_3} -\right.$$

$$\left. - \frac{\beta_1\gamma_2 + \delta_1\gamma_4}{2\gamma_1}\right)(\varepsilon_1 - \varepsilon_2),$$

where

$$\varepsilon_1 = \exp\frac{[(1+k)\alpha_1 + (1-k)\gamma_3]\,l}{2w_1},$$

$$\varepsilon_2 = \exp\frac{[(1-k)\alpha_1 + (1+k)\gamma_3]\,l}{2w_1},$$

$$k = \sqrt{1 + \frac{4\gamma_1\alpha_3}{(\alpha_1 - \gamma_3)^2}}.$$

With this, the dimensionless complex parameters are introduced in correspondence with the structure of the expressions found.

For Eqs. (5), a particular case of (4), the corresponding relationships for m, n and r have the form

$$m - n = e^{-2\alpha}, \quad nr = e^{-2\alpha}\, 2\beta(\alpha + \alpha_{11} + \alpha_{23}),$$

where the following four dimensionless parameters have been introduced:

$$\alpha = \frac{lk_1}{2w_1}, \quad \beta = \frac{lk_2}{2w_2}, \quad \alpha_{11} = \frac{lm_1}{2w_1}, \quad \alpha_{2,3} = \frac{lm_2a_3}{2w_1}.$$

2. Transient response of the first medium at the output for a step disturbance of its linear velocity from w_1 to w_1 instantaneously over the entire length of the apparatus.

The question posed is meaningful basically for heat exchange apparatus. For the initial conditions, we take the condition of stationary (steady) apparatus mode of operation.

Despite the fact that, in the given case, the problem is nonlinear, the corresponding values of m, n and r will be easily computed for all three forms of the basic equations, (1), (2) and (3). With this, we always have $\tau = 0$, m-n = 0, where m is defined from the conditions of the new steady state for each case. Thus, for example, for the simplest general equations, (1) and (2), we obtain

$$r = \frac{\alpha e^{2(\beta-\alpha)}}{m\,[\beta e^{2(\beta-\alpha)} - \alpha]}\frac{w_1}{w_1}\left(1 + \frac{w_1}{w_2}\right)(\beta - \alpha)$$

for the proper expressions for α and β for each case.

3. Transient response of the first medium at the output for a step variation of the volume of reagent applied at the input while the concentration and linear velocity are maintained.** Such a posing of the problem is meaningful for all the apparatus considered, not just for heat exchangers. As before, we take the steady-state conditions to be the initial conditions.

As in the previous case, the problem is nonlinear but, for Eqs. (1), also leads to a simple solution, which can be extended as well to the more complicated Eqs. (4) and (5).

The results of the theoretical analysis of the transient responses in moving media are found to be in correspondence with the existing experimental data, obtained both under laboratory conditions and in factories. The theoretical and experimental curves of the transient responses, as well as for individual data — obtained in parallel experiments — are in good agreement.

As an illustration, Fig. 1 shows the theoretical (1) and experimental (2) curves for the transient responses in the case of Eqs. (1) for a step disturbance of the temperature of one of the media.

IV. Transfer Functions and Block Schematics of Engineering Apparatus as Objects of Control (Plants)

On the basis of Eqs. (1)-(5) of nonstationary processes in engineering apparatus, one can find the corresponding transfer functions, and construct the block schematics. With this, one can find either exact or approximate expressions for the transfer functions.

In particular, when we have to do with basic Eqs. (2) or (1), for the case of counterflow, we obtain the block schematic shown in Fig. 2.

Here,

$$K_{11}(p) = \frac{U_{\text{out}}(p)}{U_{\text{in}}(p)} \quad \text{for } V_{\text{in}} = 0;$$

$$K_{22}(p) = \frac{V_{\text{out}}(p)}{V_{\text{in}}(p)} \quad \text{for } U_{\text{in}} = 0.$$

$$K_{12}(p) = \frac{V_{\text{out}}(p)}{U_{\text{in}}(p)} \quad \text{for } V_{\text{in}} = 0;$$

$$K_{21}(p) = \frac{U_{\text{out}}(p)}{V_{\text{in}}(p)} \quad \text{for } U_{\text{in}} = 0.$$

If, as the unit of time, we take $(\tau_1 + \tau_2)/2$, letting $q = p(\tau_1 + \tau_2)/2$, then the exact expressions for the transfer functions will have the form††

** From which one immediately can obtain the solution of the problem with the condition that the volume and the concentration of the reagent applied at the input simultaneously change by jumps.

†† For the case of general Eqs. (1), all the expressions given below are considered in the limit as $\alpha_2, \alpha_3 \to \infty$.

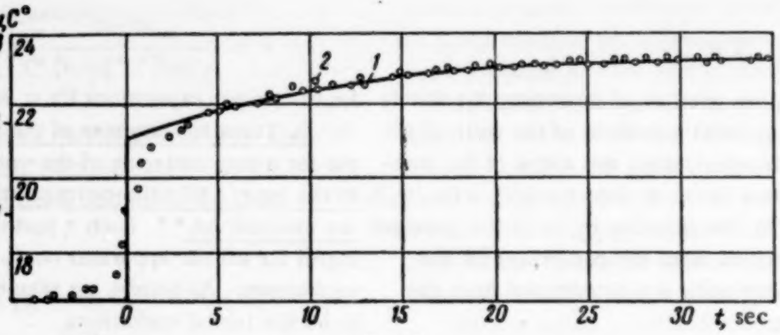

Fig. 1.

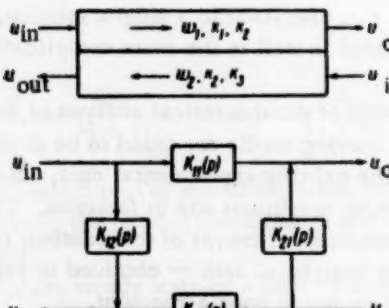

Fig. 2.

$$K_{11}(q)=e^{A(q)}W(q),\quad K_{22}(q)=e^{-A(q)}W(q)$$

$$K_{12}(q)=2\beta\frac{(1+qT_2)(1+qT_3)}{1+qT_{23}}\frac{\operatorname{sh}B(q)}{B(q)}W(q),$$

$$K_{21}(q)=2\beta\frac{w_2}{w_1}\frac{1}{1+qT_{23}}\frac{\operatorname{sh}B(q)}{B(q)}W(q),$$

where

$$W(q)=\frac{1}{\operatorname{ch}B(q)+\dfrac{C(q)}{B(q)}\operatorname{sh}B(q)},$$

$$B(q)=\sqrt{C^2(q)-4\alpha\beta\frac{(1+qT_2)(1+qT_3)}{(1+qT_{23})^2}},$$

$$A(q)=q\frac{\dfrac{w_1}{w_2}-1}{\dfrac{w_1}{w_2}+1}+\beta\frac{1+qT_2}{1+qT_{23}}-\alpha\frac{1+qT_3}{1+qT_{23}},$$

$$C(q)=q+\beta\frac{1+qT_2}{1+qT_{23}}+\alpha\frac{1+qT_3}{1+qT_{23}},$$

$$T_2=\frac{2}{1+\dfrac{w_1}{w_2}}\frac{1}{\alpha_2},\quad T_3=\frac{2}{1+\dfrac{w_1}{w_2}}\frac{1}{\alpha_3},\quad T_{23}=\frac{2}{1+\dfrac{w_1}{w_2}}\frac{1}{\alpha_2+\alpha_3}.$$

There are thus five dimensionless complex parameters in the transfer functions obtained: α, β, w_1/w_2, $\alpha_2 = l k_2/2w_1$, $\alpha_3 = l k_3/2w_1$.

The approximate representation of transfer function K_{11}, found from the corresponding expression for the transient response (6) has the form

$$\overline{K}_{11}(q)=m\frac{1+q\dfrac{m-n}{m}\dfrac{1}{r}}{1+q\dfrac{1}{r}}\exp\left(-q\frac{2}{1+\dfrac{w_1}{w_2}}\right).$$

An analogous expression is obtained for $\overline{K}_{22}$ by considering transient response (6) for the other medium.

From the approximate expressions $\overline{K}_{11}$ and $\overline{K}_{22}$ for basic Eqs. (1), (2), (4), and (5), one can easily observe the dependence of these transfer functions on the parameters and operating mode of the apparatus.

V. Definition of the Universal Characteristics of Persistence and Controllability for the Class of Apparatus Operating on the Counterflow Principle

From the theoretically obtained expressions (1) for the transient responses, one can find the characteristics of persistence and controllability of processes in engineering apparatus.

The persistence criterion S is introduced as a generalization of the plant's time constant [3]. The persistence S is defined as the magnitude of the area included between the curve of the transient response and the horizontal line corresponding to the value of the new steady state (Fig. 3).

In accordance with this definition, $S=\int_0^\infty[1-\bar{u}(t)]dt$ ‡‡

where $\bar{u}(t)$ is the expression for the transient response at the output of the apparatus (for the temperature or concentration of the reagent, respectively).

With respect to the ease of controlling a process, the positive effect of persistence, and the adverse effect of lag are well known. Consequently, as the controllability criterion R for the mode of apparatus operation, one can take the relative magnitude of the persistence (as compared with the magnitude of the lag time) $R = S/\tau$.

By thus defining the persistence and controllability of a process (S and R), we can find their general expressions for all the cases considered of basic equations and disturbing actions.

With this, using relationship (6), we find that

$$S=\frac{1}{m}\int_0^\infty ne^{-rt}dt=\frac{n}{rm}.$$

‡‡ In the particular cases, which arise frequently in technology, when the expression for the transient response has the form $y = 1-\exp(-t/T_a)$, we obtain $S = T_a$, where T_a is the plant's time constant.

In case of a dependency of the transient response on the initial conditions, the quantity S is even more meaningful, since it characterizes the persistence of the given initial steady state of apparatus operation. In the determination of the analytic expressions for the persistence and controllability characteristics in each case, the dimensionless complex parameters of the apparatus and its operating mode are introduced in the proper fashion.

We note that, in a control process, an action on the first medium is frequently effected by means of the second medium. Consequently, in defining controllability, it is reasonable to also take into account the other lag time by using the arithmetic mean $\tau = (\tau_1 + \tau_2)/2$. If, moreover, τ is taken as unity, then R = S.

We adduce several examples:

1. For disturbances of concentration (or, respectively, of temperature), we get:

a) for the case of general Eqs. (1),

$$S = R = \frac{e^{-2\alpha}}{2\alpha\beta}\,\frac{[(\alpha-\beta)e^{2\beta} + \beta e^{2(\beta-\alpha)} - \alpha]^2}{[\alpha - \beta e^{2(\beta-\alpha)}](\alpha-\beta)e^{2(\beta-\alpha)}}, \qquad (7)$$

b) for the case of general Eqs. (2),

$$S = R = \frac{n^2}{2m}\,\frac{\frac{k_1}{k_2}}{\left(1+\frac{k_2}{k_3}\right)^2 \alpha^2 e^{-2\left(1+\frac{k_2}{k_3}\right)\alpha}\left[1+\frac{\beta}{\alpha}\frac{k_1}{k_2}\frac{k_3}{k_4}\right]}$$

where

$$m = \frac{(\alpha-\beta)\,e^{2(\beta-\alpha)}}{\alpha-\beta\,e^{2(\beta-\alpha)}}, \quad n = m - e^{-2\left(1+\frac{k_2}{k_3}\right)\alpha}.$$

2. With disturbances of velocity of the first medium, we obtain, for Eqs. (1) and (2),

$$S = \frac{(\bar{\alpha}-\beta)\ {}^{2(\beta-\bar{\alpha})}}{\bar{\alpha}-\beta e^{2(\beta-\bar{\alpha})}}\,\frac{[\beta e^{2(\beta-\alpha)} - \alpha]}{\alpha e^{2(\beta-\alpha)}}\,\frac{1}{\beta-\alpha}\,\frac{1}{1+\frac{w_1}{w_2}}\,\frac{w_1}{\bar{w}_1},$$

where, in the case of Eqs. (1),

$$\alpha = \frac{lk_1}{2w_1}, \quad \beta = \frac{lk_2}{2w_2}, \quad \bar{\alpha} = \frac{lk_1}{2\bar{\bar{w}}_1},$$

and, in the case of Eqs. (2),

$$\alpha = \frac{l\bar{k}_1}{2w_1}, \quad \beta = \frac{l\bar{k}_2}{2w_2}, \quad \bar{\alpha} = \frac{l\bar{k}_1}{2\bar{\bar{w}}_1},$$

From the persistence and controllability characteristics, obtained in their general form and qua criteria, one can easily carry out their analytical and graphical analysis.

As an example, Fig. 4 shows the controllability curves R(β) for various values of α and for fixed values of the remaining parameters ($k_1/k_2 = k_2/k_3 = 0.5$, $k_3/k_4 = 2$). Fig. 5 gives the curves R(β) under the condition that there be maintained the given process completeness*** P = 1−m, m = 0.8 for k_3/k_4 = 0.5; 1; 2; 5 ($k_1/k_2 = 0.5$, $k_2/k_3 = 1$). The condition that a definite completeness of the process be given is a standard practical requirement.

In all cases, just as in those of the examples given (cf. Figs. 4 and 5), one can graphically give a comparative estimate of persistence and controllability of processes as functions of the parameters and operating modes of the apparatus. With this, one can immediately take into account any additional condition relating the possible variations of these parameters. In the example given (Fig. 5), such a condition was the specification of a definite process completeness.

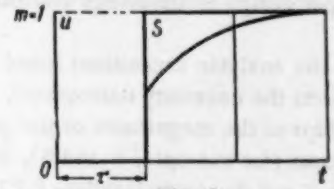

Fig. 3.

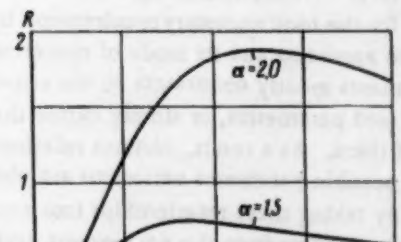

Fig. 4.

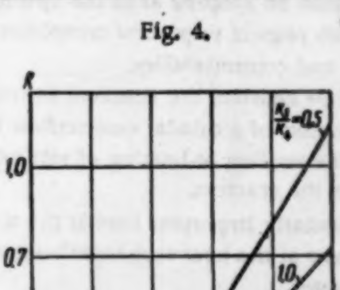

Fig. 5.

The domains of variation of α, β and other parameters for various apparatus can be established from concrete industrial data in each individual case. After this, if necessary, one can easily extend the corresponding graphs

***As the process completeness P, one takes the relative change of the input quantity after its passage through the apparatus, i.e.,

$$P = (u_{in} - u_{out})/u_{in} = 1 - m.$$

from the general analytic expressions for persistence and controllability of the process.

From the existing experimental data on heat exchangers, the values of α and β are found to lie within the limits shown on the graphs. In particular, a heat exchanger was tested in almost the conditions of actual practice, giving the figures $\alpha \approx 1.2$, $\beta \approx 0.6$, $\alpha \approx 0.85$, $\beta \approx 0.5$.

VI. Design Methodology. Example

By using the results obtained for the choice of the optimal parameters of engineering apparatus, we can take into account the requirement that the controllability be as good as possible.

The proper design methodology consists of the following:

1. From the analytic expressions found for R, one easily constructs the necessary nomograms, expressing the relationships of the magnitudes of the generalized plant parameters (for example, α and β), the process completeness P and the controllability R.†††

2. As is obvious, technological and economic conditions fix the most necessary requirements imposed on the concrete apparatus and its mode of operation. These requirements specify constraints on the erstwhile free, generalized parameters, or simply define the values of some of them. As a result, definite relationships connecting the possible parameter variations are obtained.

3. By taking these relationships into account, we choose the free quantities from the nomograms, trying for the greatest possible value of controllability R.

As a rule, depending on the conditions, a compromise solution must be adopted as to the optimal plant parameters with respect to process completeness, apparatus economy and controllability.

We now consider the concrete example of choosing the parameters of a tubular counterflow heat exchanger, used for the cooling or heating of salt solutions before they enter the reactors.

Particularly important here is the stabilization of the temperature at the heat exchanger's output prior to entry into the reactor.

1. We construct the simplest nomogram from expression (7). The nomogram curves must correspond to equal values of heat exchange process completeness.

2. Let it be required to choose the parameters for two heat exchangers with heat exchange completeness of P = = 0.1 and P = 0.6. With this, the technical data dictates that, in both cases, the generalized parameter β must be chosen only within the limits $0.6 \leq \beta \leq 1.2$.‡‡‡ With this, in accordance with expression (7), with large values of process completeness P, the magnitude of R increases sharply as β increases, and, for small values of P, it remains almost unchanged.

3. By considering the variation of the plant's controllability R on the interval (0.6; 1.2), either by nomogram or by formula (7), we arrive at the following conclusions:

a) in the given interval of variation of β, magnitude of the controllability R remains almost unchanged for the first heat exchanger. Therefore, the choice of β does not affect the plant's controlling properties[1] and, in this respect, can be made arbitrary.

b) for the second heat exchanger, the magnitude of R varies significantly in the given interval. In the given case, it is advantageous to choose the limiting value of $\beta = 1.2$, so as to provide the largest controllability of the process. It is desirable to make such a choice of β even in the case when this entails some insignificant deterioration of the other characteristics of the heat exchanger.

SUMMARY

1. The nonstationary problem of the interaction of moving media in engineering apparatus was considered.

2. A method was obtained for finding an approximate analytic expression for the transient responses in the commonest case of motion of media by the counterflow principle.

3. The characteristics of process persistence and controllability were obtained, in general form and <u>qua</u> criteria, for the broadest class of engineering apparatus operating on this counterflow principle. These characteristics have simple analytical expressions in elementary functions, and depend on a minimal number of dimensionless complex parameters, which provides ease and availability of the engineering designs.

4. Thus, possibilities were disclosed for the study of the dependence of some transient response or another on changes in system parameters for a wide class of engineering apparatus. The apparatus characteristics thus found are necessary for the solution of two problems which are important in practice: a) for the design of apparatus for an optimal steady-state operating mode with account taken of the indices of stability of this mode; b) for the simultaneous and compatible design of controller and plant.

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††† A special work [6] is devoted to the construction of such nomograms. The nomograms therein given eliminate the necessity of constructing the corresponding curves in each concrete case.

‡‡‡ After the definitive choice of β, one can determine the corresponding value of α, and from this find the diameter and number of the tubular heat exchanger's internal tubes.

[1] By the concept "controlling properties" we have in mind the properties rendering possible the attainment of the best control quality (for example, a large persistence and a small lag).

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AUTOMATIC CONTROL OF THE TURBINE DRILLING PROCESS BY MEANS OF ADAPTIVE SYSTEMS

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Moscow

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The paper considers automatic control devices, used for choosing optimal parameter ratios under changes in the external conditions of turbine drilling.

Results are adduced from the development of control schemes with respect to two parameters and of extremal type schemes wherein account is taken of the pecularities associated with the specific object of control.

An extremal device, based principally on discrete elements, is described. A semiautomatic control scheme is provided, the basis of the scheme being a scan of the technological process over a wide range of parameter variation.

The problem of automating the control of the turbine drilling process of oil and gas wells is the choice and maintenance of the optimal values of two parameters: the axial force P on the gouge and the rotational speed n of the gouge under variations in drilling conditions. With the turbine method of drilling, there is ordinarily implemented operational control only of the quantity P in the limits of 5 to 35 tons, by changes in the weight of the column of drilling pipe which gives rise to the axial force on the gouge. The turbine has the so-called "soft" characteristic, so that, with increasing axial force P, the velocity n of gouge rotation can be varied within the limits of 1200 to 1400 rpm. The partial derivative $\partial n/\partial P$ depends on the properties of the rock being drilled and on the state of the gouge.

An automatic control device is used to choose the optimal parameter ratios by adjusting the system parameters as the external drilling conditions change.

Automatic Control in Terms of One Parameter

The control system includes an automatic controller of one parameter as one of its elements. In addition to this, a one-parameter automatic controller has independen applications.

In the sequel we shall use the notation: Q is the difference between the total weight of the column of tubing Q_0 and the force Q_1 at the upper end of the column, V_s is the velocity supplied to the column's upper end and V_d is the drilling speed.

In the steady state, and with no friction, $P = Q$ and $V_d = V_s$.

Ordinary controllers maintain the value of Q, given by a setting Q_c. The magnitude of Q_c is varied by the control device or by a driller, manually.

Figure 1 shows the functional schematic of the type AVÉ automatic controller.

The force Q is measured by a transducer – a spring element, transforming force to a selsyn's angle of rotation. The angle of rotation of a second selsyn is a given setting. The difference between the angles of the transducer and setting selsyns, transformed to an ac signal, is applied to the input of an electronic amplifier; the power is then amplified by a rotary amplifier and applied to the excitation winding of a generator which supplies the dc executive motor. This motor implements the supply of the drilling instrument. Due to this, the forces P and Q are varied until such time as the difference between the angles of the transducer and setting selsyns becomes close to zero.

In the AVÉ scheme there are, not two, but four selsyns, which give the capability for both manual and mechanized (for automatic control) introduction of the settings.

Both rigid and flexible stabilizing connections are provided in the system.

Type AVÉ automatic controller was tested under industrial conditions. The tests showed that the AVÉ provides the required static and dynamic indices for the existing changes in drilling conditions, including drilling speeds close to zero. Today, design work is proceeding for several modifications of type AVÉ controllers for different drilling stands.

Automatic Control in Terms of Two Parameters

The drilling process is still insufficiently studied, which renders difficult the determination of the best law of automatic control. The development of automatic control devices proceeds step-by-step in parallel with the development of the technology.

Each of the devices developed, being based on definite technological assumptions, verifies in its turn the correctness of these assumptions. It is this wherein consists the complexity of solving the problems of optimal control and optimal technology.

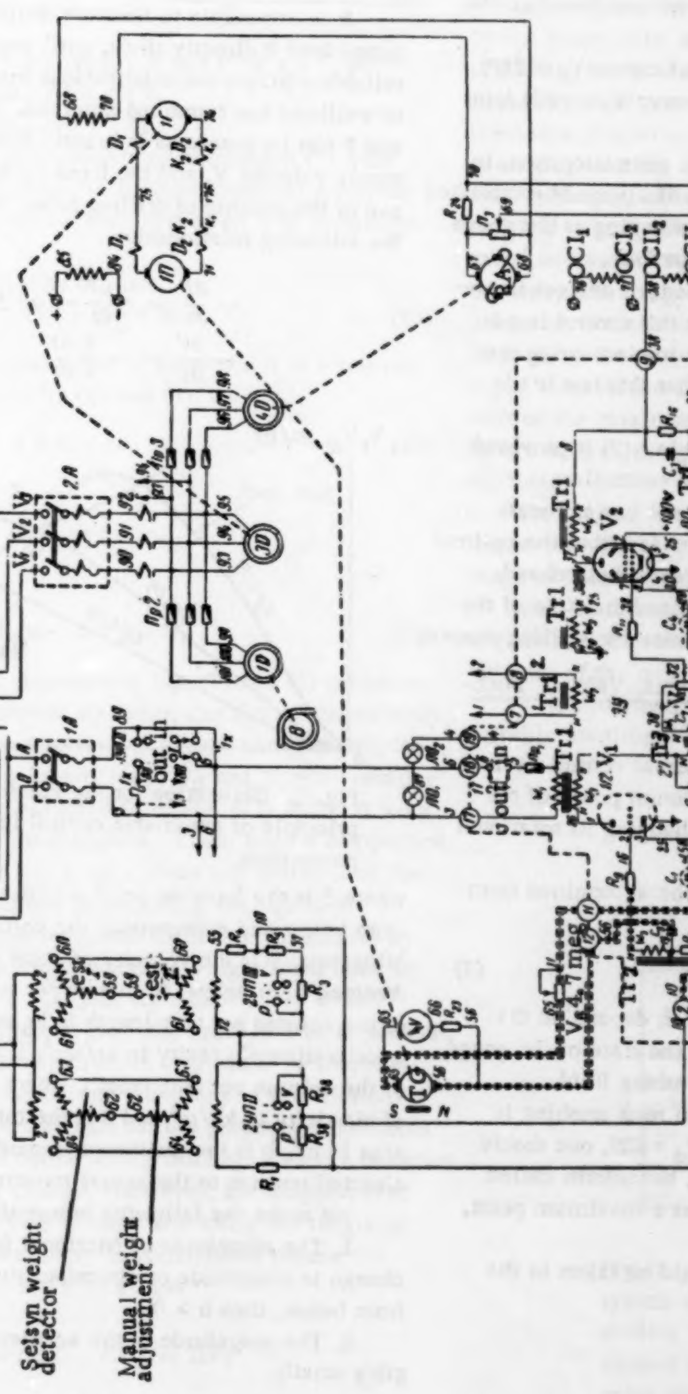

Fig. 1

The simplest system providing variation of axial force P when drilling conditions change is a device for maintaining a given relationship

$$V_s = \varphi(Q). \tag{1}$$

In a particular case, this relationship has the form

$$k_1V_s + k_2Q = C, \tag{2}$$

where k_1 and k_2 are given coefficients, and C is a value given by the setting.

The development of such a device was based on the following technological assumptions.

For different rocks, the family of curves $V_d = f(P)$ has the form shown in Fig. 2. The curve $V_s = \varphi(Q)$ joins points of optimal values of P.

If control law (1) is given to the controller, then, in the steady state for $V_s = V_d$, there will always be established a definite value of axial force P_{oi}, changing as the gouge enters rocks of different degrees of hardness.

Testing, under industrial conditions, of devices implementing control law (2) showed that this control law is unsuitable, with turbine drilling, for implementing the basic process of crushing rock, but that this law is advantageous for working wells.

Today, capability of control by law (2) is provided in several modifications of type AVE controllers.

The unsuitability of law (2) for the basic process of turbine drilling is explained by the fact that the optimal points P_{oi}, V_{oi} cannot be given correctly beforehand, since it is impossible to know beforehand the state of the gouge and other factors which determine the drilling process.

Automatic Control in Terms of Extremal Values. Technological Premises

The purpose of developing extremal control devices (ECD) for drilling is to realize maximum power of the turbine drill on the gouge without changing its rotational speed.

The turbine drill power N_1 can be determined from the formula

$$N_1 = k_3P - k_4P^2, \tag{3}$$

where k_3 and k_4 are coefficients which depend on the physical design of the turbine drill, the state of the gouge, the properties and quantity of the washing fluid.

Considering that the intensity of rock crushing is proportional to the applied power $V_d = CN$, one should assume that the function $V_d = f(P)$, henceforth called the turbine drilling curve (TDC), has a maximum point, at which $\partial V_d/\partial P = 0$.

Therefore, the control law should be taken in the form

$$\frac{\partial V_d}{\partial P} = \delta \quad (\delta \geqslant 0). \tag{4}$$

In some cases, the control system, for $\delta < 0$, is made unstable. Therefore, δ should be chosen with some margin of error. Details on this will be given below.

Turbine drilling curves $V_d = f(P)$ and $N = Pn = f_1(P)$, taken off experimentally,* essentially confirmed the assumptions made.

On Control System Structures in Connection with the Pecularities of Measurement of the Quantities and with the Presence of Noise

It is impossible to measure drilling speed V_d and gouge load P directly since, until today, the problem of reliable wireless communications from locus of drilling to wellhead has remained unsolved. The values of V_d and P can be measured indirectly from the values of the supply velocity V_s and the force Q_1 applied at the upper end of the column of drilling tube. With this, we use the following relationships:

$$-\frac{\partial F}{\partial x} = q\frac{\partial V}{\partial t} - qg \pm h, \tag{5}$$

$$-\frac{\partial V}{\partial x} = -\frac{1}{k}\frac{\partial F}{\partial t}, \tag{6}$$

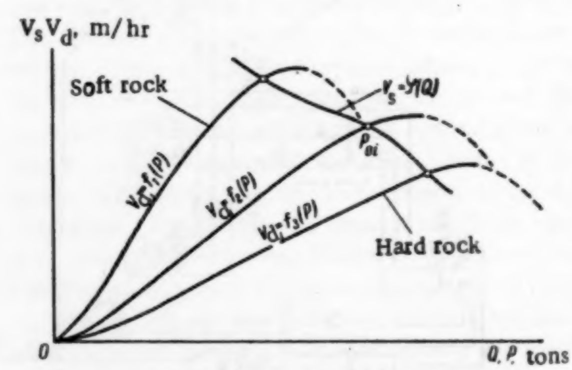

Fig. 2. Illustration for the elucidation of the principle of automatic control in terms of two parameters.

where F is the force on point x of the column, directed from below and compressing the column, measured in kilograms, V is the velocity of point x of the column, directed from below, in meters per second, q is the mass of the column per unit length in $kg{\cdot}sec^2/m^2$, g is the acceleration of gravity in m/sec^2, k = ES is the rigidity of the column per unit length, where E is the modulus of elasticity in kg/m^2 and S is the tube's cross-sectional area in m^2, h is the resistance to motion per unit length, directed counter to the actual movement.

We make the following assumptions:

1. The magnitude of frictional force h does not change in magnitude or direction; since the tube is moved from below, then h > 0.

2. The magnitude of the acceleration force is negligibly small:

* We assume that the magnitude of N is proportional to N_1.

$$q\frac{\partial V}{\partial t} \ll qg - h. \quad (7)$$

For the determination of the subsurface parameters V_d and P from the surface parameters V_s and Q_1, we obtain the following simplified expressions:

$$P = (qg - h)\,l - Q_1, \quad (8)$$

$$V_d = -\frac{l}{k}\,\frac{\partial P}{\partial t} + V_s. \quad (9)$$

Here, V_d and P are simply V and F for $x = l$, and V_s and $-Q_1$ are V and F for $x = 0$.

For $h \ll qg$, we have $P = Q$.

The limits of applicability of assumption (7) were determined on an electromechanical model of a turbine drill, connected to actual elements of an AVÉ automatic controller.

If force Q_1 varies by the law

$$Q_1 = Q_{1_0} - \alpha t, \quad (10)$$

where Q_{1_0} is the initial value of force and α is a constant; then, in accordance with (8) and (9), we get

$$P = P_0 + \alpha t, \quad (11)$$

where

$$P_0 = (qg - h)\,l - Q_{1_0}$$

and

$$V_d = -l\alpha/k + V_s. \quad (12)$$

On the model, reproducing Eqs. (5) and (6) connected to actual AVÉ controller elements, the sign of the velocity introduced by the controller setting was changed and, for various α and l, the quantities P, V_d and V_s were oscillographed. Graphs constructed from Eqs. (11) and (12) were laid on these same oscillograms. Then, from a comparison of the oscillograms and graphs, there was determined the time t from the moment of switching settings to the moment at which the replacement of Eqs. (5) and (6) by Eqs. (8) and (9) no longer entailed a significant error in the measurement of V_s. This time depends on α and l and, for $l < 2000$ meters, is of the order of 3 to 10 seconds.

The basic assumption (which may entail an essential error) is the assumption of the constancy of the frictional force h. If it is assumed that all the points of the column are translated from below during drilling, then the direction of force h does not change. However, the magnitude of h can be changed, both in time and along the length of the column, due to random unaccountable causes.

From Eqs. (11) and (12) we get

$$\Delta P = \alpha \Delta t, \quad \Delta V_d = \Delta V_s.$$

Whence control law (4) can be given in the form

$$\delta = \frac{\Delta V_s}{\alpha \Delta t}, \quad (4')$$

where the increment ΔV_s is chosen for α = const on the interval of time Δt, and $\delta \geq 0$ is a given quantity.

Averaging in the Choice of Values of ΔV_s and Δt

In the limit, the quantity $\Delta V_s/\Delta t$ is the acceleration of shaft sinking but, for its reliable determination, the quantities ΔV_s and Δt must have finite values. This is explained by the following: the process of rock crushing by the gouge must have a discrete character; the column of drilling tube, as a rule, is subject to oscillations due to pulsations of pressure of the wash fluid and also due to resonance properties. Moreover, the sinking of the column is accompanied by a change in the frictional force on the well walls (cf., the relationships shown below on Figs. 12 and 13). This makes the measurement on instantaneous speed and acceleration inapplicable, and creates the necessity of averaging the measured quantities. The minimum duration of the interval of measurement is limited by the aforementioned sources of noise, by the signal's maximum frequency, which is determined by the shift of the maximum as the rock's drill resistance changes, by the state of the gouge and by other conditions of drilling.

For the test specimen of the control devices, we chose $\Delta t \approx 10$ seconds, for ΔP = 1 to 2.5 tons, and a range of variation of $P \approx 5$ to 35 tons.

The minimum value of ΔP is limited by the dispersion of points for the determination of ΔV_d, and by the small velocity of the output in the optimum zone.

The maximum value of ΔP is limited by the admissible amplitude of oscillation in the optimum zone.

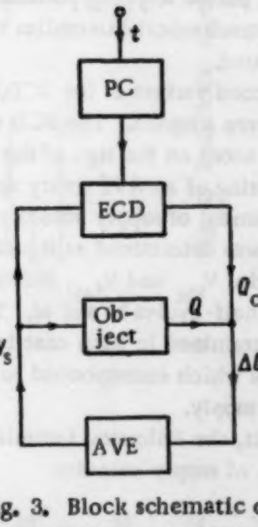

Fig. 3. Block schematic of a system with an extremal control device. ECD is the extremal control device, PC is the program commutator and AVÉ is the automatic controller.

Basic Schemes of Extremal Control Devices (ECD)

When averaging over a lengthy time interval (~ 10 seconds) is employed, it is necessary to stabilize the load on the gouge during this interval. Therefore, the system with the ECD was connected in a two-loop scheme (Fig. 3).

The first loop is a high-speed control system, used for stabilizing the load on the gouge. This loop comprises a servosystem in which the quantity Q, equal to P in the steady state, follows the given value of the setting Q_c. This loop is implemented by analog elements.

The second loop is a slow-acting, but accurate system used for the implementation of the control law $\partial V_d/\partial P = \delta$ by controlling the controller setting. This loop is implemented, in whole or in part, by discrete (digital) elements.

In the development of the ECD, the methods adopted for measuring the supply velocity V_s were to measure the increment of supply ΔS during a calibrated time interval Δt, or to measure the time increment Δt for a calibrated supply interval ΔS. Various ECD schemes were analyzed in correspondence with these methods. The simplest were the relay ECD systems. In these schemes, only two positions were provided for the organ for introducing the setting Q_c as a function of the sign.

Two variants of the relay schemes were developed, one with variations in the magnitude of setting Q_c, and the other with changes in sign of the velocity of introducing the setting Q_c.

ECD Circuit Elements and Physical Construction

In the first variant of the ECD, calibrated supply intervals were adopted. The ECD was built of basically analog electromechanical elements [2, 3] and acted on the settings of a type AN-1 electropneumatic controller by means of a pulsed stepping potentiometer. The design of the electromechanical assemblies turned out to be very complicated.

In the second variant of the ECD, calibrated time intervals Δt were adopted. The ECD was built of digital elements and acted on the sign of the velocity of introducing the setting of an AVÉ rotary controller.

The increment of supply velocity ΔV_s on the interval Δt [cf., (4')] was determined as the difference of the supply velocities $V_{s_{bi}}$ and $V_{s_{ai}}$, defined on the two measurement half-cycles bi and ai. The velocity of supply was determined in each case by the number of pulses, each of which corresponded to a definite small magnitude of supply.

As a result, the following formula was obtained for the increment of supply velocity:

$$\Delta V_{s_i} = V_{s_{bi}} - V_{s_{ai}} = \frac{n_{bi} - n_{ai}}{\Delta t}\gamma, \tag{13}$$

where γ is a proportionality factor relating magnitude of supply to number of pulses.

The measurement half-cycles begin after that moment of time when Eqs. (8) and (9) become valid.

For pulse addition, reversible counters were used. To determine the sign of the difference $\Delta V_s/\alpha\Delta t - \delta$, [cf., (4')], a nominal quantity of pulses n_N were introduced into the counter. Let $n_N = \alpha\Delta t^2\delta/\gamma$. By substituting n_N and Δ_{si} in (4'), we obtain the algebraic sum

$$\sum n = n_{bi} - n - n_N, \tag{14}$$

whose sign defines the sign of the expression $\Delta V_s/\alpha\Delta t - \delta$.

For $\delta = 0$, the scheme operates in the following manner. Let the setting be increased, and let $\partial Q_c/\partial t = \alpha > 0$. Increasing Q_c leads to an increase of Q and P.

The measurement half-cycle begins at time t_1. During the first half-cycle, $t_2 - t_1 = \Delta t$, there occurs subtraction followed by storage of the negative number of pulses $-n_{ai}$. During the second half-cycle $t_3 - t_2 = \Delta t$, there occurs the addition of n_{bi} followed by reading of the difference $\Delta n_i = n_{bi} - n_{ai}$. As a function of the sign of the difference Δn_i there is not carried out (for a positive sign) or there is carried out (for a negative sign) a switching of the sign of the velocity of introducing the setting.

For operation by the law $\partial V_d/\partial P = \delta$, where $\delta > 0$, after the first half-cycle there is introduced the nominal quantity of pulses $\pm n_N$.

The basic element of the scheme is the reversible counter, which does reading, storage and subtraction of pulses in accordance with the commands obtained from the program commutator (Fig. 4).

The assembly for setting introduction consists of a selsyn driven at a slow rotational speed by a dc motor via a reducer.

The program commutator consists of disks rotated by a synchronous motor via a reducer.

The reversible counter is a ten-place electron-tube counter. It has the built-in capability of being switched to subtraction and addition, as well as the capability of having a nominal number of pulses n_N introduced into it.

The pulse transducer consists of a selsyn, rectifier, and flip-flop with total feedback used for pulse shaping. Each rotation of the selsyn corresponds to an advance of the upper end of the drill column by a fraction of a millimeter.

Figures 5 and 6 show exterior views of the assemblies of the extremal device.

Testing and Directions of Further Improvement of the ECD Scheme

Testing of the extremal devices was carried out on the turbine drilling model and under industrial conditions, with drilling being carried out at depths up to 2000 meters.

For the tests, at the lower end of the model drilling tube column, there was simulated the turbine drilling curve (TDC) of the extremal type, close to a sinusoid within the limits of 0 to π. In addition, there was simulated dry friction, applied to several points of the drill column.

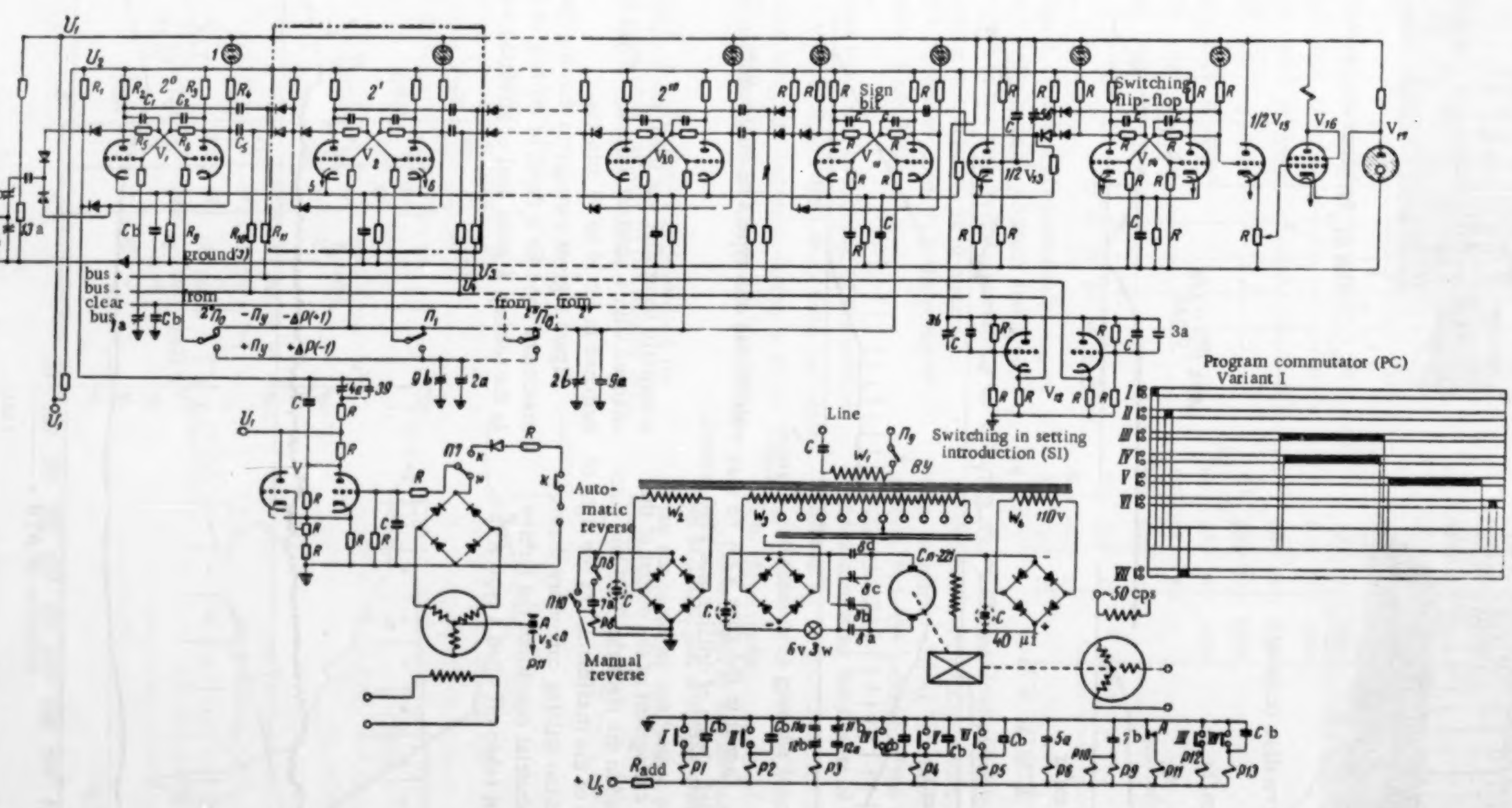

Fig. 4. Functional schematic of an ECD built of digital elements.

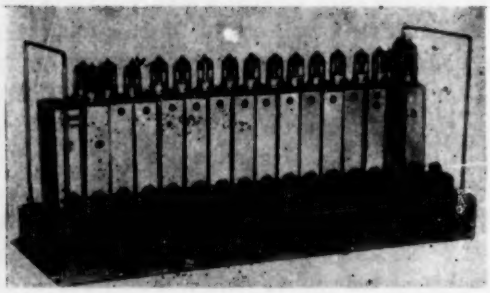

Fig. 5. Reversible counter.

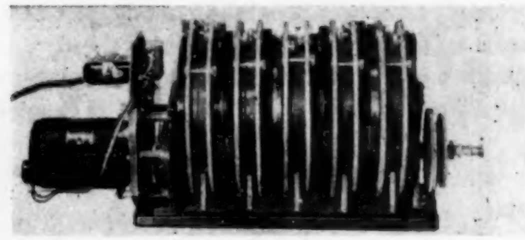

Fig. 6. Program commutator.

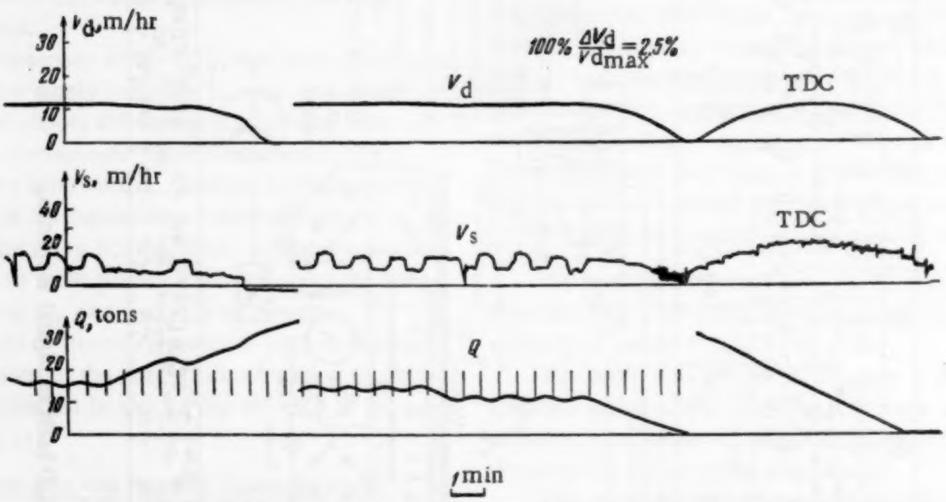

Fig. 7. Output in the zone $V_d = V_{d\ max}$ with the use of a type ATB ECD system. A model. Depth of drilling was 1000 meters.

Figure 7 shows the cartograms of the output at the maximum to the left, and to the right, the TDC and auto-oscillations in the zone of the maximum. At the right of this same figure, the turbine drilling curves are shown.

In testing under industrial conditions the turbine drilling curves were first taken off when a type AVÉ automatic controller was used, the controller setting being changed with a constant velocity. Then the ECD was switched in for 10 to 20 minutes.

Figure 8 gives cartograms showing the output in the maximum zone for a depth of drilling of 1950 meters. In the testing, a great deal of attention was given to the

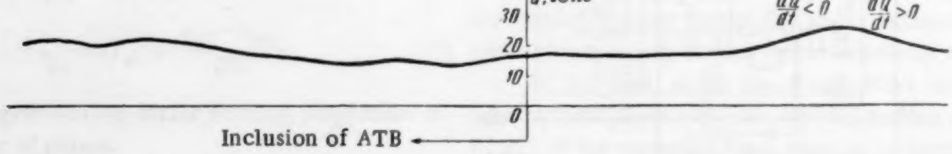

Fig. 8. Output in the zone $V_d = V_{d\ max}$ with the use of a type ATB ECD system. Industrial testing. Depth of drilling was 1950 meters (V_s in meters/hour).

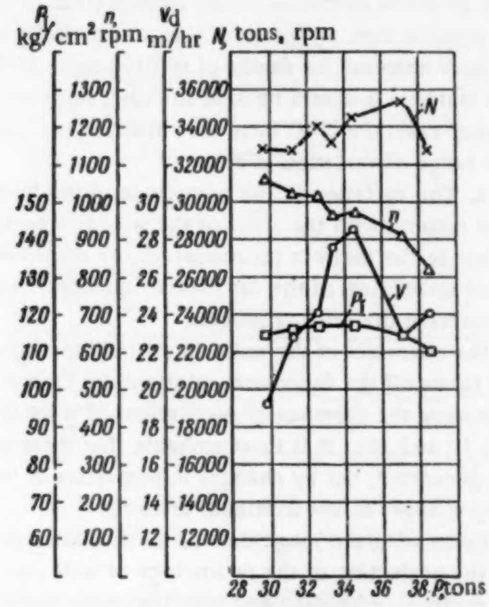

Fig. 9. Functional relationships V_d, n, N = f(P), constructed from averaged values of the data from industrial testing at a depth of 1670 meters. N = Pn in tons rpm, P_I is the pressure of the wash fluid.

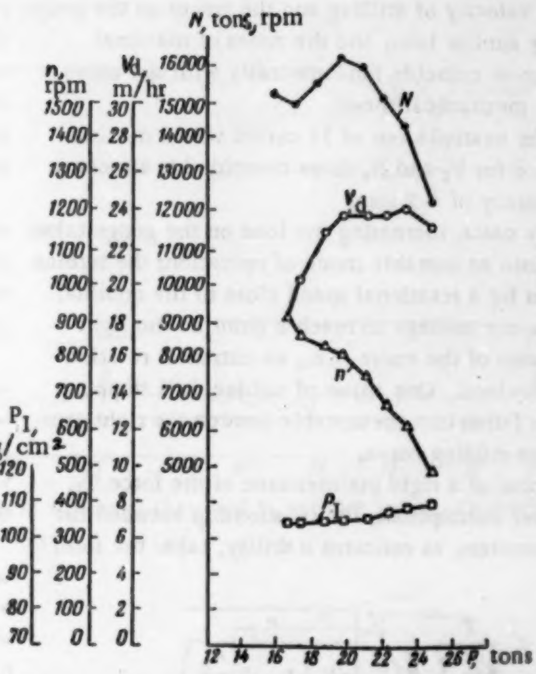

Fig. 10. Functional relationships V_d, n = f(P), constructed from averaged values of the data from industrial testing at a depth of 1900 meters. P_I is the pressure of the wash fluid.

taking off, under industrial conditions, of the turbine drilling curves $V_d = f(P)$, and their study from the point of view of determining the extremal character of the TDC, and the extent of their monotonicity on the individual segments [4].

To this end, an instrument was developed for measuring the rotational velocity of the turbine drill, and transmitting the results of the measurement to the surface.

Figures 9-11 give the functional relationships of V, n, N = f(P, t).

On the basis of the investigation and testing of the two ECD designs, one may draw several conclusions and map out plans for the further perfecting of ECD for turbine drilling:

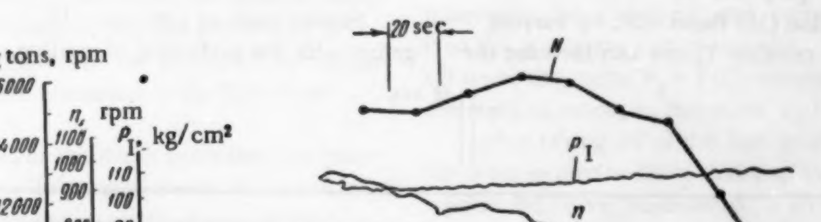

Fig. 11. Portions of the cartograms with recordings of the current values of Q, V_s and n corresponding to Fig. 10.

1. The velocity of drilling and the power on the gouge vary by very similar laws, and the zones of maximal choice of power coincide fundamentally with the zones of maximal mechanical speed.

Thus, for example, out of 10 curves where maxima were obtained for V_s and N, these coincided in six cases with an accuracy of ± 2 tons.

In many cases, increasing the load on the gouge takes the system into an unstable mode of operation; the turbine drill subsides for a rotational speed close to the nominal one, and does not manage to reach a point on the right descending arm of the curve—i.e., an extremal relationship is not attained. One cause of turbine drill stopping is that it has fallen into the unstable zone on the right arm of the turbine drilling curve.

In the case of a rigid maintenance of the force Q_1, and with other assumptions, the relationship between the critical parameters, as concerns stability, takes the form [5]

$$l_c < \frac{T}{\sqrt{\frac{q}{k}}}\sqrt{\frac{n_c}{n_{fr}\left(1-2\frac{n_c}{n_{fr}}\right)}}$$

$$\operatorname{arc\,tg}\frac{1}{c_1\sqrt{qk}}\sqrt{\frac{1}{n_{fr}n_c\left(1-2\frac{n_c}{n_{fr}}\right)}}. \qquad (15)$$

Here, l_c and n_c are the critical values of the tube column length and the turbine drill's speed of rotation, n_{fr} is the turbine drill's free-running speed, T is the turbine drill's time constant, and c_1 is a coefficient defining the rock's resistance to drilling.

This result was obtained by use of the method of Chebotarev, which was developed by him for the analysis of the stability of quasi-polynomials.

Analysis of expression (15) shows that, by varying the turbine drill's time constant T, one can increase the range of stable operation on the right portion of the TDC; it is possible that, by virtue of this, the majority of TDC will have extrema for depths of drilling up to 2000 to 3000 meters. It should be kept in mind, however, that in many cases the TDC have very slanting portions in a large range of variation of P.

2. The variation in the magnitude of the friction h of the column with the walls of the well is a basic source of noise in the indirect measurement, by means of the surface quantities, of the subsurface quantities which characterize the drilling process.

For estimates of the noise in drilling operations, there were taken off the functional relationships Q, n = $f(t)$, which show the presence of oscillations of n for Q = const (Figs. 12 and 13). It is most probable that these oscillations were generated, not by changes in properties of the rock, but by changes in the frictional force h.

Decreasing the magnitude of the friction h is connected with the perfecting of the technology of well pipe laying: with decreasing bending, and with the use of the proper wash fluids. One of the effective methods of decreasing h must be to rotate the tube column with low velocities.

It should be considered that, for vertical wells with drilling depths up to 2500 to 3000 meters, and with the proper technological operating mode, the noise engendered by the forces h will basically be less than the signal level.

3. An extremal device for drilling must be based on the computation of averaged values of velocity. The averaging time can be about 10 to 20 seconds and, in the future, must be made more precise. This makes it necessary to construct two-loop systems, similar to those described above.

4. To shorten the operation cycle of an ECD and to decrease the output time in the maximum zone, a fast-acting operative control system is of great value.

Further work on perfecting ECD must proceed together with the perfecting of drilling technology, partic-

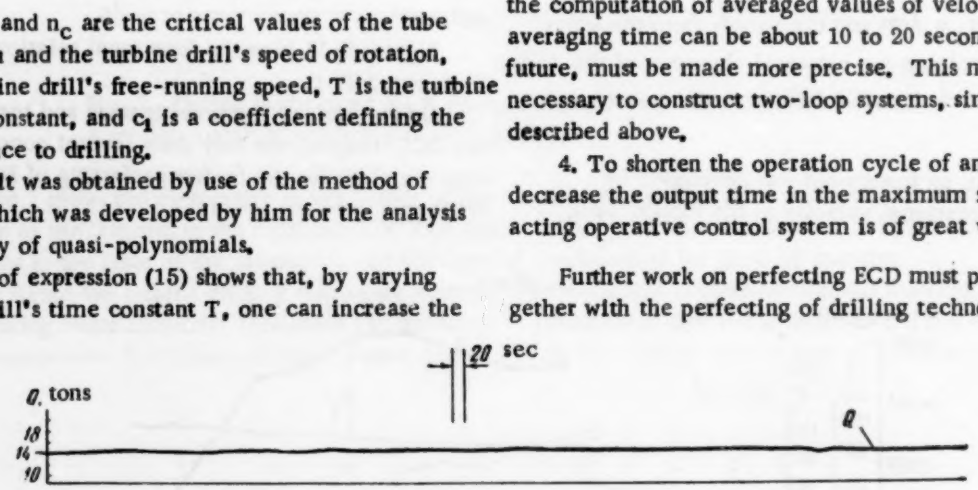

Fig. 12. Cartogram portions with recording of the current values of Q, n and V_s for Q ≈ const. V_s in meters per hour. Operational drilling. Depth of 1750 meters. Wash fluid was water.

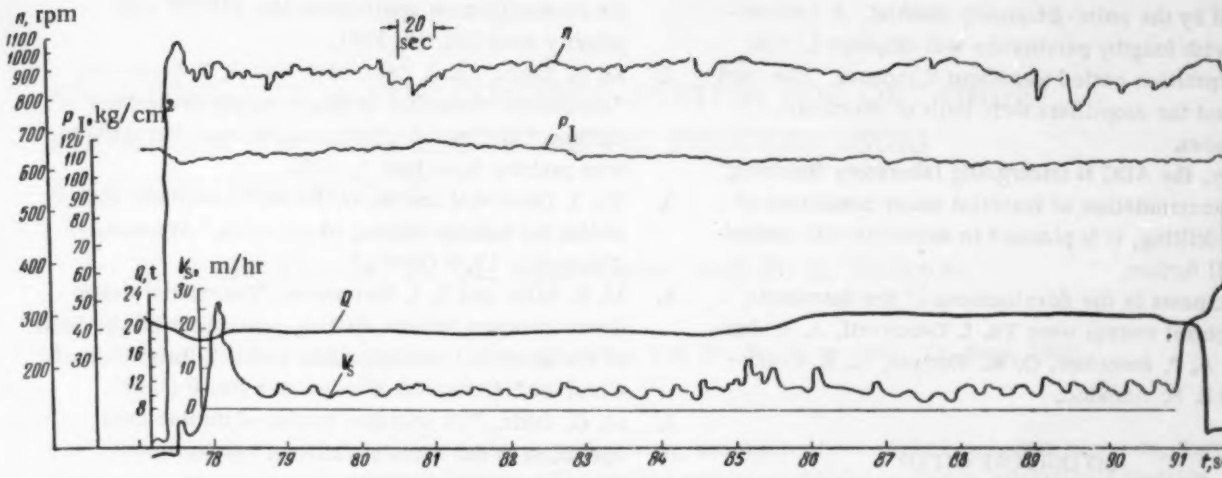

Fig. 13. Cartogram portions with recording of the current values of Q, n and V_s for Q ≈ const. Operational drilling. Depth of 2000 meters. Wash fluid was clay mortar.

ularly as it has to do with decreasing frictional noise, and increasing the zone of stable turbine drilling operation.

Under these conditions, the problem of creating a reliable extremal device for turbine drilling for vertical wells of depths of 2500 to 3000 meters, devices based on the designs developed, is a completely realistic task.

Turbine drill control by maximum mechanical velocity, using surface parameters, has still a limited use.

When the depth of drilling is increased beyond 2500 to 3000 meters, the frictional forces between the column and the well walls will increase significantly. This is particularly essential for wells with bends. This can lead to the appearance of errors in measuring the velocity of drilling, as a result of which the monotonic form of the individual segments of the turbine drilling curve will be disrupted, i.e., the curve will have several false maxima.

As the depth of drilling increases, the zone of stable turbine drill control constricts, and the probability is increased that the turbine drill will stop until the turbine drilling curve makes the transition to the right-hand descending portion.

In addition, to this, as the depth increases, the predominant role begins to be played, not by the mechanical speed of drilling, but by the quantity of rock drilled by one gouge, since the time required to replace the gouge increases sharply. Therefore, it is necessary to place a more complicated criterion than mechanical speed of drilling at the basis of control.

The reasons enumerated impelled us to begin the development of a new device for automatic drilling control (ADC).

Principles of ADC Construction

The defect in the extremal control system is its scan of the process curve "in the small".

At the basis of the ADC is a compulsory scan of the process "in the large". As applied to drilling, the scan is carried out within the limits of variation of the load on the gouge from the minimum to the maximum,

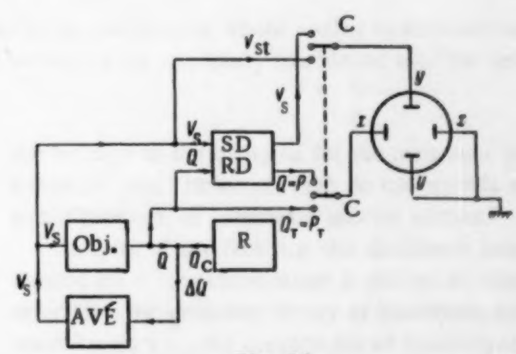

Fig. 14.

whereby the entire turbine drilling curve is taken off and stored.

The structural block schematic of the semiautomaton ADC is shown on Fig. 14. By means of an automatic data transmitter the quantity Q and, correspondingly, the load P on the gouge are varied linearly. The storage device SD stores the curve $V_s = f(Q)$ corresponding, under the assumptions made, to the curve $V_d = f(P)$.

After taking off of the first information (data) curve, the reading device RD is switched in, and periodically repeats the curve, reproducing it on the screen of a cathode-ray tube CRT. The curve thus obtained is analyzed by a driller (i.e., by a human being), using special nomograms which take into account the depth of drilling and the partially worn gouge and, from these data, the optimal value of P_{0i} is determined, this value being then set by the driller, and thereafter maintained automatically by the AVE controller. Simultaneously, current information as to the quantities P_{0i} and V_s is applied to the plates of the CRT. For this, commutator C is used. Prolonged and major deviations of the current information from the curve attest to the fact that the information has become obsolete. In this case, the old information is erased, and the process is repeated from the beginning.

In a test specimen, the storage and reading devices were made with magnetic tape. Recording was im-

plemented by the pulse-frequency method. A cathode-ray tube with lengthy persistence was employed. The reading repetition period was about 2 seconds. The commutator and the amplifiers were built of electronic vacuum tubes.

Today, the ADC is undergoing laboratory finishing; after the accumulation of material under conditions of industrial drilling, it is planned to automate this control system still further.

Participants in the development of the automatic drilling control system were Yu. I. Ostrovskii, A. S. Bondarevskii, A. P. Sosenkov, O. M. Kotlyar, M. R. Kraevskaya and B. N. Anokhin.

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ADAPTIVE AUTOMATIC CONTROL SYSTEM FOR OBTAINING ALUMINUM BY THE ELECTROLYTIC PROCESS USING A COMPUTING DEVICE

Ė. P. Sarkisyan, M. M. Agababyan, and P. S. Saakyan

Erevan

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The paper describes an adaptive automatic control system, constructed with digital computer elements, for the electrolytic process for obtaining aluminum. The control system causes the process to minimize a variable component of the cost, depending on the technological process. The search for the optimal mode is carried out in terms of one variable, namely, the position of the anode.

The control system also includes several systems for stabilizing parameters, whose setting is automatically varied as the conditions of the process change. The optimal settings were previously introduced into the controlling device's memory.

The modern industrial method of obtaining aluminum is the electrolysis of fused cryolite-alumina. In the electrolysis process, aluminum is produced at the cathode and, at the carbon anode, oxygen which, at high electrolysis temperatures, forms carbon monoxide and carbon dioxide with the carbon anode.

In theory, Faraday's law states that, in the electrolysis of fused cryolite-alumina, 0.336 g of aluminum should be produced per hour at the cathode when a current of 1 amp is used. In practice, in industrial baths embodying the current level of technological proficiency, the production of aluminum is 12 to 15% less than the theoretical value. This is due to a number of reasons, above all, to the metal solubility in the fused electrolyte and its being borne to the anode by the diffusion, convection and circulation of the electrolyte with the reverse formation of aluminum oxide.

As the result of many years of investigation and industrial use of aluminum baths, it has been established that the basic process indices – productivity and specific expenditure of electric energy – are influenced by the current density, the distance between electrodes, the temperature of the electrolyte, the concentration of alumina, the cryolite ratio, the form of the electrolyzer, the levels of electrolyte and metal in the bath, etc. These parameters change during the electrolysis process.

The control of the technological processes in present-day aluminum baths is effected manually, on the basis of rough measurements and subjective analysis which depends on the experience and individual proficiency of the operator.

It should be mentioned that in industry there are individual schemes designed to maintain constant just one of the parameters, while the others assume arbitrary values. These are automatic stabilization schemes for the voltage in the bath, and for the resistance of the interelectrode gap. However, they do not provide a solution to the problem of automatic process control.

In spite of the fact that the aluminum industry has existed for a long time, there is still no all-embracing, scientifically grounded theory of aluminum electrometallurgy, i.e., the electrolysis of fused cryolite-alumina, and no mathematical dependencies have been found between the output as a function of current, the specific expenditure of electric energy and the various parameters which determine the process under industrial conditions.

Today, obtaining mathematical relationships between the parameters of processes which occur in complicated physicochemical, electrodynamic and thermal conditions, on the sole basis of theoretical investigations, is tremendously difficult and unrealistic. The existing process of electrolysis of fused cryolite-alumina belongs to those technological processes in which such indices as productivity and specific expenditure of electric energy depend on many interrelated variable parameters, and are not found in a single-valued relationship with any one of them for arbitrary values of the remaining ones.

Consequently, the automatic control of the process of fused cryolite-alumina electrolysis can be implemented only on the basis of an analysis of the process indices, namely, the output as a function of current, and the specific expenditure of electric energy.

With the aims of increasing the efficiency of the process and of freeing the maintenance personnel from working in an atmosphere of noxious gases, there was developed an adaptive automatic control system for the electrolysis process which uses a computing device, the essence of which is described below.

The system is based on the following assumptions. The electrolysis bath is considered as an insulated thermo-

dynamical system; the series current, the alumina concentration and the composition of electrolyte in the bath are stabilized. The bath has a new design which permits complete collection of the output gases and a continuous supply of alumina. A single-period oscillation of anode motion is implemented. By the use of a computer there is carried out an automatic search for the optimal anode position, corresponding to the least value of that portion of the aluminum cost which depends on the state of flow of the process. Under definite conditions, forcing external stimuli are applied to the system and, on the basis of an analysis of the results of the process' reaction to the external stimuli, a search is automatically carried out for the optimal process conditions.

Stabilization of alumina concentration and electrolyte composition is introduced for the purpose of providing a uniform course of the process.

Stabilization of the series current is necessary to eliminate the effect on the bath's operating mode of a disturbed operating mode in the other, series-connected baths, and also to eliminate oscillations, deleterious for the electrolysis process, of the metal and disturbances of the correspondence between the form of the anode base and the surface of the liquid metal and, consequently, a nonuniform distribution of current in the electrolyte and in the anode base.

Thus, the sole advantageous and practically admissible controlling factor for the automatic search for an optimal process turned out to be just the displacement of the anode which, in the system developed, is used as the external forcing stimulus on the system.

The following requirements were imposed on the aluminum electrolyzer which was to operate in the adaptive system for automatic process control:

1) hermetic sealing, permitting complete removal of the output gases without losses and the rarefaction of atmospheric air, which is a necessary condition for the continuous determination of the indices of the process;

2) a continuous supply of alumina and fluorine salts.

At the Kanakerskii Aluminum Plant, there was developed, installed and subjected to industrial testing, an electrolyzer satisfying the requirements of automation. As the criterion of optimality of process flow there was chosen the degree of approximation to the least value of variable component of aluminum cost C_v, which depends on the course of the technological process and mirrors the variation of cost of electric energy and anode mass expended to obtain one ton of aluminum.

Figure 1 shows the block schematic for the adaptive system of automatic process control for the electrolysis of fused cryolite-alumina. According to this scheme, to the controlling device, via analog-to-digital converted AD, the values of the variable process parameters from the electrolyzer transducers are applied and, from the control console (desk), the values of the constant coefficient and stabilized parameters.

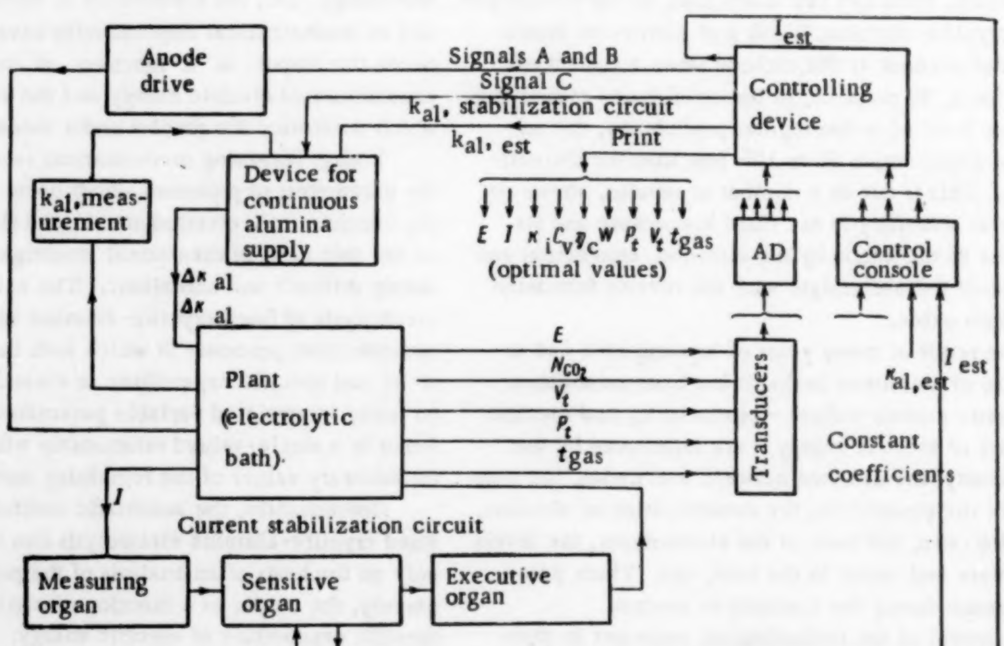

Fig. 1. Block schematic of the adaptive system of automatic process control for the electrolysis of fused cryolite-alumina. E is the voltage on the bath, I is the series current, N_{CO_2} is the percentage content of CO_2 in the output gases, P_t is the pressure of the output gases, V_t is the volume of the output gases, t_{gas} is the temperature of the output gases, Δk_{al} is a batch of alumina, Δk_{cr} is a batch of cryolite, k_{al} is alumina concentration, k_{cr} is cryolite concentration, A is raise, B is lower and C is supply alumina.

As a result of an analysis of the applied data, the control device sends a control signal to the automated electric anode drive scheme, this signal being necessary for the search for the optimal anode position. The controlling device prints out the process indices and parameters, both for the optimal anode positions and for the intervals between them.

The automated electric anode drive in the developed system provides automatic step-wise raising or lowering of the anode upon receipt of pulses from the controlling device, with the capability of regulating the magnitude of the translation step and of automatic lowering of the anode as metal is removed from the bath.

During the period of discharging metal from the bath, performed every three days and lasting only several minutes, the control device is switched out of the control system. Since variations in the temperature of the ambient medium have great influence on the mode of bath operation, the most advantageous sets of stabilizing parameter values are introduced into the controlling device for various values of the ambient medium temperature. When the temperature of the ambient medium reaches a definite value, the controlling device automatically supplies the stabilization circuit with the best values of the corresponding settings.

The system must provide stabilization of alumina concentration, which requires continuous measurement of alumina concentration whose practical implementation poses great difficulties. In connection with this, the controlling device, in addition to the functions already mentioned, is also used for the maintenance of alumina concentration constancy by determining the moment when a definite batch is spent and is sending a signal to the supply device to supply the electrolyzer with the corresponding new batch of alumina.

Figure 2 shows the functional block schematic of the controlling device, expressing the following two groups of operational modes: a) periodically repeating, and b) continuous.

The following modes appertain to the periodically repeating ones:

1. Determination of the direction of search for the optimal anode position.
2. Search for the optimal position.
3. Repetitive computation of the indices of the process in the optimal anode position followed by their print-out, and the print-out of the values of the variable parameters corresponding to the optimal anode position.
4. Periodic computation, after a definite interval of time following the establishment of an optimal anode position, of the quantity C_v and a comparison of it with the value of $C_{v,o}$ in the optimal position.

The following modes of operation are the continuous ones:

1. Generation of signals to supply the established batch of alumina to the electrolyzer bath.
2. Transmitting the proper settings to the series current and alumina concentration circuits upon changes in ambient medium temperature. It should be mentioned that the computation of the variable component of aluminum cost C_v must be performed with a very high accuracy, with an error not greater than 0.1%. This requirement is dictated both by economic considerations, and by the capabilities of technological implementation of the search for the optimal anode position under the technological conditions of the electrolysis process.

The requirements of high computational accuracy, the necessity of performing the operations of storage (memory), many logical operations, etc., led to the construction of the controlling device on the principles of electronic digital computers. The stability of the system is thus known to be guaranteed both for the control algorithm and for the conditions under which the search for the optimal anode position is carried out.

The output as a function of current, η_c, will increase only up to a definite point as the interelectrode distance increases, after which it begins to decrease monotonically. Consequently, the value of C_v connected with the voltage on the bath and the formula for output as a function of current given in the functional block schematic (Fig. 2), will have a least value for just one anode position. According to the search algorithm, when the anode is sufficiently close to the optimal position, no signal is sent to the anode drive, and anode motion is cut off, thus eliminating the possibility of anode oscillations about its optimal position.

The quality of the search which, in the system under consideration is characterized by the closeness of the anode to the optimal position, depends on the size of the anode translation step, and can be improved only by an experimental selection of the most successful size of the translation step, the capability of changing which has been built into a circuit of the automated anode drive.

In conclusion, it should be mentioned that the implementation and introduction into practice of the system described here, which differs radically from all previously existing automation systems, are connected with the achievement of greater economic efficiency. The yearly economy due to increased output of metal without an additional utilization of electric energy and also to the decreased expenditure for wages with complete automation of process control, at the Kanakerskii Aluminum Plant alone, constitutes some 10 to 15 million rubles a year for the capital outlays of less than one year.

Economy will be obtained also by simplification of the ventilation system and a decreased cost of ventilation.

The introduction into practice of the system described will also lead to the freeing of human beings from hard and unhealthful conditions, to an improvement of the atmospheric purity of the factories and the populated areas around the factories, and to a solution of the problem

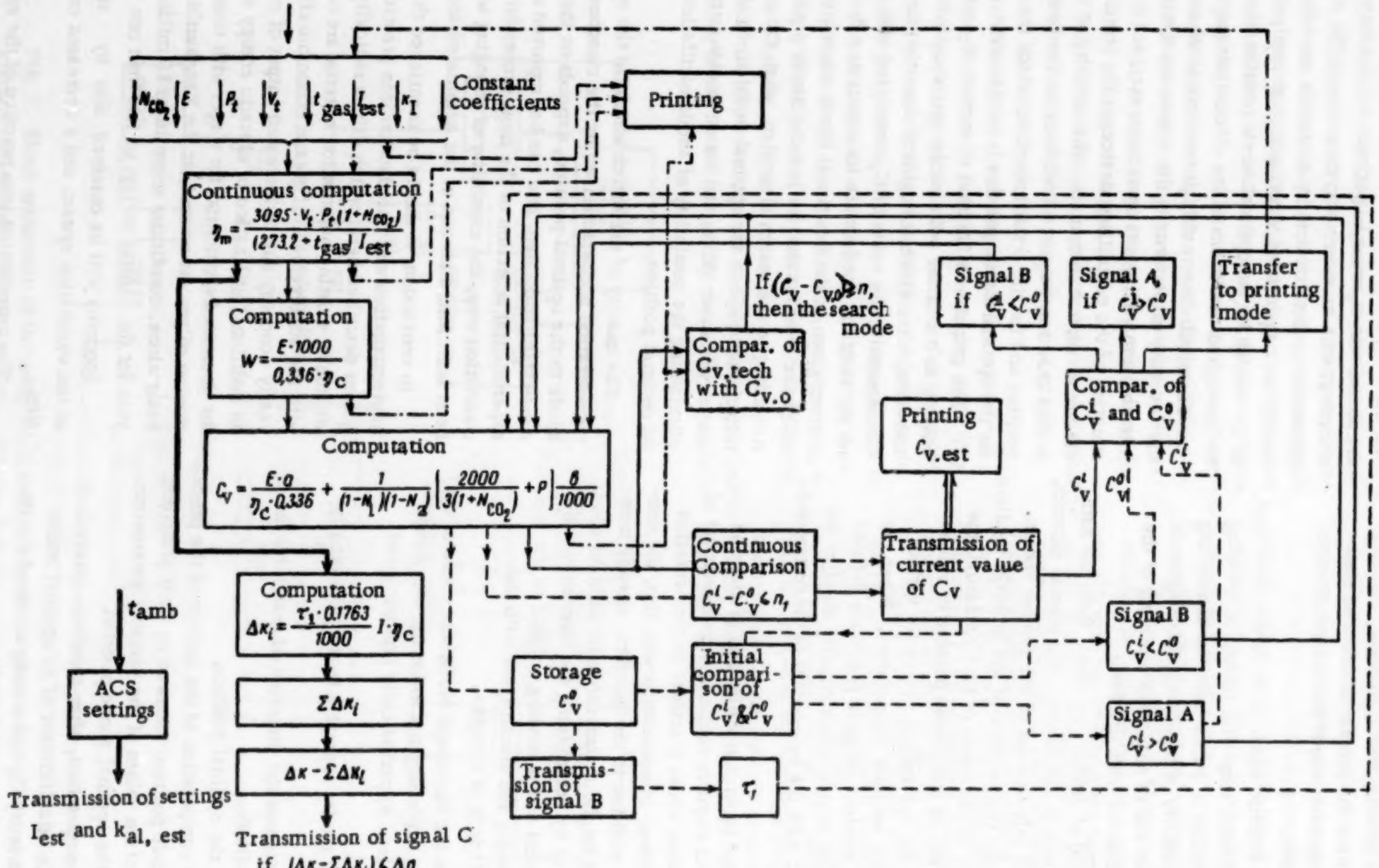

Fig. 2. Functional block schematic of the controlling device. ------- denotes the mode of determining the direction of search, ———— denotes the search mode, -·-·-· denotes printing, ━━━ denotes the continuous mode; ACS stands for Automatic Control System, C_V^o and C_V^i denote the previous and following values of the variable component of cost, respectively, τ_1 is a delay line, n is the zone of permissible deviations of cost.

of complete recovery and utilization of the noxious gases and sprays.

The system developed opens up the possibility of a profounder study of the technological process and a gathering of the experimental data whose statistical processing allows a mathematical description to be given of a complicated process. This enables one to determine the future paths and prospective development of electrolysis technology, to implement more accurate and economical designing of aluminum electrolyzers and to execute further perfecting of their physical design.

USE OF ELECTROTHERMAL ISODROME DEVICES FOR THE INTERCONNECTED CONTROL OF PRODUCTION PROCESSES

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A method is considered for the implementation of autonomous isodrome control of complicated plants by means of electrothermal devices, not requiring the complication of the controller's measuring elements. These devices are connected as identical electrothermal isodrome controllers, and differ only by the method of their connection in the control loop.

A method is described for determining the parameters of such devices, starting from the autonomy conditions, and the results are given of an experimental verification of separated and interconnected control schemes on a simulation device.

Complicated controlled plants frequently contain several controlled parameters, each of which depends on several controlling actions. Since the effect of the individually acting controllers in such objects is to worsen the quality of the control process, this leads to the necessity of using interconnected control, an advanced form of which is the autonomous control scheme suggested by I. N. Voznesenskii [1, 2].

Ordinarily, in autonomous control schemes, the communication signal between controllers is formed in the measuring elements of the leading controllers. In the general case of isodrome control, when the isodrome times are not equal among themselves, it is required, for the realization of these communications, to complicate the controller's measuring element, and to introduce a communication link allowing the derivatives of the controlled parameter to be obtained [3].

The necessity of complicating the circuit of the controller's measuring element in order to realize communication crossovers creates definite difficulty in implementing schemes for interconnected control of complex plants, since very frequently the measuring portion of a controller is simultaneously used for the purposes of automatic inspection. Moreover, the use of this communication signal is connected with the addition of a delay to the control circuit, a delay which is ordinarily attributable to the controller transducer. It is therefore of interest to use, for communications between controllers, signals which are not from the transducers, but from other elements of the control circuit.

In the present work we show the possibility of realizing autonomous control of technological processes by means of communications from the executive mechanisms of the leading controllers.

1. Determination of the Parameters of the Crossover Controller Communication Elements

We consider a plant with three controlled parameters, on each of which all the controlling elements have an effect. A skeleton scheme of this complicated control system is shown on Fig. 1. In addition to the signal of the measuring element and isodrome of its own loop, there is applied to the amplifying element of each controller a communication signal from the remaining two controllers. These external signals are applied from the

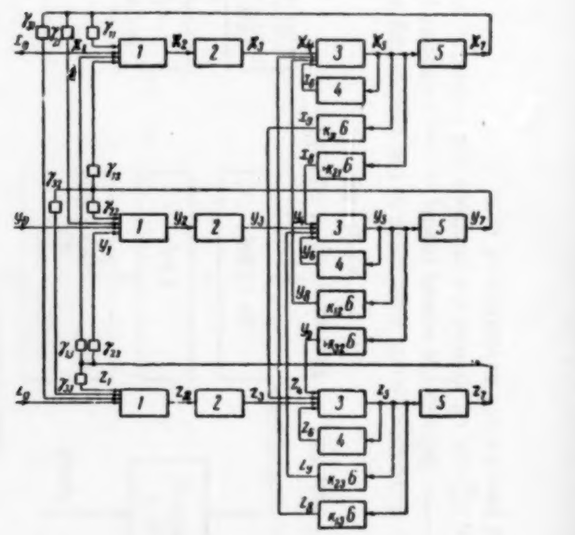

Fig. 1. Skeleton scheme of interconnected control of a plant with three controlled parameters. 1 is the plant, 2 is a measuring element, 3 is an amplifier, 4 is an isodrome, 5 is an executive mechanism with a controlling organ and 6 is a crossover communication device.

link whose input is connected to the inputs of the executive mechanisms of the leading controllers. The equations of the individual loops of the scheme of Fig. 1 can be given in the following form:

$$x_1 = \gamma_{11}x_7 + \gamma_{12}y_7 + \gamma_{13}z_7 + x_0,$$
$$x_2 = A_1x_1,$$
$$x_3 = A_2x_2,$$
$$x_4 = x_3 - x_6 + y_8 + z_8,$$
$$x_5 = A_3x_4,$$
$$x_6 = A_4x_5,$$
$$x_7 = A_5x_5,$$
$$y_1 = \gamma_{21}x_7 + \gamma_{22}y_7 + \gamma_{23}z_7 + y_0,$$
$$y_2 = B_1y_1,$$
$$y_3 = B_2y_2,$$
$$y_4 = y_3 - y_6 + x_8 + z_9, \qquad (1)$$
$$y_5 = B_3y_4,$$
$$y_6 = B_4y_5,$$
$$y_7 = B_5y_5,$$
$$z_1 = \gamma_{31}x_7 + \gamma_{32}y_7 + \gamma_{33}z_7 + z_0,$$
$$z_2 = C_1z_1,$$
$$z_3 = C_2z_2,$$
$$z_4 = z_3 - z_6 + x_9 + y_9,$$
$$z_5 = C_3z_4,$$
$$z_6 = C_4z_5,$$
$$z_7 = C_5z_5,$$

Here, x_2, y_2, z_2 are the controlled parameters, x_7, y_7, z_7 are the controlling actions, x_0, y_0, z_0 are the external disturbances, γ_{ij} (i, j = 1, 2, 3) are known coefficients of the plant which take into account the influence of the controlling elements on the controlled parameters. We denote by A_1, B_1, C_1 the operators of the individual objects of control, whose forms depend on the properties of these objects. The operators of the remaining links of the control circuit equal

$$A_2 = a_2,\quad A_3 = a_3,\quad A_4 = \frac{a_4}{1+pT_{I1}},\quad A_5 = \frac{a_5}{pT_{c1}},$$
$$B_2 = b_2,\quad B_3 = b_3,\quad B_4 = \frac{b_4}{1+pT_{I2}},\quad B_5 = \frac{b_5}{pT_{c2}}, \qquad (2)$$
$$C_2 = c_2,\quad C_3 = c_3,\quad C_4 = \frac{c_4}{1+pT_{I3}},\quad C_5 = \frac{c_5}{pT_{c3}}.$$

The equations of the communication devices have the form

$$x_8 = K_{21}x_5,\quad x_9 = K_{31}x_5,\quad y_8 = K_{12}y_5,$$
$$y_9 = K_{32}y_5,\quad z_8 = K_{13}z_5,\quad z_9 = K_{23}z_5, \qquad (3)$$

where K_{12}, K_{13}, K_{21}, K_{23}, K_{31}, K_{32} are unknown operators of the communication devices which must be determined. We obtain the following system of equations from (1) and (3):

$$x_5 = A_3x_4 = A_0(x_3 + K_{12}y_5 + K_{13}z_5),$$
$$y_5 = B_3y_4 = B_0(y_3 + K_{21}x_5 + K_{23}z_5), \qquad (4)$$
$$z_5 = C_3z_4 = C_0(z_3 + K_{31}x_5 + K_{32}y_5),$$

where

$$A_0 = \frac{A_3}{1+A_3A_4},\quad B_0 = \frac{B_3}{1+B_3B_4},\quad C_0 = \frac{C_3}{1+C_3C_4}, \qquad (4')$$

System (4) can be put in the form

$$Ax_5 - K_{21}y_5 - K_{13}z_5 = x_3,$$
$$-K_{21}x_5 + By_5 - K_{23}z_5 = y_3, \qquad (5)$$
$$-K_{31}x_5 - K_{32}y_5 + Cz_5 = z_3,$$

where

$$A = A_0^{-1},\ B = B_0^{-1},\ C = C_0^{-1}. \qquad (5')$$

We obtain the following expressions from (5) for x_5, y_5, z_5:

$$x_5 = \frac{1}{\Delta}(D_1x_3 + D_2y_3 + D_3z_3),$$
$$y_5 = \frac{1}{\Delta}(D_4x_3 + D_5y_3 + D_6z_3), \qquad (6)$$
$$z_5 = \frac{1}{\Delta}(D_7x_3 + D_8y_3 + D_9z_3),$$

where, by Δ, we denote the determinant of system (5), equal to

$$\Delta = \begin{vmatrix} A & -K_{12} & -K_{13} \\ -K_{21} & B & -K_{23} \\ -K_{31} & -K_{32} & C \end{vmatrix}, \qquad (7)$$

and, by $D_1, \ldots, D_9$, we denote its cofactors, equal to

$$D_1 = BC - K_{32}K_{23},\quad D_2 = K_{12}C + K_{32}K_{13},$$
$$D_3 = K_{12}K_{23} + BK_{13},\quad D_4 = K_{21}C + K_{31}K_{23},$$
$$D_5 = AC - K_{31}K_{13},\quad D_6 = AK_{23} - K_{21}K_{13}, \qquad (8)$$
$$D_7 = K_{21}K_{32} + BK_{31},\quad D_8 = AK_{32} + K_{31}K_{12},$$
$$D_9 = AB - K_{21}K_{12}.$$

We note that for system (5) to have a solution, it is necessary that its determinant not equal zero:

$$\Delta \neq 0. \tag{9}$$

From (1) and (6) we get

$$
\begin{aligned}
x_7 &= \frac{1}{\Delta} A_5 (D_1 x_3 + D_2 y_3 + D_3 z_3), \\
y_7 &= \frac{1}{\Delta} B_5 (D_4 x_3 + D_5 y_3 + D_6 z_3), \\
z_7 &= \frac{1}{\Delta} C_5 (D_7 x_3 + D_8 y_3 + D_9 z_3).
\end{aligned} \tag{10}
$$

Taking into account the expressions for the controlling parameters, we obtain, from (1) and (10), the following relationships for the controlled parameters:

$$
\begin{aligned}
x_2 = A_1 \Big\{ \frac{1}{\Delta} [(M_1D_1 + M_2D_4 + M_3D_7)\, x_3 + (M_1D_2 + M_2D_5 + M_3D_8)\, y_3 + \\
+ (M_1D_3 + M_2D_6 + M_3D_9)\, z_3] + x_0 \Big\}, \\
y_2 = B_1 \Big\{ \frac{1}{\Delta} [(M_4D_1 + M_5D_4 + M_6D_7)\, x_3 + (M_4D_2 + M_5D_5 + M_6D_8)\, y_3 + \\
+ (M_4D_3 + M_5D_6 + M_6D_9)\, z_3] + y_0 \Big\}, \\
z_2 = C_1 \Big\{ \frac{1}{\Delta} [(M_7D_1 + M_8D_4 + M_9D_7)\, x_3 + (M_7D_2 + M_8D_5 + M_9D_8)\, y_3 + \\
+ (M_7D_3 + M_8D_6 + M_9D_9)\, z_3] + z_0 \Big\},
\end{aligned} \tag{11}
$$

with the notation

$$
\begin{aligned}
M_1 = \gamma_{11}A_5,\ M_2 = \gamma_{12}B_5,\ M_3 = \gamma_{13}C_5,\ M_4 = \gamma_{21}A_5,\ M_5 = \gamma_{22}B_5, \\
M_6 = \gamma_{23}C_5,\ M_7 = \gamma_{31}A_5,\ M_8 = \gamma_{32}B_5,\ M_9 = \gamma_{33}C_5.
\end{aligned} \tag{12}
$$

We obtain the following autonomy conditions from (11):

$$
\begin{aligned}
M_1D_2 + M_2D_5 + M_3D_8 = 0, \\
M_1D_3 + M_2D_6 + M_3D_9 = 0, \\
M_4D_1 + M_5D_4 + M_6D_7 = 0, \\
M_4D_3 + M_5D_6 + M_6D_9 = 0, \\
M_7D_1 + M_8D_4 + M_9D_7 = 0, \\
M_7D_2 + M_8D_5 + M_9D_8 = 0.
\end{aligned} \tag{13}
$$

In accordance with (8), the communication operators being sought, K_{12}, K_{13}, K_{21}, K_{23}, K_{31}, K_{32}, enter into the expressions for D_1-D_9. We consider a method of determining these operators, starting from the autonomy conditions. These autonomy conditions (13) can be given in the following form:

$$
\begin{aligned}
\Delta_1 &= \begin{vmatrix} A & -K_{12} & -K_{13} \\ M_1 & M_2 & M_3 \\ -K_{31} & -K_{32} & C \end{vmatrix} = 0, \\
\Delta_2 &= \begin{vmatrix} A & -K_{12} & -K_{13} \\ -K_{21} & B & -K_{23} \\ M_1 & M_2 & M_3 \end{vmatrix} = 0, \\
\Delta_3 &= \begin{vmatrix} M_4 & M_5 & M_6 \\ -K_{21} & B & -K_{23} \\ -K_{31} & -K_{32} & C \end{vmatrix} = 0, \\
\Delta_4 &= \begin{vmatrix} A & -K_{12} & -K_{13} \\ -K_{21} & B & -K_{23} \\ M_4 & M_5 & M_6 \end{vmatrix} = 0, \\
\Delta_5 &= \begin{vmatrix} M_7 & M_8 & M_9 \\ -K_{21} & B & -K_{23} \\ -K_{31} & -K_{32} & C \end{vmatrix} = 0, \\
\Delta_6 &= \begin{vmatrix} A & -K_{12} & -K_{13} \\ M_7 & M_8 & M_9 \\ -K_{31} & -K_{32} & C \end{vmatrix} = 0.
\end{aligned} \tag{14}
$$

We consider determinants Δ_1 and Δ_2, which have two identical rows in common. It is well known that the condition for a third-order determinant to equal zero is the equality, or proportionality, of the elements of two of its rows or columns [4]. On the strength of this property, we obtain the following relationship between the common rows of these two determinants*:

$$\frac{A}{M_1} = -\frac{K_{12}}{M_2} = -\frac{K_{13}}{M_3}. \tag{15}$$

From (15) we obtain the values of the operators:

$$K_{12} = -A\frac{M_2}{M_1}, \quad K_{13} = -A\frac{M_3}{M_1}. \tag{15'}$$

By an analogous consideration of determinant pairs Δ_3, Δ_4 and Δ_5, Δ_6, we get

$$K_{21} = -B\frac{M_4}{M_5}, \quad K_{23} = -B\frac{M_6}{M_5}, \tag{16}$$

$$K_{32} = -C\frac{M_8}{M_9}, \quad K_{31} = -C\frac{M_7}{M_9}. \tag{17}$$

By using the expressions obtained for the communication operators between the controllers, we can show that the

*This solution is unique, since the proportionality of the other rows in determinants Δ_1 and Δ_2 leads to a zero value for the determinant of system (5), which contradicts (9).

requirement that the determinant of system (5) not equal zero reduces to the meeting of the condition

$$\begin{vmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{vmatrix} \neq 0. \tag{18}$$

This condition is usually met in actual objects of control (plants).

By substituing the values of A, B, C and $M_1, \ldots, M_9$ from (5') and (12) in (15'), (16) and (17), we get

$$\begin{aligned}
K_{21} &= -\frac{\gamma_{21}}{\gamma_{22}} \frac{a_5 T_{c2}(1+b_3 b_4)}{T_{c1} b_5 b_3} \frac{(1+pT'_{I2})}{(1+pT_{I2})}, \\
K_{31} &= -\frac{\gamma_{31}}{\gamma_{33}} \frac{a_5 T_{c3}(1+c_3 c_4)}{T_{c1} c_5 c_3} \frac{(1+pT'_{I3})}{(1+pT_{I3})}, \\
K_{12} &= -\frac{\gamma_{12}}{\gamma_{11}} \frac{b_5 T_{c1}(1+a_3 a_4)}{T_{c2} a_5 a_3} \frac{(1+pT'_{I1})}{(1+pT_{I1})}, \\
K_{32} &= -\frac{\gamma_{32}}{\gamma_{33}} \frac{b_5 T_{c3}(1+c_3 c_4)}{T_{c2} c_5 c_3} \frac{(1+pT'_{I3})}{(1+pT_{I3})}, \\
K_{13} &= -\frac{\gamma_{13}}{\gamma_{11}} \frac{c_5 T_{c1}(1+a_3 a_4)}{T_{c3} a_5 a_3} \frac{(1+pT'_{I1})}{(1+pT_{I1})}, \\
K_{23} &= -\frac{\gamma_{23}}{\gamma_{22}} \frac{c_5 T_{c2}(1+b_3 b_4)}{T_{c3} b_5 b_3} \frac{(1+pT'_{I2})}{(1+pT_{I2})},
\end{aligned} \tag{19}$$

where

$$T'_{I1} = \frac{T_{I1}}{1+a_3 a_4}, \quad T'_{I2} = \frac{T_{I2}}{1+b_3 b_4}, \quad T'_{I3} = \frac{T_{I3}}{1+c_3 c_4}. \tag{20}$$

The following relationships usually hold in isodrome controllers of production processes:

$$a_3 a_4 \gg 1, \; b_3 b_4 \gg 1, \; c_3 c_4 \gg 1;$$
$$T_{Ij} \geqslant (10-100)\, T'_{Ij}. \tag{21}$$

By taking (21) into account, we can present the communication operators in the form

$$\begin{aligned}
K_{21} &= -\frac{\gamma_{21}}{\gamma_{22}} \frac{a_5}{T_{c1}} \frac{T_{c2}}{b_5} \frac{b_4}{1+pT_{I2}}, \\
K_{31} &= -\frac{\gamma_{31}}{\gamma_{33}} \frac{a_3}{T_{c1}} \frac{T_{c3}}{c_5} \frac{c_4}{1+pT_{I3}}, \\
K_{12} &= -\frac{\gamma_{12}}{\gamma_{11}} \frac{b_5}{T_{c2}} \frac{T_{c1}}{a_5} \frac{a_4}{1+pT_{I1}}, \\
K_{32} &= -\frac{\gamma_{32}}{\gamma_{33}} \frac{b_5}{T_{c2}} \frac{T_{c3}}{c_5} \frac{c_4}{1+pT_{I3}}
\end{aligned} \tag{22}$$

$$\begin{aligned}
K_{13} &= -\frac{\gamma_{13}}{\gamma_{11}} \frac{c_5}{T_{c3}} \frac{T_{c1}}{a_5} \frac{a_4}{1+pT_{I1}}, \\
K_{23} &= -\frac{\gamma_{23}}{\gamma_{22}} \frac{c_5}{T_{c3}} \frac{T_{c2}}{b_5} \frac{b_4}{1+pT_{I2}}.
\end{aligned} \tag{22 (cont.)}$$

From (22) one easily deduces the structure of the communication operators for plants with n controlled parameters:

$$K_{ji} = -\frac{\gamma_{ji}}{\gamma_{jj}} \frac{E_{5i}}{E_{5j}} E_{4j}, \tag{23}$$

where E_4 and E_5 denote, respectively, the isodrome operators and the operators for the controlling organ with an executive mechanism for the individual controllers.†

The communication operators of (22) can be realized by electrothermal devices of the isodrome type whose heater windings are connected in the output circuit of the leading controller's amplifier, and whose thermocouples are connected at the input of the driven controller's measuring element. The time constant of this communication device must equal the isodrome time of the driven controller, and the transfer factor of this link depends solely on the ratios of the parameters of the two control contours: driven and driving.

The method described for the realization of the connections (communications) between the controllers in order to obtain autonomous control allows one to eliminate the delays of the driven controllers' measuring elements, and to increase the dynamic control accuracy. Realization of these communications by means of electrothermal isodromes with even tuning of the isodrome times [5] is particularly valuable for the autonomous control of inertial plants.‡

2. Experimental Verification on a Simulator of the Interconnected Control Scheme

For the purpose of verifying the effectiveness of the method suggested for realizing connections in interconnected control schemes, an experimental investigation was carried out on a simulator for systems of individual and of interconnected control. In its technological features, the simulator is a model of one of the portions of the drying-absorbing section of a plant for sulfuric acid production by the contact process.

Figure 2 shows the block schematic of the simulator. To monohydrate absorber I, with a closed sprinkling cycle, there is applied acid Q_1 with concentration S_1 and acid Q_2 with concentration S_2. To tower II, there is fed monohydrate Q_4 of concentration S_3, and water Q_6. Dried acid, of quantity Q_2 and concentration S_2, is applied to pump V

† The validity of formula (23) was also verified by the author for a plant with the number n = 4 of controller parameters.

‡ For isodrome times up to 500 seconds, autonomy can also be realized by the dynamic connections of the VTI system [6].

in supply tank III and, from there, to absorber I. The continuous separation of the process output is realized by flows Q_5 and Q_7.

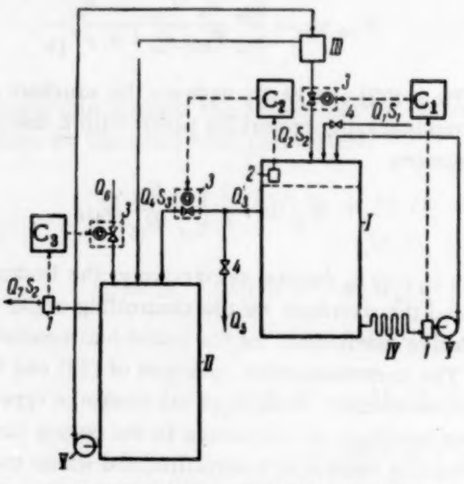

Fig. 2. Block schematic of the simulator. I is the absorber, II is the tower, III is the supply tank, IV is the cooler, V is the pump, C_1 and C_3 are the controllers of acid concentration, C_2 is the controller of acid level; 1 is the concentration measuring transducer, 2 is the level measuring transducer, 3 is the controlling valve with executive mechanisms, 4 is the disturbing organ.

In this simulator, there was an automatic maintenance of constant concentration S_3 and acid level H in absorber I, and acid concentration S_2 in tower II. Acid level in this tower was kept constant by an overflow drain.

With individual operation of the controllers, controller C_1 of monohydrate concentration controls the quantity of dried acid Q_2 applied to absorber I, monohydrate level controller C_2 controls the quantity of monohydrate Q_4 applied to drying tower II, and controller of dried acid concentration C_3 controls the amount of water Q_6 applied to the drying tower II. For the dynamics of the controlled parameters, the following differential equations are valid:

$$\begin{aligned}\Delta S_3 &= A_1(\alpha_1\Delta S_2 + \alpha_2\Delta Q_2 + \alpha_3\Delta Q_1),\\ \Delta H_1 &= B_1(\Delta Q_1 + \Delta Q_2 - \Delta Q_4 - \Delta Q_5),\\ \Delta S_2 &= C_1(\alpha_4\Delta Q_4 + \alpha_5\Delta S_3 + \alpha_6\Delta Q_6),\end{aligned} \tag{24}$$

where

$$A_1 = \frac{e^{-p\tau_1}}{1+pT_1},\quad B_1 = \frac{1}{pF},\quad C_1 = \frac{e^{-p\tau_2}}{1+pT_2},$$

$$\alpha_1 = \frac{Q_2^\circ}{Q_3^\circ},\quad \alpha_2 = \frac{Q_1^\circ(S_2^\circ - S_1^\circ)}{(Q_3^\circ)^2},\quad \alpha_3 = \frac{Q_2^\circ(S_1^\circ - S_2^\circ)}{(Q_3^\circ)^2}, \tag{24'}$$

$$\alpha_4 = \frac{Q_6^\circ S_3^\circ}{(Q_4^\circ + Q_6^\circ)^2},\quad \alpha_5 = \frac{Q_4^\circ}{Q_4^\circ + Q_6^\circ},\quad \alpha_6 = \frac{Q_4^\circ S_3^\circ}{(Q_4^\circ + Q_6^\circ)^2}.$$

Here, we have adopted the following notation: p is the differentiation operator, T_1 and T_2 are the time constants, respectively, of absorber I and tower II; τ_1, τ_2 and τ_3 are the lags of the controlled parameters; F is the image surface of the acid in tower II. A superscript of zero denotes the corresponding quantity in the system's equilibrium state.

Analysis of the right members of Eqs. (24) shows that the deviation ΔS_3 from the actual variations of ΔS_2, as well as the deviation ΔS_2 from the actual variations of ΔS_3, are less than the changes of these controlled parameters from other disturbances. By neglecting these influences in the given investigation, we obtain the following equations for the simulator inputs in general form:

$$x_1 = \gamma_{11}x_7 + x_0,\ y_1 = \gamma_{21}x_7 + \gamma_{22}y_7 + y_0, \qquad z_1 = \gamma_{32}y_7 + \gamma_{33}z_7 + z_0, \tag{25}$$

where x_1, y_1, z_1 are the plant's inputs, x_7, y_7, z_7 are the controlling flows, $\gamma_{11} = \alpha_2$, $\gamma_{21} = 1$, $\gamma_{22} = -1$, $\gamma_{32} = \alpha_4$, $\gamma_{33} = \alpha_6$, $x_0 = \alpha_3\Delta Q_1$, $y_0 = \Delta Q_1 - \Delta Q_5$ and $z_0 = 0$.

The apparatus used on the actual plant was set up on the simulator. All controllers were electrical with electrothermal isodromes and with constant servomotor speeds. The optimal parameters (internal connections) of the individual controllers were determined from the conditions that the transient responses have damping degree $\psi = 0.8$ and minimal squared deviation of the controlled parameter [7]. The simulator parameters are given in Table 1.

The operators of the electrothermal devices of the controllers' crossover communications were calculated by the methodology presented previously.

According to (22), with account taken of the coefficient matrix of the simulator and the parameters of the individual control circuits, we have

$$K_{12} = 0,\quad K_{13} = 0,\quad K_{21} = 6.05\frac{1+0.191\,p}{1+2.1\,p},$$
$$K_{23} = 0,\quad K_{31} = 0,\quad K_{32} = 60.5\frac{1+0.277\,p}{1+3.6\,p}. \tag{26}$$

The actual controller connections were realized by electrothermal isodromes whose operators were equal to

$$K_{21} = \frac{6.05}{1+2.1\,p},\quad K_{32} = \frac{57.5}{1+3.5\,p}. \tag{27}$$

The method of testing was the following: Two forms of external disturbances were communicated to the simulator: a) the quantity of acid Q_1 applied to absorber I was varied; b) the quantity of acid Q_5 withdrawn "from stock" was varied. These disturbances were communicated to the simulator both for individual operation of the controllers and for the interconnected control scheme. The results of the tests are given in Table 2.

The simulator tests show that, with interconnected control, the dynamic deviations of all the controlled parameters were less by 20% than those with individual control. The same verification of an interconnected con-

TABLE 1

Controller	Plant parameters					Controller parameters						
	τ, min	T_a, min	$\frac{\tau}{T_a}$	a_1, $\frac{\% H_2SO_4}{1/min}$ (cm/1/min)	a_2, $\frac{mv}{\% H_2SO_4}$ (mv/cm)	a_3, mv^{-1}	a_4, mv	a_5, l/min	T_c, min	T_I, min	$\frac{T_I}{T_a}$	K_0
Concentration controller of absorber I	0.8	8.4	0.094	−0.23	18.9	6.0	12.3	1.47	0.27	3.5	0.42	15.9
Level controller of absorber I	0.2	6.42	0.0312	21.9	0.393	1.62	5.75	1.63	0.284	1.97	0.314	50
Concentration controller of tower II	1.9	3.65	0.52	0.076	48.6	0.42	28.6	2.18	0.57	3.65	1.0	1.66

TABLE 2

	Individual control		Interconnected control		$\frac{\delta_2 \cdot 100}{\delta_1}$
	mean parameter value	mean deviation (δ_1)	mean parameter value	mean deviation (δ_2)	
Concentration controller of absorber I	0.65% H_2SO_4	0.0076% H_2SO_4	0.65% H_2SO_4	0.00628% H_2SO_4	82.6
Level controller of absorber I	33.6 cm	0.5 cm	33.7 cm	0.4 cm	80.0
Concentration controller of tower II	0.275% H_2SO_4	0.021% H_2SO_4	0.276% H_2SO_4	0.0169% H_2SO_4	80.5

trolled assembly under production conditions in a chemical plant showed a decrease in dynamic deviations of the controlled parameters of 33%.

SUMMARY

The paper demonstrated the possibility of obtaining autonomy in interconnected control schemes for technological processes by means of simple electrothermal devices analogous, in their physical design, to electrothermal isodromes. The initial data for determining the parameters of these devices are the parameters of the leading and driven controllers. The paper gives the method for determining these parameters. An experimental verification of the method suggested for realizing cross-over communication on a simulator showed the capabilities of increasing dynamic control accuracy. The advantages of this method of realizing autonomy in interconnected control schemes for technological processes are:

a) simplicity of introducing autonomous or interconnected control in individually acting controllers;

b) no necessity to complicate the controllers' measuring elements;

c) capability of obtaining large time constants, which is particularly important for inertial plants;

d) simplicity of physical design and reliability of operation.

These advantages allow one to recommend the introduction of electrothermal communication devices in autonomous control systems.

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CONTROL OF CHEMICAL PRODUCTION PROCESSES BY INDICATORS OF PRODUCT QUALITY

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As applied to chemical production, automatic control, in the overwhelming majority of the cases, reduces to stabilization of the processes by maintaining the constancy of indirect parameters: temperature, pressure, discharge, level, etc. Such control satisfies the requirements of a given yield and quality of the end product only under the condition that the composition and quality of the raw material are constant. If these conditions are disturbed, the necessity arises of manually resetting the data transmitter mechanisms of the majority of controllers. More complete automation of a process can be achieved only by control in accordance with the indicators determining the quality (composition) of the selected intermediate and end products. With this, the task of process optimization still rests with the operator. The transition to complete automation can be made if the optimization is implemented automatically.

Below we provide several examples of control by qualitative indicators.

A typical process in chemical industry is continuous fractionating. The requirements of obtaining fractionating products of a strictly given composition, with a simultaneous intensification of the process and an increase in its economy, can be fulfilled only with complete automation of the fractionating column. The use of automation is the more important when the components to be separated have boiling points which are close together, and when there is withdrawal of intermediate fractions.

In this case, a beneficial effect can be achieved with cascaded control with the simultaneous use of quality (composition) controllers of the end and intermediate products.

As is well known, for stable operation of any fractionating column, it is necessary to have an equality of the material and heat flows entering and leaving the column. With this, the consumption of heat for separation must be minimal. Moreover, for the withdrawal of distillate, vat residue and intermediate fractions with a minimum of admixtures, it is necessary to maintain a definite form of the composition distribution curve along the height of the column.

We consider a scheme for the automation of a fractionating column as applied to the rectification of alcohol (Fig. 1). In the initial product which is to be rectified, the constitution by weight is 48% ethyl alcohol, 50% water and 2% higher alcohols. The rectification task is to obtain a distillate whose weight composition must be 89-90% ethyl alcohol and not more than 0.1% higher alcohols, with a vat residue (fusel water)* containing no more than 0.01% ethyl alcohol, and intermediate fractions containing 10-12% higher alcohols.

The scheme can also be recommended for the separation of other mixtures.

To control the supply of raw material (the intermediate product of the low-boiling fraction), discharge controller D is used, its setting being established by the production personnel as a function of the presence of raw material and the required productivity of the column.

For the control of the composition of the vat product (fusel water) and the composition of the intermediate fractions – the higher alcohols – three-stage assembly K_{fw} is used. The first stage of the control assembly acts on the valve installed on the steam supply line from the boiler. The rate of steam discharge is established automatically from the output quantity of the second stage which measures the temperature in the steam area of the column's fifth plate. The dependence of the amount of higher alcohols accumulated in their tap-off zones (the third, fourth, fifth and sixth plates) on the temperature in the steam area of the fifth plate (this temperature equals 96 to 98°C) has been theoretically established and experimentally verified. In its turn, the assignment of the temperature is automatically established from the output quantity of the transducer measuring the alcohol loss in the fusel water (third stage).

Operation of controlling assembly K_{fw} proceeds in the following manner: The controller's temperature data transmitter on the fifth plate is set to its least value, the controller's alcohol loss data transmitter is set for minimal

*"Fusel water" ("fuzel'naya voda") is apparently the Russian term for the mixture of fusel oil and water left as residue after distillation of the alcohol [Publisher's note].

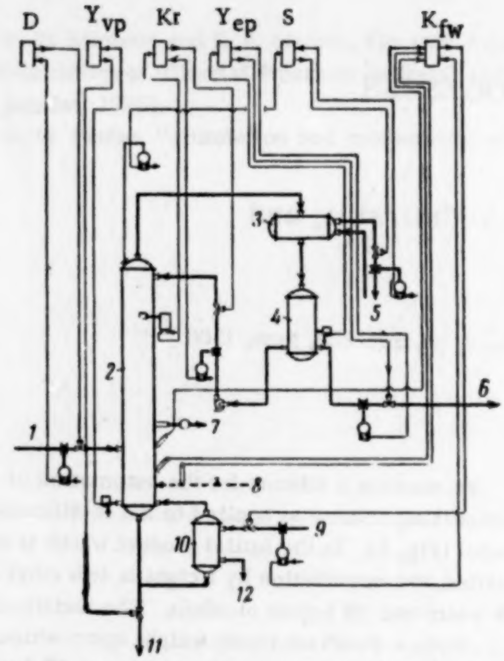

Fig. 1. Schematic of an automated alcohol fractionating column. 1) Intermediate product of low-boiling fraction, 2) column, 3) condenser, 4) storage tank, 5) cooling water, 6) rectified alcohol, 7) fraction of higher alcohols, 8) saturated water vapor (for measuring losses), 9) heating steam, 10) boiler, 11) fusel water (vat product), 12) steam condensate.

loss. If the actual loss turns out to be greater than the admissible value, the temperature assignment is automatically changed, increasing the assigned temperature value on the fifth plate. As a result, the application of steam to the "concentrated" vat product is increased.

The three-stage controller just described makes it possible to control the contents of the vat product and the intermediate fractions by controlling the application of steam as a function of the changes of two parameters, and by establishing the required tap-off of the intermediate fractions. The presence of this cascaded controller virtually eliminates the disturbing stimuli which arise due to oscillations in steam pressure. As a result of its operation, the assembly automatically expends a minimal quantity of steam; a further decrease in steam supply leads to an increased loss of alcohol in the tapped-off fusel water. Withdrawal of the intermediate fractions occurs from the zone of maximum concentration.

For the control of the required composition of the final product (the distillate), cascaded control is also used. Two-stage controller Kr consists of a controller of reflux discharge which acts on the end product return valve. The assignment of reflux discharge is automatically established by the end product composition transducer. This transducer is installed on the tenth plate from the top where, for an insignificant change in composition, there occurs the greatest change in the transducer's output quantity.

The equality of the thermal flows entering and leaving the fractionating column is maintained by the elasticity of the steam in the column by means of two-stage control assembly S, consisting of a discharge controller which acts on the supply of cooling water. The assignment of the discharge is automatically established from the pressure transducer. The balance of the material flows is maintained by level controller Y_{vp} in the vat column, acting on the withdrawal of the vat product. In addition, control is provided for the withdrawal of distillate from the tower by end product controller Y_{ep}.

With the use of the control system just described, the column operates completely automatically. If the operator has increased the supply of raw material for rectification, then, in the column, there will be a decrease in temperature (thanks to the introduction of the additional quantity of raw material), the temperature controller of the three-stage controller operates, and the assignment of steam supply to the column vat is increased. As a result of the increased supply of steam, the pressure in the column is raised. With this, the pressure transducer of controller S changes the assignment of water discharge, increasing the supply, i.e., the supply of steam and water will be put into correspondence with the increased load. The withdrawal of distillate and vat product is also increased automatically. With the introduction of an additional quantity of raw material into the column, the concentration of alcohol on the plates is increased. With the action of the rectificate composition transducer, the supply of reflux is decreased, the level in capacity of the end product is increased, and withdrawal of rectificate is increased. With an increase in raw material supply, there is also an automatic increase in the withdrawal of the higher alcohol fractions. Obviously, the scheme will operate analogously if there is an increase in alcohol content of the raw material supplied.

To implement the scheme just described, there were developed special quality indicator transducers for determining rectificate, vat product and higher alcohol fractions composition.

The operation of the instrument for distillate composition is based on a comparison of the buoyancies of the vapors of standard fluids and the vapors found in the column. The standard fluid fills a hull whose inner surface is a bellows; the hull is mounted in the column. The pressure difference experienced by the bellows is transmitted, via a special frictionless gasket, to the pneumatic compensation system. The pressure of the compensating air is proportional to the difference in buoyancy, i.e., to the composition of the vapor at the corresponding point of the column. The instrument can be used for binary mixtures. For multicomponent mixtures, the instrument is useful only in the case when the content of all the com-

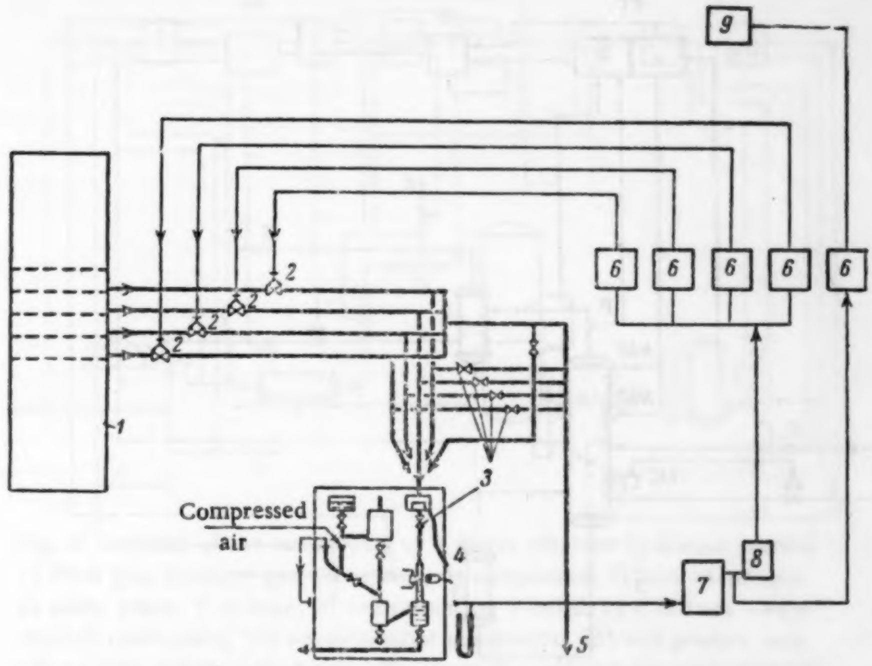

Fig. 2. Schematic of the instrument for measuring concentration of the higher alcohols. 1) Column, 2) valve membrane, 3) stopcock, 4) photoelement with lamp, 5) higher alcohol fraction, 6) storage device, 7) pressure measurer, 8) controlling block, 9) concentration measurer.

ponents (apart from the investigated ones) remains invariant.

The transducer for determining the composition of the vat product (the instrument for determining the loss of alcohol in the fusel water) works on the basis of the phenomenon that the boiling temperatures of the pure high-boiling component and the vat residue are different. If, in the vat area of the fractionating column, there is introduced an insignificant amount of saturated water vapor (high-boiling component), then, from the magnitude of temperature difference between the steam just introduced and the steam already existing in the column, one can determine the content of the highly volatile component (alcohol). For this, the pressure of the introduced steam must equal the pressure in the column, for which the delivery of the steam is implemented via a valve and the condensation chamber. The temperature difference is measured by resistance thermometers (200 ohms) in conjunction with a standard electronic bridge scaled for one-ohm impedance differences. Thermometers are installed in the condensation chamber and in the column vat.

The instrument for determining the composition of higher alcohols operates periodically with a cycle of 5 to 6 minutes on the principle of stratification of the higher alcohols in a saturated solution of sodium chloride (Fig. 2). The command instrument (a timer) every five minutes automatically extracts a specimen of definite volume from the corresponding plate and mixes it with a proportioned volume of sodium chloride solution. After mixing, stratification occurs; the higher alcohols, being lighter, are distributed higher. Thanks to the choice of the proper dyes, the layer of higher alcohols is sharply distinguished by its color. Compressed air, automatically supplied after stratification, forces the mixture into a measuring container where a device with a photoelement reacts to the passage of the plane of stratification of the higher alcohols from the remaining components of the solution. With this, there is ascertained, and stored, the air pressure, which is proportional to the volume occupied by the higher alcohols. The stored pressure acts on the controlling valve installed at the outlet from this plate. Simultaneously with the storing of the pressure, there occurs a decantation of the fluid from the instrument and the withdrawal of a specimen from another plate. A cycle during which there is established the necessary withdrawal from the four tables occupies from 20 to 25 minutes. Moreover, the instrument determines and fixes the mean content of the higher alcohols in the common collector, so that a full cycle takes about 30 minutes. Since the redistribution of the higher alcohols occurs slowly, cyclical operation with a 30-minute period is fully justifiable.

The control system just described was implemented at one of the factories and operated successfully for more than a year and a half. As the result of its introduction, the expenditure of steam and water per ton of raw material was decreased by approximately 4 to 5%, the productivity of the aggregate was increased by 5%, disruptions of the technological regimen were eliminated, and, in particular,

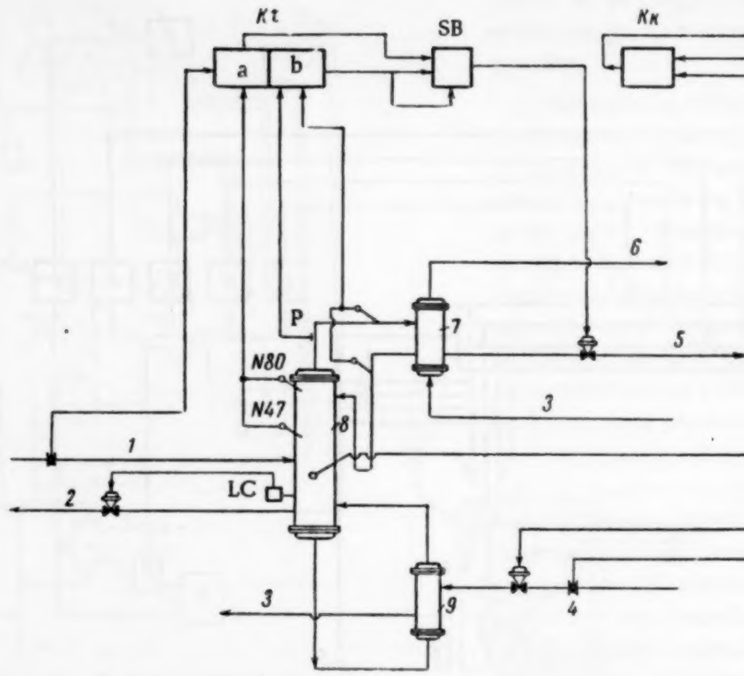

Fig. 3. Schematic of an automated column for hydrocarbon separation. 1) Ethane-ethylene group, 2) ethane fraction, 3) propane-propylene fluid, 4) gaseous propane-propylene, 5) ethylene fraction, 6) propane-propylene vapor to the compressor, 7) condenser, 8) column, 9) boiler.

there was a decrease in the losses of alcohol in the fusel water.

On the same principles there was constructed a control system by quality indicators for another fractionating process, intended for the separation of hydrocarbon mixtures (Fig. 3).

The rectifier is supplied with an ethane-ethylene fraction with an admixture of methane and propylene. The fraction has a nonconstant composition. The distillate withdrawn is the ethylene fraction with a minimal content, while the vat product is the ethane fraction with a minimal ethylene content. The pressure in the column is 22 kg/cm^2.

The column vat control assembly Kk, consisting of a heat carrier discharge controller and a vat composition transducer, varies the supply of heat carrier to the column.

Column top controller Kt, consisting of the ethylene fraction (distillate) discharge controller and the distillate composition transducer, varies the withdrawal of the distillate.

Equality of the flows of entering and withdrawn materials is maintained by level controller LC, acting on the withdrawal of ethane. Equality of heat flows is controlled by pressure transducer P, acting on controller Kt and then on the withdrawal of ethylene.

The quality of the upper product (the distillate) is determined by the temperature difference between two points fastening parts of the column. For different disturbing stimuli, worsening the distillate composition, an increase in the content of ethane in the ethylene occurs more intensively on the lower plates, i.e., "contraction" of the column begins, leading to an increase in the temperature difference. Conversely, with an increase in saturation of the column by ethylene, a decrease in the temperature difference occurs. It should be mentioned that pressure oscillations in the column have little effect on the magnitude of the temperature difference. The temperature difference is determined by resistance thermometers in conjunction with one type ÉMD instrument, calibrated for a measurement range of 0 to 5°C (the range of temperature differences). Experimental results have established that the most characteristic points are plates 47 and 80. The output quantity of the temperature difference instrument changes the assignment on the ethylene withdrawal controller. In view of the large capacity of the column and the large magnitude of the transient and transfer lags in the scheme, there were introduced additional correcting pulses as functions of the speed of variation of the magnitude of the load on the column, i.e., as functions of the speed of variation of the fundamental disturbing stimulus. These pulses are formed by means of a lead block and the controlling block and, in conjunction with the pulses for the temperature difference between plates 47 and 80, act on the assignment of the ethylene withdrawal controller (assembly a). It should be taken into account that, when a mixture containing methane is supplied to the column, the instrument for the temperature difference between plates 47 and 80 begins to

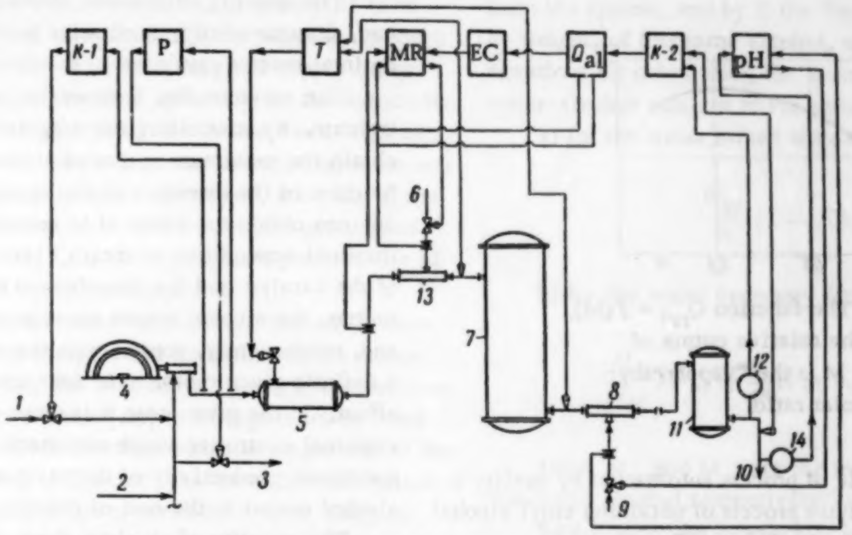

Fig. 4. Scheme of the automation of a direct ethylene hydration process. 1) Fresh gas, 2) return gas, 3) exhaust, 4) compressor, 5) heat exchanger, 6) water vapor, 7) reactor, 8) neutralization T-bend, 9) alkalized water-alcohol condensate, 10) water-alcohol condensate, 11) end product tank, 12) specific gravity transducer, 13) mixing T-bend, 14) concentration pH transducer.

give incorrect indications; the presence of methane also worsens the conditions for condensation of ethylene in the dephlegmator, which leads to an increase of pressure in the column. In view of the impossibility of condensing methane due to the absence of a supply of a freezing agent, it is drawn out from the column together with the ethylene fraction, i.e., the withdrawals from the column are increased apart from the dependency on the saturation status of the ethylene. To determine the methane content in the ethylene fraction, there is used the temperature difference of the phlegm and the upper part of the column, which is increased by the presence of methane. However, the indication of temperature difference can also be increased without the appearance of methane (for example, for a decrease in the amount of phlegm). Therefore, it is necessary to change the ethylene withdrawal controller assignment as a function of the temperature difference between the phlegm and the top of the column only with the condition that there has been a simultaneous rise in the pressure in the column. For this, the pulses from the instruments (transducer for the temperature difference between the phlegm and the top of the column, and the pressure transducer) are applied to the right side of controller Kt (assembly b) and, if a definite magnitude is reached, act on the switching block SB, thereby connecting the data transmitter for the ethylene withdrawal controller with the output from the transducer for the temperature difference between the phlegm and the top of the column (or of the pressure).

Thus, with normal column operation, the assignment of the ethylene withdrawal controller is established as a function of the quality of the upper product, with corrections for the speed of change of the load, and for the appearance of methane in the column, as a function of the methane content with a simultaneous increase of pressure in the column.

Equality of the quantities of heat entering and leaving the column is controlled by controller Kt from signals applied from pressure transducer P.

As the transducer for end product composition, a recording chromothermograph is used. So that the chromothermograph could be used as a controller, a special attachment was developed [2] which implements the following functions: fixing of the ethylene's peak amplitude on the chromatogram, which approximately corresponds to its concentration, by volume, in the end product; 2) transformation of the peak amplitude to a proportional magnitude of compressed air pressure (within the limits of 0 to 1 atm); 3) storing this air pressure until the results of the subsequent analysis are obtained.

The attachment consists of a ratchet mechanism with a rheostat transducer, and electromagnetic relay and a type ÈMD electronic balanced bridge; the attachment's operation is controlled by a timer.

As a function of the ethylene content in the end product, the output quantity (pressure) of the controlling chromothermograph attachment is changed; this pressure is applied to the data transmitter of the heat carrier (propane-propylene vapors) discharge in the fractionating column boilers.

The system just described, installed in one of the synthetic alcohol factories, turned out to be workable, and a prolonged industrial test is currently in progress.

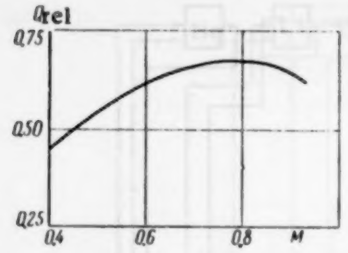

Fig. 5. The function $Q_{rel} = f(M)$. Q_{rel} is the relative output of alcohol, M is the "vapor-ethylene" molar ratio.

The next example of process automation by quality indicators is the catalytic process of obtaining ethyl alcohol by the direct hydration of ethylene (Fig. 4).

Concentrated ethylene with an admixture of ethane and methane (the forward gas) is mixed with the gas which did not react in the reactor (the return gas) and is supplied, via a system of heat exchangers, to an adiabatic reactor. After entry into the reactor, the gas is diluted in a given ratio by live steam in the mixing T-bend. In the reactor, at a definite temperature, there occurs the catalytic transformation of the steam-gas mixture to ethyl alcohol. The unreacted gas is separated in separators, is mixed with concentrated ethylene and applied to the compressor intake, where it is compressed and then is recirculated through the system.

Used as the catalyst is phosphoric acid on a solid carrier, with part of the acid being carried off by the gases, so that the steam-gas mixture leaving the reactor is subjected to neutralization by the alkalized water-alcohol condensate. After neutralization, the water-alcohol condensate is separated from the steam-gas mixture in the system of separators, after which it is directed to the fractionating column for fixing.

The activity of the catalyst used drops during a definite period of use, so that the necessity of replacing it arises, and therefore, the process of obtaining alcohol by the direct hydration method is carried on periodically.

The systems, used in the majority of factories, of controlling individual unconnected parameters (temperature and pressure before the reactor, steam-ethylene ratio, etc.) do not provide the maintenance of an optimal operating mode, for which one attains the maximum output of alcohol per ton of ethylene fed through with a minimal expenditure of water vapor.

For optimization of the process, an automation scheme was suggested which is based on the existence of an empirical relationship between the quantity of absolute alcohol obtained from direct hydration systems and the molar ratio of steam vapor to the circulating ethylene (Fig. 5). This relationship was obtained in NII synthetic alcohols on a reactor model, and was reinforced by further theoretical investigations.

The quantity of absolute alcohol is that quantitative, and, simultaneously, qualitative indicator which defines optimal reactor operation in the direct hydration system.

The curve on Fig. 5 shows the presence of an extremum. By controlling the magnitude of M, one can obtain the maximum output of alcohol whereby, as a function of the direction of the approach to the extremum, one can obtain the value of M corresponding to the minimal expenditure of steam. Due to the change of state of the catalyst and the disturbances of the process parameters, the alcohol output curve is continuously shifted, and, consequently, control with the aim of maintaining a definite given molar ratio does not give the necessary effect. In the given case, it is more effective to use an extremal controller which automatically maintains the maximum productivity of the reactor in terms of absolute alcohol output as the cost of changing the molar ratio.

The quantity of absolute alcohol obtained also depends on the reactor temperature, in that an increase in the reaction temperature causes the transformation of the ethylene to proceed more intensively and, consequently, increases the output of alcohol, but, with this, the catalyst is depleted; and there is a corresponding shortening of its useful life. Depletion of the catalyst is characterized by the temperature in the lower portion of the reactor, while the degree of transformation of the ethylene is characterized by the temperature in the reactor's upper portion. The temperature in the reactor's upper portion must increase as a function of the time of catalyst operation. In practice, there exists a temperature range inside of which there occurs a resetting of the data transmitter every four or five days. With an increase of temperature in the reactor's lower portion, the assignment for its upper portion must necessarily be decreased.

The automation scheme must also provide for control of the pressure and concentration of the ethylene applied to the reactor. In addition, it is necessary to control the intake of alkalized water-alcohol condensate in the neutralization T-bend, with the aim of maintaining a constant alkalinity (pH value) of the mixture obtained in the reactor.

In correspondence with all this, the scheme proposed for automating the process of direct ethylene hydration (Fig. 4) contains the following elements.

Controller MR, maintaining the molar "steam-ethylene" ratio and acting on the valve for the steam supply to the mixing T-bend. To the assignment chamber of controller MR there is applied the output coordinate of extremal controller EC. To the input of controller EC there is applied a signal from the assembly measuring the output of (obtained) absolute alcohol Q. Concentration measurer K-2 is intended for the introduction of corrections to the indications of assembly Q_{al}.

In actual production, there occur changes in the supply of ethylene and, from this, oscillations in the obtaining of absolute alcohol. As a result, with the presence of lags in the system, there occurs a "false" translation of the ex-

tremal controller which, essentially, increases the time necessary for finding an optimum of the process. In some intervals of time, the changes in ethylene supply occur more frequently than the time necessary for the automatic determination of an optimum. To create more favorable operating conditions for the extremal controller, there was adopted, as the quantity to be optimized, the ratio of the quantity of absolute alcohol obtained to the quantity of circulating ethylene, i.e., the aggregate's productivity of alcohol per ton of circulated ethylene.

Two-pulse temperature controller T is used for controlling the reactor temperature by acting on the steam supply valve feeding the heat exchanger. The temperature assignment for the reactor's upper portion is set by hand from the graphs. Over an entire run, the magnitude of the assignment is changed three or four times; for deviations of the temperature in the lower portion from the assigned value, there is an automatic correction made of the magnitude of the assignment for the reactor's upper portion. Controller T consists of two electronic potentiometers whose output signals are applied to the controlling block.

Pressure controller D, consisting of standard measuring elements, acts on the valve installed on the gas "exhaust" line.

Ethylene concentration controller K-1 varies the supply of the forward gas in the circulation system. An infrared gas analyzer, based on the selectivity of the absorption effect of hydrocarbon gases in the infrared portion of the spectrum, is used as the controller's transducer.

Control of the hydrogen-ion concentration in the water-alcohol condensate is effected by an electronic pH meter with a pneumatic output, in conjunction with highly alkaline electrodes. Maintenance of the pH of the concentration is implemented by changes in the quantity of the alkalized water-alcohol condensate introduced into the neutralization T-bend. In connection with the circumstance that the proper temperature must be given to the alkalized condensate, a two-pulse pH controller is used after the neutralization T-bend, i.e., there is a temperature correction introduced.

To implement the control scheme, it is necessary to measure continuously the quantity of absolute alcohol obtained from the hydration system. Direct measurement of the quantity of water-alcohol condensate, with its subsequent concentration correction, fails to give the requisite effect because of the liquid's high temperature. Moreover, a variable quantity of alcohol is continuously introduced into the hydration system in the form of the alkalized water-alcohol condensate.

To determine the absolute quantity of alcohol, one can use an equation obtained from a consideration of the balance of the water introduced into the reactor and the water contained in the water-alcohol condensate after the reactor. If we denote by W the weight in tons of the water vapor introduced into the reactor, then this same weight of water will be obtained at the tank's output. Denoting by W_{al} the weight in tons of absolute alcohol obtained from the system, and by K the fractional concentration, by weight, of this same alcohol, we obtain the following equations for calculating the quantity of water in the water-alcohol mixture at the system's output:

a) for the water mixed with alcohol

$$\frac{W_{al}}{K}(1-K);$$

b) for the water necessary for obtaining the alcohol

$$\frac{M_w}{M_{al}}W_{al} = \frac{18}{46}W_{al} = 0.39\,W_{al}.$$

Here, M_w and M_{al} are the molecular weights of water and alcohol, respectively.

By equating the quantity of water vapor used with the water in the water-alcohol mixture, we obtain

$$W_{vap} = \frac{W_{al}}{K}(1-K) + 0.39\,W_{al}; \quad (1)$$

or, after transformation,

$$W_{al} = \frac{W_{vap}K}{1-1.61K}. \quad (2)$$

In view of the absence of a device for automatically solving Eq. (2), it can be transformed to a multinomial. For this, we expand expression (2) in a series of powers of the difference $\Delta K = K - K_a$, where $K_a = 0.105$ is the actual mean concentration by weight.

By using Taylor's formula, and limiting ourselves to the first two terms, we obtain

$$W_{al} = 0.1122\,W_{vap} + 1.14\,W_{vap}\Delta K. \quad (3)$$

By substituting, in the second term of Eq. (3), the mean expenditure of steam, equal to 11.25 tons/hour, we get

$$W_{al} = 0.1122\,W_{vap} + 12.74\Delta K. \quad (4)$$

Automatic solution of Eq. (4) can be implemented by pneumatic adding elements of the MAUS system. Pneumatic signals are applied to the adder from the steam expenditure and from concentration measurer K. The concentration of the water-alcohol condensate is determined from the specific gravity for constant temperature, with the condition that the hydrogen-ion concentration, the pH, be maintained constant.

In the measurement of the discharges of ethylene and steam and the concentration of the water-alcohol condensate, there occur momentary oscillations of these quantities, so that there arise synchronous distortions of the value of the quantity to be optimized. To eliminate these distortions, the values of concentration and steam and ethylene discharges are averaged by means of integrators which permit the determination of total value over a definite interval of time, i.e., a quantity proportional to

the average value of the measured parameter during that same interval of time. An integrator consists of an aperiodic link with positive feedback and an adding link. The aperiodic link is a series connection, of a variable pneumatic impedance (a throttle), and a pneumatic capacitance; the link's gain can be changed by changing the throttle opening.

For the automatic determinations of the quantity of absolute alcohol obtained and of the quantity to be optimized, it is necessary to take into account that the changes in the discharges of the circulating gas and water vapor applied to the mixing T-bend occur at the beginning of the technological scheme of the process, whereas the measurement of water-alcohol condensate concentration occurs at the end. Lag time, determined experimentally, is about 8 to 10 minutes. Therefore, in the measurement and control systems, there is introduced a time shift between the application of signals from the ethylene and steam discharge transducers and the alcohol concentration transducer. For this there are used pneumatic devices, developed in the IAT AN SSSR [3], which permit the storage of the discrete quantities put out by the integrators.

In the scheme used for integration with account taken of a time shift (Fig. 6), it is provided that the output pressures from the discharge meters for ethylene DT-2 and for steam DT-1, and from concentration determination element K-2, are applied to the corresponding integrators with 10-minute integration times. The integrated value of concentration from integrator IN-3 is applied directly to the device for discharge measurement DM, and the results of the integrations of the ethylene and steam discharges are applied from integrators IN-1 and IN-2 to storage devices S-2 and S-4. The integrated values are

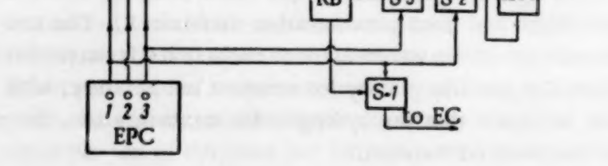

Fig. 6. Integration scheme with a time shift. V is a valve; DT-1 and DT-2 are the discharge transducers for steam and ethylene, respectively; IN-1, IN-2, IN-3 are integrators; S-1, S-2, S-3, S-4, S-5 and S-6 are storage devices; DM is the measurer of absolute alcohol discharge; RI is the recording instrument; RB is the ratio block; EC is the extremal controller; EPC is the electropneumatic command instrument.

retained in these storage elements for 10 minutes, and are then transmitted to elements S-5 and S-6, whence, with a 10-minute shift with respect to the integral value of the concentration, they are applied from S-5 to the RB block, and from S-6 to the DM block. After transmission of the stored values to the second set of storage elements, the first set is cleared for reception of the new integral values.

The twinning of the storage elements is designed to eliminate oscillations of the integrated values during the switching period. Each 10.5 minutes valves V are opened automatically, and the pneumatic capacitors are discharged into the atmosphere, i.e., the integrated value is cleared to zero. After closure of valves K, the integrators are again prepared for new integrations. All switching operations are carried out by the electropneumatic command instrument EPC in accordance with a given graph.

As the extremal controller in the proposed scheme, it was suggested that one use a type ÉPR pneumatic series controller with a stored maximum. However, in connection with the complexity of the control problem, it turned out to be necessary to introduce the following changes:

1. On the curve of the function "relative output of alcohol – molar ratio" (Fig. 5), there does not in practice exist a clearly expressed extremum point and, consequently, it is impossible to use a controller with storage of the maximum, since a controller of this type will transfer to this maximum. In the case given here, it is necessary to provide a comparison of each succeeding value, not with the maximum value, but with the preceding one (to use a controller of the step type).

2. Experimental data, taken off from an operating hydration system, showed that during a system run there occurs a continuous shift of the maximum in arbitrary directions, so that a working point lying on the curve's left arm actually falls on its right side, which leads to a sharp increase in steam discharge to the mixer for one and the same output of alcohol. Therefore, such an extremal control system is necessary for which the approach to the extremum, for any direction of its translation, is always carried out on the side of the least discharge of steam (a controller with nonreturn to the maximum).

3. Since the quantity to be optimized is applied discretely to the optimizer and does not change during a 10-minute period, the output pressure from controller EC will continuously change, attaining an inadmissible magnitude. It is necessary to provide a limitation on the time of change of the output pressure.

4. The "steam-ethylene" ratio varies only within definite limits, given by the technological regulations, since it is necessary to establish a limit on the pressure at the controller's output.

In the scheme described, the transition to a step-type controller was implemented by installing a block for the comparison of the last magnitude p_{last} with the previous one p_{prev}. This block is so adjusted that a change in direction of the controller's output pressure (reverse)

occurs, not only when $p_{last} < p_{prev}$, but also for $p_{last} \leq$ $\leq p_{prev} + \Delta p$, where Δp is a previously given small pressure.

The simultaneous installation of two comparison blocks (last magnitude with the maximal and with the preceding magnitudes) andlogical elements for conjunction and disjunction permitted the implementation of approach to the maximum from the side of the least expenditure of steam. The limitations introduced into the controller were implemented by means of a signalling relay and logical disjunction elements, and the step-function output from the controller was achieved by installing a pneumatic interruptor, controlled by command instrument EPC.

Use of this scheme permitted the transition to a completely automatically controlled direct ethylene hydration aggregate. Preliminary tests showed the capability of essentially increasing the output of alcohol. The use of the automation scheme turned out to be particularly effective at the ends of the system's runs — output was increased by 15 to 25%; the increase in alcohol output at the beginning of the run was about 5 to 8%. The figures just cited were obtained by integrating the curves of alcohol output at the beginning and end of runs with the control system switched in, and switched out. It was simultaneously found that the optimal magnitude of the "steam-ethylene" molar ratio is lower than that established by the chart of the technological regimen. Thus, for the optimal recommended molar ratio M = 0.75, the actual M, found by the extremal controller, was about 0.65 to 0.68. The transition to the lower ratio gives a significant economy of steam without simultaneously shortening the useful life of the catalyst.

Testing on the efficiency of use of the entire system continues.

SUMMARY

1. The workability was verified, for several typical processes, of an automatic control system using instruments for quality indicators.

2. The possibility was proven of completely automatic control of these processes for variable quality and quantity of raw materials.

3. Special quality transducers were developed which provide an accuracy in measuring product composition on the order of 0.1%.

4. The possibility was demonstrated of creating quality transducers from standard measuring elements connected in logical schemes.

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DEVELOPMENT OF APPARATUS FOR THE AUTOMATIC CONTROL OF BOILER AGGREGATES

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The principles of constructing electronic sampled-data systems of automatic control are presented.

A VTI contact controller system is considered. The path is pointed out for the further improvement of sampled-data control systems.

The development of the means of automatic control of boiler aggregates engaged the attention of many specialists in the Soviet Union more than 30 years ago. The first serious successes were obtained in the direction of developing hydraulic control systems with jet amplifiers.* There were constructed trial specimens of various electrical systems, including photoelectronic,† induction-electronic‡ and also those based on the use of a contact measuring instrument (including a controller with a "falling arc"). ** Work on automation of boiler aggregates was broadly developed after 1945. The basis for this was the aforementioned hydraulic control system, controller systems with "falling arcs" and the newly developed "electromechanical" control systems† which were used as standards in all new boiler aggregates. Further efforts, directed toward the development of more highly perfected controllers, led to the development of an "electronic" system** which by degrees displaced the other designs and is today the standard for the new powerful stands.

We now turn to the consideration of modern electronic automatic control systems. The following principles underlie the systems' construction: parallelism of inspection and control, electrical methods of action, sampled-data character of the executive mechanisms' control. Prolonged, massive industrial testing of electronic controllers verified the correctness of the choice of these principles, which were also retained for the succeeding perfected electronic systems.

1. The Principle of Parallelism

Common to all the developed means of automatic boiler aggregate control considered is the principle of parallel action of the automation and inspection devices.

In a number of branches of industry, a different system of inspection and control has been used until now, a system in which the automatic controller is connected in series with the secondary inspection instrument (the so-called "instrument" system). In this variant, a disturbance of the function of the inspection instrument leads to a complete disruption of the control system as a whole.

With parallel action of independent inspection and control systems (in the so-called "apparatus" type systems), the reliability is significantly improved. In this case, one arm is completely subsidiary to the second. The probability of disturbance of the automatic control arm is significantly lower here, since a fewer number of simpler elements are included in it. Increased reliability is furthered by the specialization of the functions of the control and inspection systems, which reduces the technical requirements imposed on the apparatus of each system separately, and leads to a simplification of the apparatus.

2. Electrical Methods of Action

Latterly, the development of control systems for boiler aggregates has confidently followed the path of using electrical means.

The use of electric energy for implementing the functions of signal transmission from measuring elements to the controller and beyond, to the executive mechanism-for purposes of amplifying signals, transforming them, forming the control laws—allowed one to unify homogeneous elements independently of the purpose of the controller, virtually removing limitations on the physical placement of the elements.

The use of electric energy removes the need to impose requirements on the qualitative state of the carriers of auxiliary energy, requirements which arise, for example, in hydraulic and pneumatic systems, and which lead to the necessity of employing additional equipment.

The capability of employing sensitive, low-inertia measuring elements possessing, as a rule, little energy, provides a high quality in the automatic controllers. The use of such elements presupposes the presence of high-gain

* Developed by the "Teploavtomat" Plant.

† Developed by the Central Boiler-Turbine Institute (TsKTI).

‡ Developed by the All-Union Electrical Engineering Institute (VÉI).

** Developed by the All-Union Heat Engineering Institute (VTI).

amplifiers in the controller, since the controller's output element — the executive mechanism — must possess great mechanical energy. This requirement is easily met by certain electrical (for example, electronic) amplifiers which, at the same time, have little inherent inertia. The use of electronic amplifiers simultaneously opens up broad possibilities of introducing simple elements into the controllers for shaping the control laws.

One has succeeded in building a controller in which high reliability of operation is joined with diversity of function, universality — with simplicity of manufacture, the convenience of adjustments collected on one panel of the calibrating organ — with stability of the adjustment parameters, a significant range of variation of these parameters with a high sensitivity and significant controller output power.

3. Block Schematics

A large role in the development of a control system is played by the choice of the optimal block schematic for the controller. In particular, it is of interest to compare the three characteristic schemes shown on Fig. 1.

The scheme of Fig. 1a assumes the presence of amplifier 2, acting on an astatic executive mechanism and feedback device 4, forming the device's control law, which shunts the amplifier and the executive mechanism. The capability of remote control of the executive mechanism is provided by means of device 5.

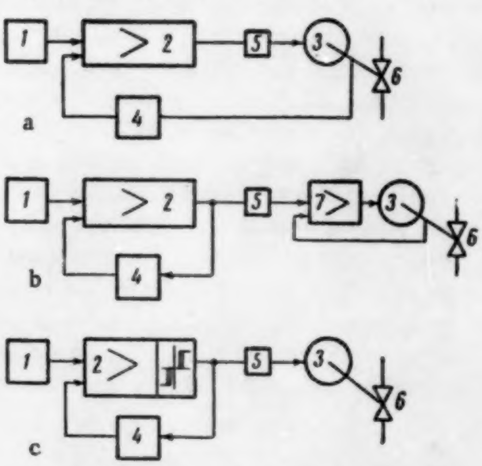

Fig. 1. Controller block schematics. a) A feedback path shunts the amplifier and an astatic executive mechanism, b) the feedback path shunts only the amplifier, while the astatic executive mechanism is transformed to a static one by the use of an auxiliary amplifier, c) the feedback loop shunts an amplifier with a relay characteristic; the executive mechanism is astatic with a constant velocity, 1) a transducer, 2) an amplifier, 3) an executive mechanism, 4) a feedback device, 5) a remote control organ, 6) a controlling organ, 7) an auxiliary amplifier.

In the scheme of Fig. 1b, forming of the control law is implemented by feedback device 4†† in a loop which shunts only amplifier 2. A system is formed which forms the control law in the form of an electric voltage (current) which is then processed by static executive mechanism 3. Since the output quantity generated by the forming device (properly speaking, by the controller) carries insignificant energy, auxiliary amplifier 7 has to be introduced into the executive mechanism. The ordinarily used astatic executive mechanism is transformed to a static one (i.e., to a servosystem) by shunting it, together with the additional amplifier, by a static feedback path.

In the scheme of Fig. 1c, an amplifier with a relay (trigger) characteristic is used, which also permits formation of the control law within the limits of controller 2 proper, but without the necessity of shunting the executive mechanism by a feedback circuit (as in the scheme of Fig. 1a), and without the necessity of using a second amplifier (as in the scheme of Fig. 1b). In contradistinction to the first two variants, however, there is now the specific requirement imposed here that the executive mechanism possess a constant speed. A comparison of the block schematics from the point of view of satisfying the basic requirements of their use — reliability of operation, implementation of the given control law, convenience of adjustment, maintenance, etc. — leads to the following conclusions.

The greatest reliability of operation can be achieved with a controller constructed in accordance with the scheme of Fig. 1c. The scheme of Fig. 1a is complicated by the presence of the external feedback path, disturbance of which leads to a sharp translation of the controlling organ. The scheme of Fig. 1b is complicated by the presence of the second amplifier and the second feedback circuit; disturbance of the forward path from the controller to the executive mechanism in this case leads to a sharp displacement of the controlling organ. The transition from remote to automatic control of the executive mechanism in the schemes of Figs. 1b and 1c also, as a rule, gives rise to sharp displacements of the controlling organ.

Implementation of the given control law may be executed with somewhat greater accuracy in the scheme of Fig. 1b, and, particularly, in the scheme of Fig. 1a. However, when the controlling organ reaches an extreme position here, there occurs a disruption of the control law, as a result of which the controlled quantity experiences a significant deviation after the return of the controlling organ to the range of control.

In all cases, one may achieve approximately equivalent convenience of adjustment and maintenance.

The cost of the apparatus turns out to be approximately equivalent for the variants of Fig. 1a and 1c, and is higher for the variant of Fig. 1b. In cases when it is necessary for one controller to control several identically displaced

†† Formation of the control law can also be carried out in the amplifier's "forward" path without the use of feedback, but this entails great difficulty in its realization.

controlling organs, the cost of the variant constructed according to the scheme of Fig. 1b may turn out to be comparable to, and even less than, the cost for the other two variants. However, this advantage is accompanied by the defects that arise when the controlling organ, having reached an "end" position, returns to the range of control.

4. Electronic Controllers of the VTI System

With account taken of the considerations presented earlier, there was developed a reasonable universal electronic control system put in production in 1951. As the basic variant of this system, the structural scheme of Fig. 1c was realized, developed in more detail in the skeleton scheme of Fig. 2. The primary instruments 1, 2 and 3 measure the necessary quantities. The primary instruments provide, at their outputs, a dc or ac current proportional to the measured quantity. Signals from the primary instruments are applied to the electronic controlling instrument EC, containing a measuring device, an amplifier and a feedback device. In measuring device 5 there is implemented algebraic summation of the signals of the primary instruments and from manual data transmitter 4. The total voltage $\underline{x}$ from the measuring device is applied to amplifier 6. The output signal $\underline{y}$ is fed to feedback device 7, shunting the amplifier, and is also directed to the external circuit. In the executive circuit there are provided: switch 8 from automatic to remote control, remote control key 9, final amplifier (magnetic starter) 10, and executive mechanism 11 with shaft position indicator 12. The lower portion of Fig. 2 shows how the controller (connected in the PI and PID modes) processes a unit step of the input quantity $\underline{x}$.

In a complete electronic system there are provided diverse primary instruments (transducers) checking temperature, pressure, pressure drops, mechanical translations, ac current strength, oxygen content in gases, etc., the corresponding controlling instruments, executive mechanisms and auxiliary elements. Different combinations of instruments permit the implementation of simple and of complex (including connected) schemes for the realization of different control laws, etc.

Figure 3 shows a simplified schematic of one of the most widely distributed combinations, namely, a type EC-III (ER-III) electronic controlling instrument permitting simultaneous connection of from one to three primary instruments with induction transformers and acting on one executive mechanism. Controllers of this type are used, for example, for the automation of boiler aggregate supply, for controlling the ratios of steam-air and heat-air, for controlling evacuation of the fire boxes, etc.

The controller's measuring device contains the power supply for the three induction transformers and a potentiometer, filling the role of a manual data transmitter. Together with the transformer supplying its winding, each induction transformer forms an equilibrium bridge in the diagonal of which a voltage divider is connected, enabling the necessary fraction of the signal from the given transducer to be used. The data transmitter circuit forms an analogous bridge as well.

The algebraic sum of the voltages from the transducers (zero in the steady state) is applied to the electronic device which contains an input transformer and a two-stage balancing phase-selective amplifier with a relay output, shunted by a negative feedback path of RC elements.

The amplifier is supplied with a variable anode voltage. Its first stage is connected in a parallel scheme and is distinguished by the presence of a capacitor which shunts the anode load resistor. From the output of the first stage is obtained a smoothed dc voltage whose sign depends on the phase of the input voltage. This gives the amplifier the property of noise resistance, i.e., a small functional dependence of the output voltage on the presence, in the signal to be amplified, of reactive components of the fundamental phase and higher harmonics. The amplifier normally operates, for example, with a reactive signal at its input whose amplitude is some ten times greater than the controller's zone of insensitivity. The use of the given amplifier permitted the avoidance of a serious disadvantage of ordinary ac amplifiers, namely, their too facile saturability for small noise levels. In its turn, this permitted the controller to be connected with primary instruments provided with the simplest, most reliable, induction trans-

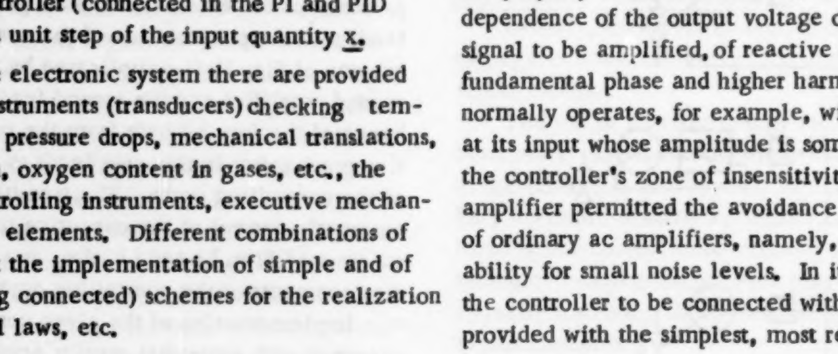

Fig. 2. Skeleton scheme of an electronic controller. 1, 2, and 3) primary instruments (transducers), 4) data transmitter, 5) measuring device, 6) amplifier, 7) feedback device, 8) operation mode switch, 9) remote control key, 10) magnetic starter, 11) executive mechanism, 12) shaft position indicator.

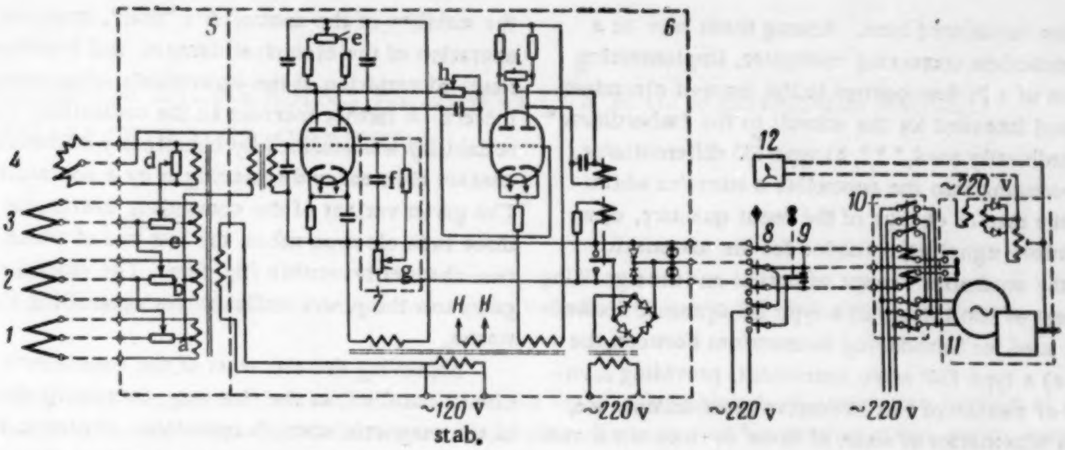

Fig. 3. Simplified schematic of electronic controller EC-III. Adjustment organs: a, b, c) sensitivity transducers, d) measuring device zero corrector, e) amplifier zero corrector, f) replaceable "isodrome time" resistor, g) "communication speed", h) "damper", i) "insensitivity zone". The remaining nomenclature corresponds to that of Fig. 2.

formers which produce signals with noise of the character cited.

There is an electric damper connected between the first and second stages by means of which the deleterious effect of pulsations of the quantity to be controlled, which are observed in some cases, may be eliminated.

The second stage is also a balancing scheme. In the diagonal of the bridge formed by the two triodes of the second vacuum tube and by two load impedances, there is connected a divider which supplies the controlling winding of the reversible polarized relay, shunted by a capacitor to eliminate contact vibrations. An additional improvement in the operating conditions of the relay's contacts, plus the provision of a loop-type characteristic of the relay element, is attained by the use of a second winding, the so-called "holding" winding, which conducts current during the periods of contact making, and also by the use of an arc-arresting device, namely, semiconducting rectifiers shunting the external load. The relay contacts control the transmission of current pulses from the auxiliary rectifier in the external circuit to the signal lamps, and also to the feedback device.

The feedback device consists of one or two (in other controller modifications) series-connected RC networks, supplied by a regulated fraction of the output voltage pulse. The output voltage of this device is applied to the floating grid of the amplifier's first-stage vacuum tube.

The controller's external circuit contains a reversible three-phase magnetic starter which directly controls the motor of the executive mechanism. Capability of remote control has been provided.

The executive mechanism is a three-phase asynchronous motor with a two-stage reducer. The physical design of the mechanism includes a brake for decreasing the effect of "run-out", a hand wheel for manual (local) control, a single-pole terminal switch and a rheostat transducer for remote indications of the output shaft position. Modifications of the mechanism provide output torques of 25 and 100 kg for a shaft rotation of 90° in 30 seconds.

Depending on the modifications of the feedback device, the controller, under definite conditions, approximately implements P-, PI or PID ‡‡ control laws with a wide range of tuning parameter adjustments.

In addition to the basic devices already cited, the complete apparatus contains a group of auxiliary instruments which extend the technical capabilities of the

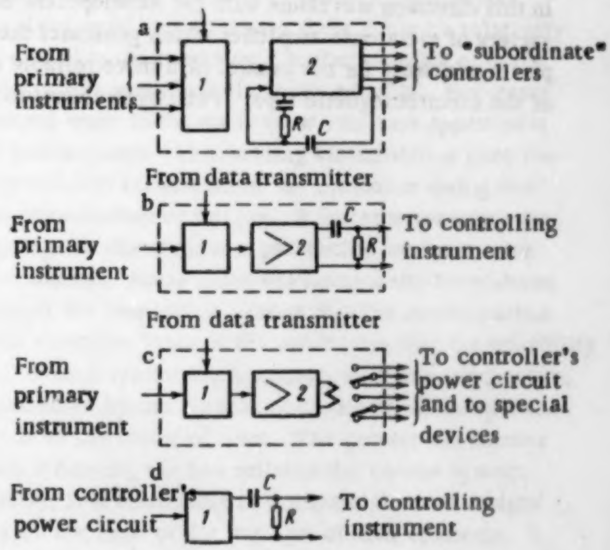

Fig. 4. Skeleton schematics of certain auxiliary devices of the electronic control scheme. a) Type EKR contactless correcting controller, b) type ED differentiator, c) type EOS limiter-signaller, d) type DS dynamic connection device, 1) measuring device, 2) amplifier.

‡‡ Translator's note: The foregoing letters stand for: Proportional, Integral and Derivative.

control system considered here. Among these are: a) a type ÉKR contactless correcting controller, implementing the formation of a PI-law control in the form of electrical quantities, and intended for the stimuli to the "subordinate" controllers ordinarily used,*** b) type ÉD differentiator, used for introducing into the controller a stimulus which depends on the rate of change of the input quantity, c) a type ÉOS limiter-signaller, intended for the automatic limiting of the controller's range of action and the signalling of the moment of limitation, d) a type DS dynamic connection device, used for introducing connections between the controllers, e) a type ÉSP servo instrument, providing identity of paths of motion of two executive mechanisms, etc. The skeleton schematics of some of these devices are shown in Fig. 4.

5. Further Development of Electronic Control Systems

Prolonged use tests of more than ten thousand electronic controllers permitted their weak points to be made manifest. About 90% of all the cases of disrupted normal controller functioning were due to failures of the controlling instrument amplifier's output element – polarized relay RP-5. The remaing 10% of the failures were almost uniformly distributed between the magnetic starter and the executive mechanism. An insignificant number of failures occurred in the remaining elements.

Further perfecting of the apparatus for electronic automation was executed in several stages.

The first problem was to increase the reliability of performance of the polarized relay. An important step in this direction was taken with the development of a new variety of electronic amplifier which generates the output power necessary for the control of a more reliable relay of the electromagnetic type. This, by the way, increased the stability of the controller's "null", improved the operation of the electrical damper, and broadened the range of variation of the controller's adjustment parameters. A further increase in the controlling instrument's reliability was attained by completely eliminating the contact element and replacing it by a contactless one. The given variant of the controlling instrument contains three twin electron tubes, the last two of which form the two-channel reversible flip-flop. The amplifier directly generates the power sufficient for controlling the magnetic starter.

Improving the elements of the controller's power circuit entailed, as the first step, increasing the reliability of the magnetic starter's operation. However, its replacement by contactless elements – terminal magnetic amplifiers – provides a real gain only when this is simultaneously accompanied by a radical change in the physical design of the executive mechanism, which must require moderate electric power and must possess low inherent inertia.

The development of such an executive mechanism led to the creation of several models of electromechanical and electrohydraulic types. All of them operate in conjunction with fast-acting, one-stage terminal magnetic amplifiers, process sufficiently well-defined pulses of duration greater than 0.1 second, are self-braking, and are equipped with organs for remote and local control.

The combination of flip-flop controlling instrument with terminal magnetic amplifier plus "sampled-data" executive mechanism forms a high-reliability control system which answers to the various requirements of automation in energetics and in other branches of industry.

***This corresponds to element 2 in the previously considered scheme of Fig. 1b.

METHODS OF INCREASING RELIABILITY AND RESPONSE SPEED OF STEAM TURBINE CONTROL

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The question as to the choice of reliable control element design is considered. A description is given of a hydrodynamical scheme for steam turbine control and of its elements: self-centering piston, pump, accelerator, etc.

The reliability of operation of an automatic control system and its individual assemblies essentially determines the possibility of automating some object or another. Indeed, automation is called upon to replace human labor by the more accurate "labor" of an automaton. But, with unreliable means of automation, the labor of the machine operator is frequently replaced by the more highly specialized labor of the automata maintenance men, which makes the automation of the given process meaningless.

However, the concept itself of "reliability of automata operation" is not accurately defined.

The reliability of automata operation and automatic control system operation depends basically on the dynamic properties of the automaton or system, on the reliability of the physical design and on the quality of the manufacture and choice of the materials.

In this paper, the attempt is made to establish criteria which would allow comparisons of automata designs in terms of their reliability. With this, questions of dynamic properties, quality of manufacture and quality of materials are excluded from consideration because, first of all, a large number of works are devoted to these questions and, secondly, it is desired to give special prominence to the necessity of creating reliable physical designs of automatic devices.

By the reliability of operation of some mechanism or another is understood its trouble-free use. It is necessary to establish some criteria which would allow the provision of the possible occurrence of troubles due to faults in the automaton's physical design. Without such criteria, comparisons of designs in terms of their reliability would require many years of testing of the competing designs. And even under these conditions the comparison would be rendered difficult due to troubles in the automata for reasons indirectly related to their physical design (technological errors, faulty maintenance, etc.).

If one analyzes the statistics of control system troubles, then one can turn one's attention to the disturbances in similar physical assemblies. Such assemblies are joints, springs, membranes, contacts, radio tubes, etc.

The assemblies enumerated have one common idiosyncrasy – the causes of failure accumulate in them. In the course of time, joints wear. Springs under high tension break from fatigue. Membranes, just as springs, break from fatigue. Contacts burn out. In radio tubes, emission ceases, etc. The designer's problem in using these elements consists in this, that he must create for them such operating conditions that disruptions of their normal functioning will be completely eliminated or put off for the longest possible time. When, for random causes, difficult operating conditions arise, the disruptions occur in precisely these most vulnerable assemblies.

On many of the turbines of old design produced by the Leningrad Metal Factory, there is used a relatively little-turned speed controller. In the majority of cases, this controller operates sufficiently reliably. But cases are known when increases in rotor vibration appeared in three power plants. This loading circumstance gave rise to a systematic breakdown of the controller spring due to the phenomenon of fatigue. It became necessary that the springs be changed, as a preventive measure, once in four months. Many more examples could be adduced to support the proposition advanced. The consideration of such examples leads to the conclusion that the reliability of any control system as, moreover, with any mechanism, is determined by the number of elements whose operation depends on the factor of time. The greater the number of such elements, the less reliable the control system. Certainly, it is impossible to compare physical designs solely on the basis of the numbers of such elements. It is necessary to take into account the conditions of their operation as well. Improvement of these conditions permits the reliability of such elements, and of the system as a whole, to be increased.

The reliability criterion just formulated allows one not only to estimate the quality of the physical design, but also makes it possible to choose the most reasonable methods of improving it, and allows one to avoid creating designs which, while new, give no practical increase in reliability.

In this paper we present methods for increasing reliability and speed of response of steam turbine control systems. The selection and development of these methods is based on the reliability criterion suggested. Thus, the contents of this paper are, as it were, the working out of an example of the use of the given criterion for creating rational control systems. Although the paper is devoted to a particular question, its results also have a more general value.

I. A Reasonable System of Steam Turbine Control — a Hydrodynamical One

The multiformity of physical designs of control system assemblies for steam turbines is determined, to a significant degree, by patent restrictions and by the preferences evolvedby different designers for various requirements. One group consider principally engineering sophistication, another considers compactness, a third, convenience of maintenance, etc. A consequence of such a diversity of approaches is the difficulty of choosing a best solution. We shall make the attempt to analyze the existing speed controller designs on the basis of the reliability criterion.

The physical designs of the ordinary centrifugal controllers with hinge joints are insufficiently reliable due to wear of the joints. This is supported by the fact that, in repairing turbines, it is necessary, as a rule, to overhaul the hinge joints. An increase of reliability is attained by eliminating the hinge joints, i.e., by making the transition to unjointed controllers. This path was taken by Westinghouse, Allis-Chalmers and also in the developments of MÉI, the Stalin LMZ, and many other factories and organizations.

But it is impossible to separate the physical design of the controller from the other assemblies of the control system. A unified approach to the solution of the problem always gives better results.

In many contemporary control systems for steam turbines, oiled servomotors are used. Consequently, the reliability of the control system depends on the reliability of the oil supply. The most reliable drive for the oil pumps is the drive from the turbine's principal shaft (such a drive is received by the entire turbine works). With this, the operation of the pump depends on the number of rotations of the aggregate, which permits the pump to be used, in the given case, as a sampled-data transducer of the speed controller.

Oiled servomotors, as a rule, are controlled by cylindrical valves. If a cylindrical valve is used as a piston, receiving pressure impulses from the pump, then a control system with one amplifier will consist of a pump, a valve, and a servomotor. All these elements are necessarily independent of the type of control system and, consequently, no matter what speed regulator might be used (mechanical, electrical, etc.), it would be an extra link of the scheme. Naturally, the removal of excess links always increases system reliability, if, with this, no deterioration of control quality ensues.

Such a control system was used by Westinghouse, Sultser and Simmonds-Shukkert, and by the Kirovskii Plant in the USSR, which supports the correctness of the logical deduction just made. However, all the aforementioned factories subsequently turned from such simple control schemes. An analysis of the results of operation of these systems under working conditions shows that the basic reasons for the troubles in their functioning are:

1) increase of insensitivity of the spring-loaded piston-valve;

2) oil pressure pulsations at the pump's output;

3) unsatisfactory form of the pump's characteristics.

The example just considered shows that the reliability criterion suggested is necessary but not sufficient. It was emphasized earlier that its use permitted the choice of the theoretically most reliable control system, but this sytem must, in addition, possess high sensitivity, accuracy and speed of response. However, this criterion permits us to mark out the path to perfecting a control system. If it is assumed that the basic functional schematic of a control scheme is correctly chosen, then the path to its further development is also clear. In the case under consideration, the chosen scheme becomes workable if, without lowering reliability and complicating the scheme, one succeeds in eliminating the observed causes of trouble. We present methods below which permit the problem thus posed to be solved, thanks to which the simple control scheme created turned out to be completely workable.

1. Use of a self-centering piston. The previously known methods of increasing the sensitivity of piston mechanisms turned out to be inapplicable in the given case. With the use of an intermittent piston, static friction is replaced by kinetic friction, but this reduces the friction coefficient by no more than a factor of two, which is inadequate for contemporary requirements on the quality of controller operation. The use of rotating pistons reduces the friction coefficient by a large factor, but when the piston is loaded by a steel spring, it forces a significant loading on the supporting bearings, reducing the reliability of the piston mechanism.

The problem was solved by the creation of a self-centering piston [1, 2].

On the outer surface of such a piston two series of notches are made (Fig. 1). In each notch (I, II, III and IV) liquid is supplied from the delivery cavity via a specially drilled aperture f_0 of small diameter. From the notches, the liquid flows via the gap between the piston and the cylinder. To prevent the fluid from falling from the delivery cavity into the notches, there is provided on the piston's surface, in addition to drill-hole f_0, a drain gutter a, from which the fluid is directed to the reduced pressure cavity by a longitudinal drain of relatively large cross-sectional area.

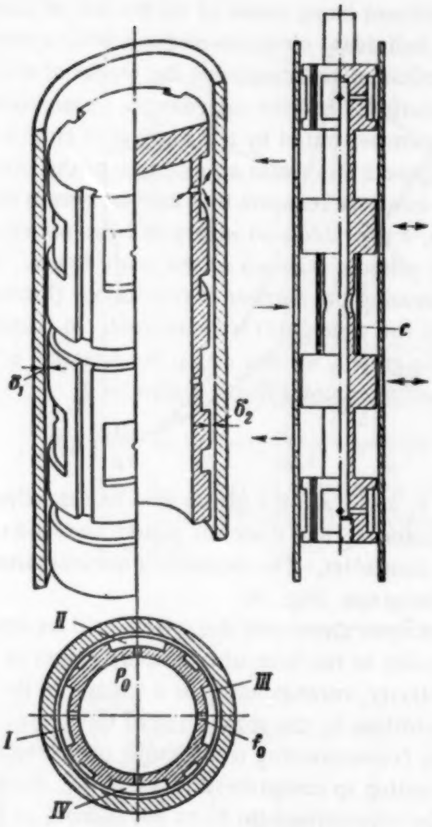

Fig. 1. Self-centering piston.

If external forces press the piston to the cylinder wall, then, in notch I, positioned on the side of the lesser gap δ_1, the pressure P_1 is increased, so that the resistance to the flow of liquid from the notch is increased. In notch III, on the opposite side, the liquid pressure falls due to the decrease of the resistance to flow through the enlarged gap δ_2. A pressure difference, P_I-P_{III} appears which, in overcoming the action of the external forces, tends to return the piston to a coaxial position with respect to the cylinder. The maximum pressure difference will occur when the piston touches the cylinder wall. In this case, in the notch on the side of the null gap, virtually the full fluid pressure P_0 will exist and, on the opposite side, the minimum possible (corresponding to the maximum gap δ). Under these conditions of fluid supply to notch I, the pressure in it will be close to P_0 by virtue of the fact that gap δ_1 equals zero. Consequently, the pressure difference will be larger, the lower the pressure P_{III} in notch III. It is therefore necessary to attain a decrease in resistance to the flow of fluid from the notch, and to prevent fall of fluid from the delivery cavity to the notch apart from drill-hole f_0.

The fluid pressure in any notch is determined from the relationship

$$\frac{P}{P_0} = \frac{B}{P_0}\left[\sqrt{1 + 2\frac{P_0}{B}} - 1\right], \qquad (1)$$

where P is the pressure in the centering notch, and P_0 is the pressure in the delivery cavity.

$$2B = \left[\frac{12\mu f_0}{\delta^3\left(\frac{l}{m} + \frac{l}{m_1} + \frac{2b}{L_{av}}\right)\sqrt{1,12\rho}}\right]^2. \qquad (2)$$

In expression (2), μ is the dynamic coefficient of viscosity, in kg·sec/m²; f_0 is the area of the delivery drill-hole in m²; δ is the gap between cylinder wall and piston at the locus of the notch in meters; ρ is the fluid density in kg·sec²/m⁴; l/m, l/m_1 and $2b/L_{av}$ are the ratios of width of leakage flow to the length of the path to the reduced pressure cavity.

Experimental verification [3] supported the high efficiency of such a method of decreasing the friction coefficient.

The special feature of this method is that it can be used for any medium — liquid or gas.

2. Pulsations of oil pressure at the pump's output. A study of the operation of hydrodynamical control under working conditions showed that the pressure at the pump's output pulsates independently of the changes in number of rotations. With this, the period of the oscillations of pressure is so great that the piston taking the pressure and, following it, the principal servomotor, are able to follow the changes in pressure.

A detailed study of this question [4] showed that the causes of the pulsations are vortex formations in the oil

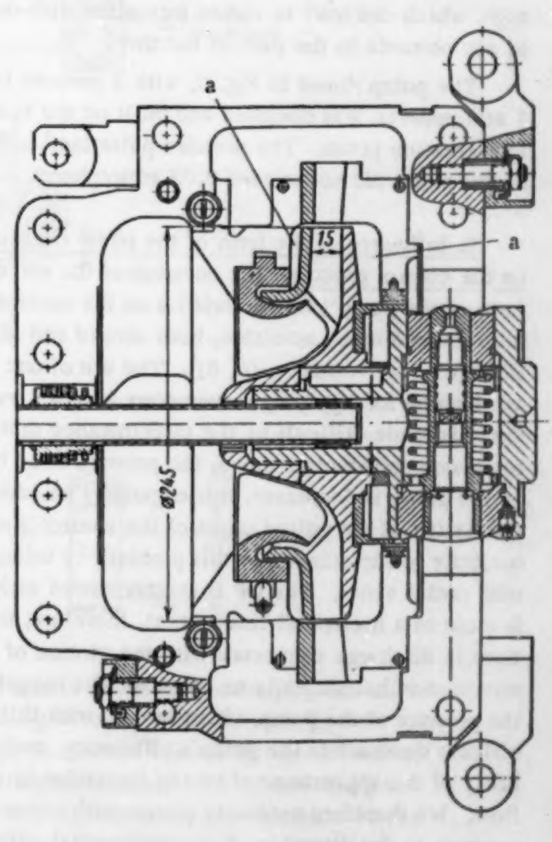

Fig. 2. Oil pump.

flow. Discovery of the reasons for the pulsations permitted the development of measures to eliminate them.

To eliminate vortex formation before input to the wheel, a well-streamlined input chamber is used (Fig. 2). Then, before the input to the vane channel itself, there is employed a directing apparatus with radial vanes. To eliminate the effect of leakage from the lateral chambers on the entering oil flow, there was used a special hooked form of packing. Such a packing squeezes the leakage flow to the lateral disk wall and eliminates its effect on the basic flow. The use of such a form allows the use of packing with relatively large radial gaps, which increases the reliability of operation of the pump, the disk of which is fitted directly onto the turbine shaft.

Vortex formation in the vane channel of the pump is reduced with a decrease in angle of attack. We therefore adopted an angle of attack no greater than 3 to 6°.

Disturbances of the flow at the pump's output can arise due to the action of parasitic flows in the pump's lateral chambers a (cf., Fig. 2). To eliminate this effect, it is more advantageous to use a directing vane apparatus at the pump's output, and not the helices which are the most widely disseminated in contemporary pumps. The vane directing apparatus has an advantage over the helix also in the respect that it eliminates the possibility of vortex formation at the output of the directing apparatus. For this, it is only necessary to have a sufficiently free chamber on it and, correspondingly, a small velocity of flow. In a helix, the velocity of flow is comparatively high, which can lead to vortex formation with the presence of any obstacle in the path of the flow.

The pump shown in Fig. 2, with a pressure head of 7 atmospheres, was designed and built on the basis of the investigation given. The pressure pulsations at the output of this pump did not exceed 0.03 atmospheres.

3. Influence of the form of the pump characteristics on the control process. The question of the effect of the form of the pump's characteristics on the control process has received much attention, both abroad and in the USSR (cf., for example, [5, 6]). The use of one and the same pump for supplying servomotors and as a pulse transducer is made difficult by the circumstance that, when the piston servomotors move, the pressure head built up by the pump is decreased; this engenders an additional translation of the pulsed organ of the control system. The majority of factories solve this problem by using a pump with radial vanes. The QP characteristic of such a pump is close to a horizontal line so that, therefore, the variations in discharge connected with the motion of the piston servomotors has virtually no effect on the magnitude of the pressure at the pump. But there is, with this, a significant decrease in the pump's efficiency, and the possibility of the appearance of vortex formation in the oil flow. We therefore used only pumps with vanes curved contrary to the direction of circumferential velocity.

To consider the possibilities of using such a pump, with account being taken of all the sets of interactions of the individual elements of the control system, we used the method of superimposing the graphs of the pump characteristics and the impedances for two positions for the system separated by the interval of time Δt [7].

Figure 3 illustrates an example of the investigation of the transient response in a control system with servomotors, a two-sided oil supply and cut-off valves. The control scheme is shown on the same figure. The process of decreasing the number of revolutions (increasing the load on the aggregate) is considered. In correspondence with the graphs, we can set up the equation of motion of piston servomotor 3 and controller 1:

$$F_S \frac{dz}{dt} = F_C \frac{dx}{dt} + \left(\frac{\partial Q_1}{\partial p} + \frac{\partial Q_2}{\partial p}\right) \Delta P_2 , \tag{3}$$

where F_S and z are the piston area and translation of the servomotor, F_C and x are the piston area and translation of the controller. The remaining nomenclature is clear from the graph (Fig. 3).

Analysis shows that the system will be dynamically stable only in the case when the increment of pump productivity, corresponding to a change in the number of revolutions by the magnitude of the degree of nonuniformity (corresponding to a change of aggregate load from free-running to complete) $\Delta p_2 = \Delta p_{max}$, must be greater than the expenditure (in liters per second) of the oil applied to the servomotor for maximum raising of the valve. In [7] a detailed analysis of four variants of the control scheme was given.

If, as the principal servomotors, one uses servomotors with circulating valves (the Brown-Bovary system), then it is most advantageous to use valves with double throttling [8] in which not only the supply, but also the drainage, of oil is throttled. In these valves the aperture profile can be so chosen that, for slow motion of the valve, the pressure of the fluid in the line between the apertures will change for an unchanged discharge of oil via the valve. With this, there will be the least effect of servomotor motion on pressure pulses.

In other systems we recommend the use of a pump with two disks, wherein one is used for supplying the servomotors, and the other as a pulse transducer. In this case there is virtually no action of the servomotor on the pressure pulse.

4. Two-injector oil supply scheme. A centrifugal pump does not possess the property of automatic intake, i.e., upon rarefaction in the intake housing and the entry of air into it, the supply of fluid from the pump ceases.

To increase the reliability of operation of a centrifugal pump, various auxiliary devices are used, providing a tributary at the input to the pump. Using the reliability criterion, we chose an injector method of creating a tributary at the pump's input (Fig. 4). The injector has no moving parts and therefore, as a mechanism, is very reliable. This method was suggested and developed by the

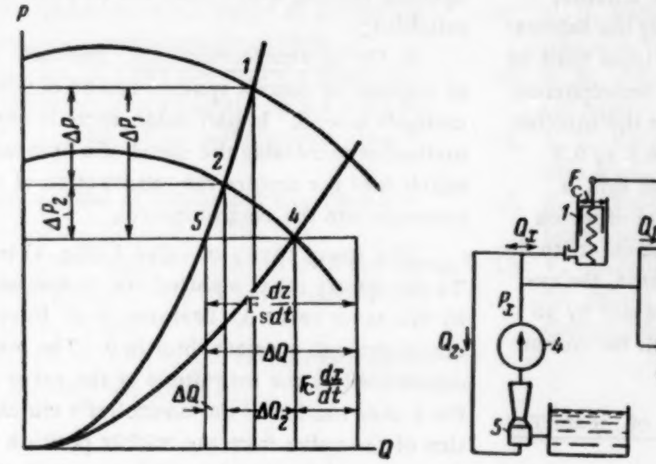

Fig. 3. Effect of servomotor piston motion on impulse pressure. 1) Piston controller, 2) valve, 3) servomotor, 4) pump, 5) injector, 6) constant throttle.

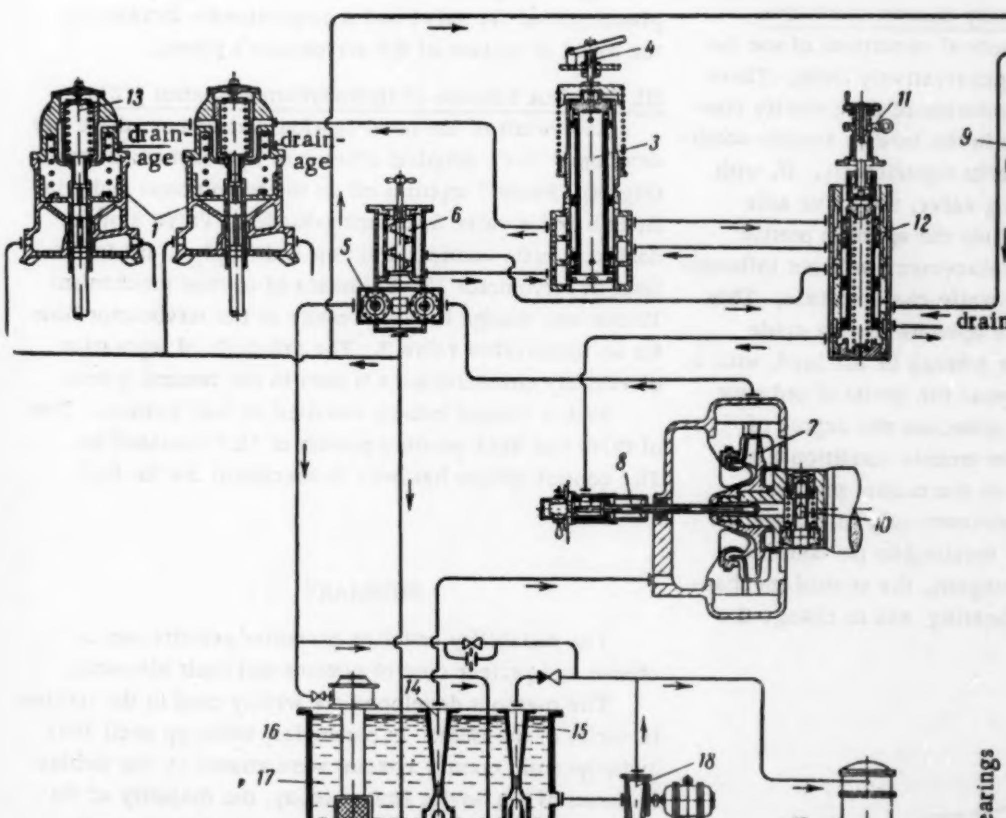

Fig. 4. Simplest schematic of a hydrodynamic control scheme. 1) Throttle valve servomotor, 2) cut-off valve servomotor, 3) piston controller-valve, 4) mechanism for checking velocity of automaton for safe increase of rotations, 5) spherical reverse valve, 6) filter, 7) oil pump, 8) manual control for cut-off valve, 9) intermediate amplifier for the cut-off valve drive, 10) velocity safety automaton, 11) control mechanism, 12) valve control mechanism for the controlling and cut-off valves, 13) intermediate amplifier servomotor for the throttled valve drive, 14) injector support, 15) injector lubricating point, 16) auxiliary turbopump, 17) oil reservoir, 18) spare pump with an electric drive, 19) oil cooler.

Westinghouse Company. In the Westinghouse scheme, the tributary injector simultaneously supplies the lubrication system. Therefore, the pressure at the input must be chosen rather high (on the order of 1 to 1.5 atmospheres) which increases the expenditure of power on the injector. For reliable pump operation, a tributary of 0.2 to 0.3 atmospheres is sufficient. In connection with this, a scheme with two injectors was suggested, one of which supplies oil to the lubrication system and the other, to the input of the pump (Fig. 4). With such a scheme, the expenditure of power on the oil supply is decreased by 15 to 25% [9, 10]. Such schemes are used by all the turbine factories in the USSR.

II. Methods of Increasing Speed of Response of Steam Turbine Control Systems

An increase in speed of response without a decrease in reliability can be obtained only with the condition that the use of new methods does not require the introduction of additional elements to the control system.

1. Separation of the velocity characteristic into static and dynamic [11]. In normal conditions of use the frequency of the ac line changes relatively little. There is a correspondingly small translation of the velocity controller's clutch. With a break in the load (a trouble condition), the controller's clutch shifts significantly. If, with a servomotor with a circulating valve, the valve axle bearing is made immovable, then the aperture profile beyond the limits of small displacements will not influence the form and curvature of the static characteristic. This ordinarily closed portion of the aperture can be made sufficiently wide, and then, for a break in the load, with a displacement of the valve beyond the limits of ordinary oscillations, a large aperture opens, and the degree of nonuniformity of control (under trouble conditions) is sharply decreased, which speeds the control process.

In the given case, it is necessary only to change the physical design of the control mechanism (in controllers made by the Brown-Bovary company, the control mechanism includes the valve axle bearing) and to change the

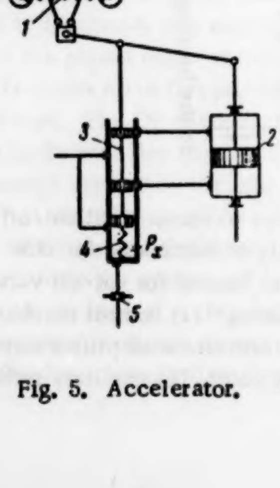

Fig. 5. Accelerator.

aperture profile, which does not decrease the system's reliability.

2. Use of accelerators [12]. Increases in the speed of response of control systems can be obtained by other methods as well. In particular, there is suggested a method of increasing the speed of a hydraulic servomotor which does not require the introduction of additional elements into the control system.

The lower cavity of valve 3 (Fig. 5) is made closed. To this cavity oil is supplied via an aperture controlled by this same valve 3. Draining of oil from the cavity occurs through constant throttle 5. The pressure P_x is determined by the magnitude of the valve displacement. For a slow motion of the controller's clutch, the deviation of the valve from the middle position is small, so that the action of the accelerator has a small effect on the deviation of the valve. For a rapid motion of the clutch, the deviation of the valve from the center position will be large, and there will be a corresponding increase of pressure P_x, which gives rise to an additional, significant displacement of the valve and a proportionate increase in the speed of motion of the servomotor's piston.

III. Simplest Scheme of Hydrodynamic Control [13]

As a result of the investigation carried out, there was developed the simplest scheme of hydrodynamic control (Fig. 4). Pump 7 supplies oil to the servomotors and, via filter 5 under valve 3, is spring-loaded. Valve 3 of the double throttle controls draining and supply of oil in the lines of servomotor 13. By means of control mechanism 12 one can change the oil pressure in the servomotor lines for an immovable valve 3. The principle of separation of velocity characteristics is used in the control system.

Such a control system was used on four turbines. Two of them had back pressure powers of 12.3 thousand kw. The control system has been in successful use for four years.

SUMMARY

The reliability criterion presented permits one to choose and perfect control systems and their elements.

The methods developed are widely used in the turbine factories of the USSR. In particular, while up until 1941 hydrodynamic control systems were unused by the turbine factories of the Soviet Union, today, the majority of the factories use this system, and almost all of them use the two-injector lubrication system.

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COMPLEX AUTOMATION OF AN OPEN-HEARTH FURNACE WITH GAS HEATING

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A system is described for the automatic control of an open-hearth furnace's thermal operating mode. The system consists of eight control loops connected via the plant. The questions of measuring several controlling parameters are considered.

The paper discusses the attempt to automate open-hearth furnace No. 14 of the Nizhne-Tagil'skii Metallurgical Combine.* The furnace has a capacity of 380 tons, and is fired by coke and blast-furnace gases. For the torch candling coal tar or anthracene oil are used, while oxygen is used for the intensification of the combustion process. There is today no boiler-reclaimer furnace.

This paper deals with the automation only of the furnace's thermal operating modes; stimuli to the physico-chemical processes occurring in the tank are not taken into account.

In the development of the automation scheme, the following basic idiosyncrasies of the open-hearth process were taken into account. In the automation of an open-hearth furnace it is impossible to select the combustion and thermal loads on the basis of some previously worked-out program, since the furnace properties continuously change during the course of the run; the scheme itself must choose the regimen which is optimal at the given moment. To shorten the duration of the smelting process (the blow) it is desirable to maximize the thermal load on the furnace. In the existing furnace designs this entails an increase in thermal efficiency. However, an increase in the thermal load is limited by the following factors:

1. The temperature of the regenerator's crown and upper checkered brickwork must not be higher than a limit due to the durability of the refractory material.
2. There must be provided complete combustion of the fuel in the working area.
3. Driving out of the flame from the working area must be limited, both by the magnitude of the heat losses, and by the durability of the front wall.

Measurements

For normal operation of a system for open-hearth furnace automation, it is necessary to measure, correctly and reliably, the aforementioned parameters on which limitations are imposed. Most important also are the questions of the dynamics of the measurement process. Use of the ordinary methods of measurement turned out to be inapplicable for furnace automation, except for the method for measuring the temperature of the regenerators' upper checkered brickwork by means of radiation pyrometers with electronic potentiometers. Use of these radiation pyrometers for measuring the crown temperature turned out to be impossible. Lengthy experience shows that it is impossible to provide reliable usage of this measurement in connection with the frequent slagging of the tuyere apertures, the formation of "icycles" in the pyrometer's field of vision, and in connection with the distortions due to tongues of flame reaching into the pyrometer's field of vision.

A significantly more reliable crown temperature measurement was successfully provided by the subcrown photopyrometer of the Central Laboratory for Automation of the Ministry of Construction of the RSFSR (Russian Soviet Federated Socialist Republic). The basic part of this instrument is an antimony-cesium vacuum photoelement installed in a water-cooled tuyere inside the furnace's working space directly under the crown. The photoelement's peep hole has a diameter of 0.9 mm and is continuously blown off by compressed air. The disturbances connected with obstructions of the peep hole, icicle formation and flame effects are virtually nonexistent here.

The measurement of pressure under the crown under conditions of actual use is implemented with sufficient reliability. The enclosed devices can operate without overhaul or cleaning for periods of several months.

In [1] doubts were expressed as to whether it were possible to consider the pressure under the crown as a

*The work on the complex automation of an open-hearth furnace was carried on jointly by the All-Union Research Institute for Metallurgical Heat Engineering (under the direction of V. N. Timofeev and G. I. Shirokov), the industrial-engineering enterprises of the Uralmetallurgavtomatik (under the direction of M. M. Gordon and B. G. Gutner) and by the Nizhne-Tagil'skii Metallurgical Combine (under the direction of G. A. Petrov and I. N. Mikhailov).

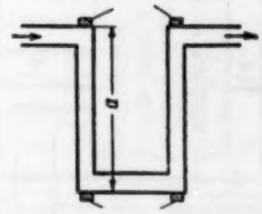
quantity characterizing the gas outflow and brick loosening action from the furnace's working space. Contradicting this, the attempt to employ automation on furnace No. 14 showed that this quantity quite accurately characterizes the pressure regimen in the working space and that, therefore, the scheme should not be complicated by the introduction of auxiliary pulses for controlling gas outflow and brick loosening.

Type RDM-35 compensation bell recorder-controller, heretofore used for controlling the pressure under the crown, turned out to be unfit. The controller's sensitive element must react to pressure changes with maximal speed. In addition, it must filter out the pulsations of pressure engendered by the irregularities of the combustion process. The period of the pressure pulsations connected with the irregularities of the combustion process is about 3 to 4 seconds. The impulse tube, together with the measuring instrument (the sensitive element) constitutes an oscillatory link. It is important to match the parameters of the impulse tube with those of the measuring instrument such that the period of the free oscillations of the sensitive element plus the impulse tube is about 6 to 7 seconds. This can be achieved by using a membrane sensitive element and a large-diameter impulse tube ($1\frac{1}{2}$ to 2 inches), for a length of line of some tens of meters. From the point of view of filtering out the pulsations, an installation of the instrument too close to the furnace is undesirable. A corresponding instrument with a membrane sensitive element was constructed at the Nizhne-Tagil'skii Metallurgical Combine. For the inspection of incompleteness of combustion, it is necessary to carry out a continuous analysis of the flue gases. The difficulty of this measurement is widely known. For an open-hearth furnace, the problem is solved in the following way. The intake points are positioned on the air regenerators, close to the damstone walls between the slag chamber and the regenerator, i.e., where the flue gases are sufficiently well mixed. The flue gas to be analyzed is drawn off by means of a steam ejector pump. The mixture of gas and steam is sprinkled with water, causing the steam to condense, and the moisture thus formed is removed by means of a two-stage centrifugal separator. The dried and cleaned sample, under a pressure of several hundred millimeters of water column, is applied to a type MGK-348 magnetic gas analyzer transducer, which analyzes the specimen for oxygen content. The air-cooled enclosed tuyere in the regenerator crown is periodically (for each dumping) blown off by steam, thus preventing it from clogging. The transport time for the gas specimen from the ejector pump to the gas analyzer transducer is about 15 to 17 seconds for an impulse line some 45 to 50 meters in length. The diameter of the impulse tube is one-half inch.

Two gas analyzers are installed on the furnace, for the right and left sides of the furnace separately. The moving system of the gas analyzers' secondary instruments are switched in only when the sample flue gases flow around the corresponding transducer. Switching in of the moving systems is implemented with a delay of two minutes after the beginning of dumping. Thanks to this, the gas analyzer does not record the transient combustion mode associated with the reversing of the furnace. In the control process, it is always that gas analyzer around whose transducer the flue gases flow that participates.

Another essentially new measuring instrument is a design of the Uralmetallurgavtomatik, a pulsed ultrasonic discharge meter for coal tar and anthracene oil. The instrument is not subject to clogging, since there is no place in it where the fluid would be stationary and where, therefore, there could accumulate sludge settling from the fluid. There are no moving parts in the instrument's transducer.

Crystal transducer



Crystal receiver

Fig. 1.

The instrument's basic element (its transducer, which is found in the coal tar conduit) is a U-shaped channel drilled in the metallic body (Fig. 1). On the front ends of the transducer's tube there are installed piezoelectric crystals, capable of transforming ultrasonic oscillations to electric waves and vice versa. Two of the crystals are transducers of the ultrasonic vibrations, and two of them are receivers.

A pulse sent by the crystal transducer arrives at the crystal receiver after the time $T = a/c$, where $\underline{a}$ is the distance between the crystals and $\underline{c}$ is the velocity of sound in the fluid. The ultrasonic pulse received is transformed to an electric voltage, amplified and applied to the crystal transducer, which sends the following ultrasonic pulse, etc. This method, as is known from the literature [2], provides temperature error compensation.

If the fluid is moving with velocity $\underline{v}$, then the intervals between pulses in the two channels of the transducer will be different. In one case, this interval will equal $T_1 = a/(c + v)$ and, in the other, $T_2 = a/(c-v)$.

Consequently, the pulse trains in the two transducer channels will have different frequencies. In the first case, this frequency will equal

$$f_1 = (c + v)/a;$$

in the second,

$$f_2 = (c-v)/a.$$

The difference of these frequencies

$$f_1-f_2 = f_d = 2v/a$$

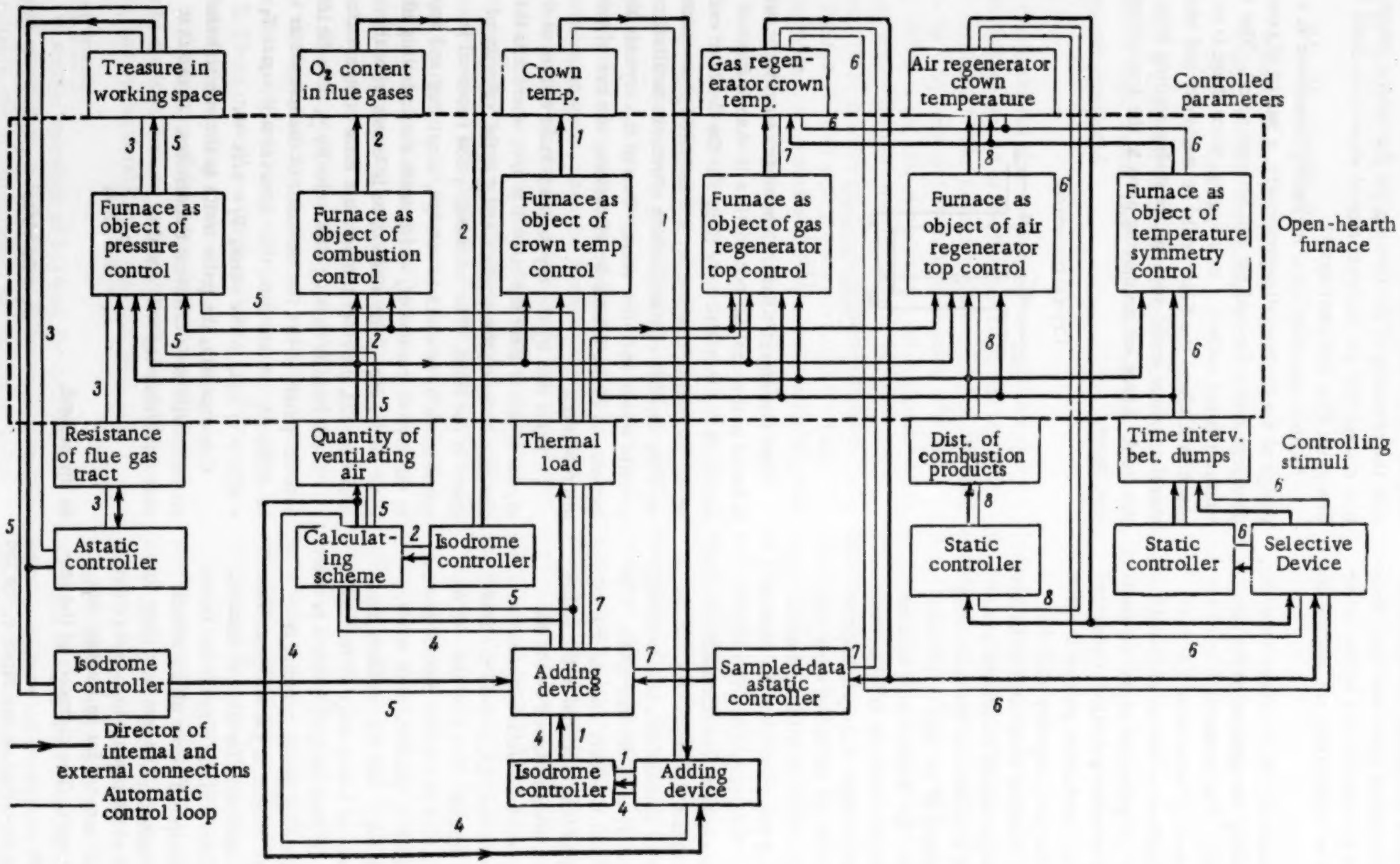


Fig. 2. Simplified block schematic of the open-hearth furnace's automatic control scheme. 1) Direction of internal and external connections, 2) automatic control loops.

depends only on the speed of the fluid and does not depend on the velocity of sound. With this, the result of the measurement does not depend on the temperature. Thus, the frequency f_d is a measure of the discharge rate.

The instrument's electronic scheme isolates the frequency f_d and transforms it to a dc voltage proportional to this frequency. The dc voltage is applied to the electric potentiometer, graduated in discharge units.

The transducer's U-shaped design permitted the channel diameter to be made sufficiently small, so that the instrument can measure small discharges of fluid. The range of the instrument's scale is zero to 1000 kg/hour. The fact that both transducer channels are drilled in one metallic body provides virtual equality of temperature in the channels even in the absence of discharge. This is important for the stability of the instrument's null.

Automatic Control

A simplified block schematic of the automatic control scheme for the open-hearth furnace's thermal regimen is shown in Fig. 2. The individual loops of this multiloop system are denoted on the figure by the numbers appended to the lines.

The control process proceeds in the following manner: in the filling period, when the temperature of the crown of the working space and the upper checkered brickwork of the regenerators is low, and there is no production of gases in the vat, pressure control in the working space is implemented by the rotation of the flue gas damper with a vertical axis, which controls the hydraulic jet of the astatic controller (control loop 3). The oxygen content on the checkered brickwork of the air regenerators at this time ordinarily does not fall below the given magnitude (6%), and the quantity of ventilating air is determined by the pneumatic computing scheme in correspondence with the quantity and caloricity of the fuel applied to the furnace. The computing scheme applies ventilating air with a coefficient of excess of $\alpha = 1.1$ before oxygen deduction. The thermal load is maximal at this time. The assigned value of oxygen concentration – 6 % – was determined empirically. In this case, on the gas checkered brickwork where there is virtually no outflow air, there is no CO in the flue gases.

If there is, in the charge, an uncovered oil waste, then the percentage of oxygen in the flue gases begins to decrease even during the course of charging. In this case, the gas analyzer, acting on a type IP-130 isodrome controller (control loop 2) corrects the operation of the computing scheme and adds air so that the assigned percentage of oxygen in the flue gases will be maintained. With this, the flue gas damper moves closer to an open state.

During the pig iron priming, the gas analyzer so increases the supply of air that the flue gas damper is opened completely. Then control of the furnace working space pressure enters into play by changing the thermal load, i.e., by changing the discharge of coke oven gas and, consequently, of air. This control is implemented by a type 04 isodrome controller (control loop 5). This controller acts on the adding device which controls the thermal load data transmitter, i.e., the discharge of coke oven gas. The discharge of blast-furnace gas remains constant during the entire smelting run, the resin is applied after draining of the slag from the furnace, and its discharge remains constant until the end of the smelting run. The discharge of oxygen is given by a steel mill hand.

With vigorous action in the vat, the amount of gas escaping from the boiling metal becomes so large, ordinarily, that the ventilator is completely filled, and the oxygen content in the combustion products remains low. In this case, when the computing scheme, as corrected by the gas analysis, requires more air than the ventilator can give (with account taken of the oxygen applied to the furnace), then operation of control loop 4 begins. The special differential manometer of this loop (not shown on the figure), which reacts only to drops greater than the corresponding full ventilator load, begins, by means of type IP-130 isodrome controller, to act on the adding device controlling the data transmitter of the coke gas controller. As a result, the thermal load is decreased. In this case, the discharge of ventilating air always remains maximal, and the discharge of coke oven gas is reduced to a very small amount or even to zero.

As the generation of gas from the vat decreases, the limiting actions concerned with incompleteness of combustion (loop 4) and working space pressure (loop 5) begin to weaken, and the thermal load is increased. However, there is a simultaneous increase in the crown temperature. The rising of this temperature leads to the formation of an input signal to loop 1, the action of which also limits the thermal load down to the end of the smelting run. This limitation is implemented by means of the adding device, common to loops 1 and 4, via a type IP-130 isodrome controller, the one which acts to limit the thermal load as a function of combustion incompleteness. In case of overheating of both gas checkered brickworks, control loop 7 can also limit the thermal load. This loop works on the principle of sampled-data control.

For stable operation of the entire system, it is necessary to match the speeds of action of the individual loops. The fastest system is that which controls the pressure in the working space by means of the flue gas damper. The time of a complete damper rotation is 10 seconds. Pressure control by means of thermal load variation is somewhat slower since there are several delays in the control loop from the pneumatic computing scheme. Still slower is the system for correcting air supply as the result of gas analysis; the time of the double isodrome controller, i.e., the time during which the executive mechanism duplicates its traverse due to astatic stimuli, is about one minute. Finally, the slowest control systems are those for thermal load as a function of crown temperature and incompleteness of combustion. The doubled time here is about 2 minutes.

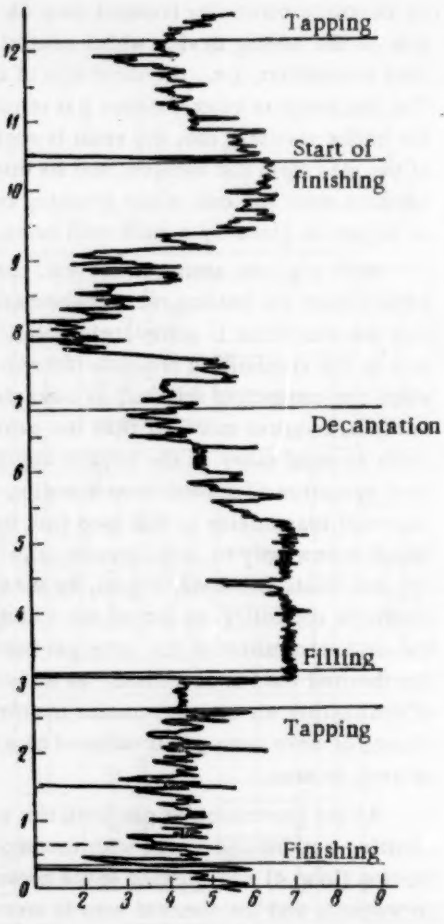


Fig. 3. Discharge of coke oven gas during a smelting run (scale is 0 − 10,000 m³/hour).

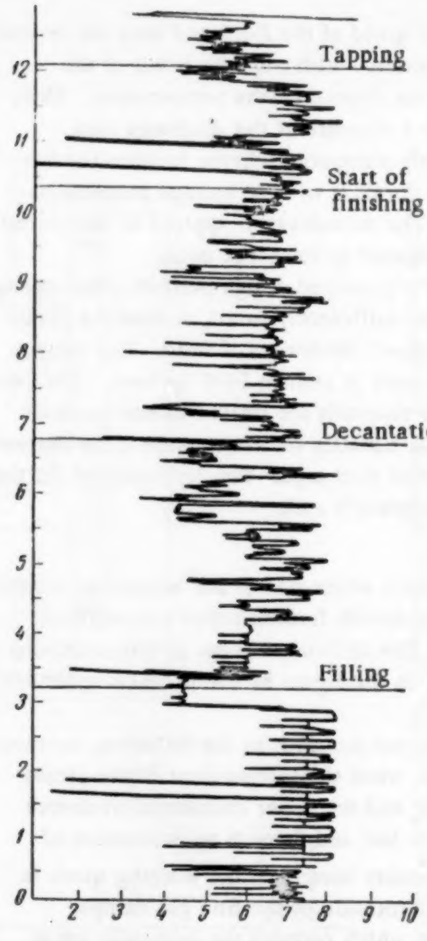


Fig. 4. Air discharge during a smelting run (scale is 0 − 50,000 m³/hour).

Only slightly related with the control loops just enumerated are the assemblies for controlling the upper temperatures of the air and gas checkered brickworks and for automatic dumping. These receive pulses from an integral time relay which operates with alternating pulses as functions of upper temperatures of the air and gas checkered brickworks. The alternation occurs each 20 seconds.

The integral time relay is a static controller of the temperature symmetry of the upper checkered brickworks of the regenerators (control loop 6). By means of temperature measurements of the top of the air checkered brickwork, control is effected by the damper's static distribution law. If the temperature approaches the assigned magnitude, than this damper begins to be throttled (control loop 8). Thanks to this, the quantity of flue gases passing through the air checkered brickworks is decreased, and their temperature begins to rise.

If the temperature of one of the gas checkered brickworks approaches the limiting magnitude, the command to dump is applied sooner than would be done by the integral relay, apart from the static controller. If the temperature of both gas checkered brickworks approaches the limiting value, there begins the discontinuous decrease of the thermal load by the action of control loop 7, as was discussed earlier.

Figure 3 is a diagram of coke oven gas expenditure during one smelting run. It is clear that, during the time of violent vat operation (from 7 : 30 to 9 : 00), the discharge of gas falls almost to zero, then increases somewhat and, in the last period of the run, during the finishing period, again decreases (on Fig. 3, from 10:00 to 12:00). The diagram of air expenditure has a completely different form (cf. Fig. 4). During the period of violent boiling, when the discharge of coke oven gas is small, the expenditure of air is very large, which is also provided by the combustion of the carbon monoxide released from the boiling metal at this time. With this, the flue gas temperature, during the entire smelting run, oscillates within virtually constant limits (Fig. 5). This oscillation is related to the reversal of the flame, and must not be eliminated by the system of automation.

It is not necessary to change the adjustment of the scheme during the furnace run. As the properties of the

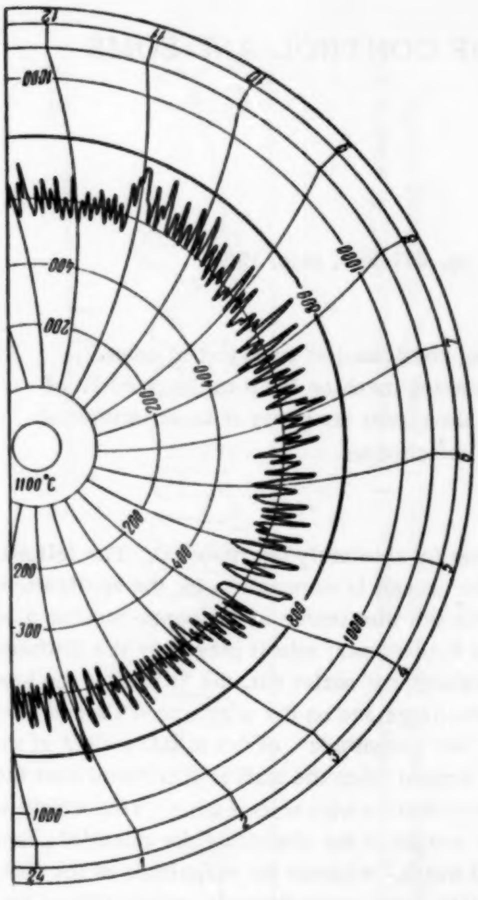


Fig. 5. Flue gas temperature in the course of a smelting run.

furnace worsen, there is an automatic change in the significance of the individual control loops. Thus, for example, as the draft worsens, there is a decrease in the operating time of the controller which controls the pressure rotating the damper and an increase in the operating time of the controller which limits the load as a function of the pressure in the furnace's working space.

Complete combustion of the carbon monoxide released from the vat, and the maintenance of the maximum thermal load admissible at each moment, provide a lowering of the specific expenditure of the nominal fuel with a simultaneous increase in productivity of the furnace. A comparison of smelting runs made with, and without, inclusion of the automation scheme showed that furnace productivity, thanks to automation, increased with a simultaneous lowering of the specific discharge of the nominal fuel.

It is also obvious that, with automatically controlled operating modes, the furnace's strength must be increased thanks to lesser temperature oscillations inside the refractory surface. However, the automated furnace is operating today only on its second operating period, so that it is impossible to make quantitative deductions as to the effect of the introduction of automation on length of service life.

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A PAPER-MAKING MACHINE AS AN OBJECT OF CONTROL AND SOME WAYS TOWARD ITS COMPLEX AUTOMATION

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The statistical and dynamic characteristics of a paper-making machine, considered as an object of control, are determined. It is shown that the complex automation of a paper-making machine leads to the problem of autonomous control. Possible ways to realize autonomy are cited, and the results are given of an experimental investigation of a system for automatically controlling the weight of 1 m^2 of paper.

Contemporary schemes for automating paper-making provide for control of its basic technological variables, but the interrelationships of these variables via the controlled object are not taken into account in the schemes put forth. Automation at such a level may be defined as partial, since it does not exclude the mutual influences of the controlled quantities in the object's dynamic regimen.

With the intensification of the production process which accompanies the increase of speed of paper sheet up to 500 to 600 m/min, partial automation cannot provide normal use of a paper-making machine. Under these conditions, effective control of the latter is only possible with complex automation, which must not only provide automatic control of all the basic portions of the process, but must also take into account the interrelationships of the corresponding controlled quantities.

The problem of complex automation requires a different approach, new in principle, to construction of automatic control systems for the productive flow. Such an approach assumes first of all, the development of a method for investigating and describing a paper-making machine as an object of control. This question, as is well known, has not yet been considered in the literature. The data obtained may be used for the exposition of the principles of constructing an automatable complex, and also for the development of the technical means for realizing it. The present paper is devoted to these questions.

1. Method of Investigation

Figure 1 shows a simplified schematic of the paper-making machine. The high-concentration stuff from the machine stuff chest is supplied to the supply tank. From there, the stuff is directed to the mixing pump in a suction pipe which also supplies recycled water. With the mixing, low-concentration stuff is formed, which is then supplied to the head box via the washing and screening equipment. At this step of the process, the fundamental role is played by the maintenance of the constancy of the concentration which determines the content of the paper sheet by absolutely dry material. The following step in the process is connected with the application of the stuff to the wire screen of the paper-making machine. Here, the fundamental role is played by the discharge of stuff through the outlet slit, the "slice" of the head box. On this discharge and on the wire screen's speed of motion, depends the magnitude of the initial weight of the layer which is formed when the stuff is distributed over the registering portion of the wire screen table. This weight is made up of the weight of the absolutely dry material plus the weight of water. Whereas the magnitude of the first of these weight components depends principally on the regimen of the preceding steps in the technological process, on all the following steps only removal of water occurs, and virtually no change in the content of absolutely dry material occurs. This underlines the possibility of using, to derive the differential equations of the object of control, the equations of its material balance with respect to each of the aforementioned weight components.

The values of the coefficient of the differential equations can be obtained analytically by means of the paper-making machine's static characteristics. Such characteristics must be given in the form of mathematical relationships, expressing the basic indicators of the regimen at the individual portions of the process as functions of the operative variable technological factors. For this in individual cases, for example, for the drying part, it turns out to be impossible to obtain the static characteristics in a form convenient for calculation. This latter circumstance imposes definite limitations on the analytic method of investigating a paper-making machine as an object of control, and requires that experimental methods be used.

2. Derivation of the Equations of the Object of Control's Dynamics

The material balance equations of the paper-making machine with respect to the absolutely dry material and with respect to moisture content have the forms, respectively,

$$G_d - G_s - G_w = 0, \qquad (1)$$

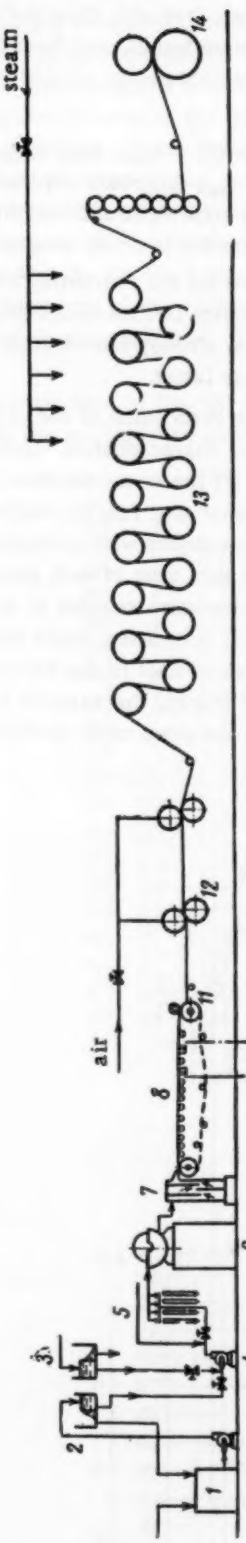

Fig. 1. 1) Machine stuff chest, 2) high-concentration stuff, 3) recycled water, 4) mixing pump, 5) low-concentration stuff, 6) washing and screening equipment, 7) head box, 8) wire screen table, 9) registering portion, 10) suction portion, 11) couch roll, 12) pressing portion, 13) drying portion, 14) calenders.

where G_d is the specific weight of discharge of the absolutely dry material when filled, G_s is the same, but in the mixing device with the paper stuff, G_w the same, but in the mixing device with recycled water;

$$G_0W_0 - q_r - q_{su} - q_p - q_D - G_0W_k = 0, \tag{2}$$

where G_0 is the initial weight of the layer formed when the stuff is distributed on the screen, in g/m², W_0 is the moisture content of the stuff applied to the screen, W_k is the moisture content of the finished paper, q_r, q_{su}, q_p, and q_D are the dessication speeds with 1 m² on, respectively, the registering part, the suction cabinet and the couch roll, the pressing and the drying portions.

The material balance equations, when expressed in terms of finite increments, take the form

$$\frac{dG}{dt} = C^*\Delta G_s + G_s\,\Delta C^* - C\Delta G_d - G_d\Delta C, \tag{3}$$

where G is the content of absolutely dry material in 1 m² of finished paper, C^* is the concentration of stuff in the machine stuff chest and C is the concentration of stuff in the head box;

$$\frac{dW_k}{dt} = W_0\Delta G_0 + G_0\Delta W_0 - \Delta q_r - \Delta q_{su} - \Delta q_p - \Delta q_D - W_k\Delta G_0 - G_0\Delta W_k. \tag{4}$$

The terms of Eqs. (3) and (4) can be expressed in terms of the static characteristics, obtained from an analysis of the physical features of the process, estimates of the interactions of the material fluxes, and mathematical processing of the regularities already known in the theory of paper production [1]. By carrying out the corresponding substitutions, and then going over to relative quantities, we obtain

$$\begin{aligned}
&(A_{111}s + A_{110})\,x_1 + (A_{131}s + A_{130})\,x_3 + A_{141}sx_4 = \\
&\quad = B_{110}\mu_1(t-\tau') + f_1(t-\tau'), \\
&-(A_{211}s + A_{210})\,x_1(t-\tau'') + (A_{222}s^2 + A_{221}s + A_{220}) \\
&\quad x_2 - A_{230}x_3(t-\tau'') + \\
&\quad + A_{240}x_4(t-\tau'') + A_{250}x_5' = \\
&\quad = B_{220}\mu_2(t-\tau_1) + f_2(t-\tau_1), \\
&\quad - A_{310}x_1 + (A_{331}s + A_{330})\,x_3 = \\
&= \mu_3(t-\tau'), \quad A_{440}x_4 = \mu_4, \quad A_{551}sx_5 = \mu_5;
\end{aligned} \tag{5}$$

where x_1, x_2, x_3, x_4, x_5 are the relative values of the controlled quantities of the weight of 1 m², the moisture content of the paper, the discharge of stuff from the head box when filled, speed of the wire screen's motion and pressing pressure, respectively; μ_1, μ_2, μ_3, μ_4, μ_5 are the relative values of the controlling actions applied, respectively, by the stuff valve on the suction side of the mixing pump, the valve on the steam trunks to the drying portion, the valve on the supply trunk taking stuff from

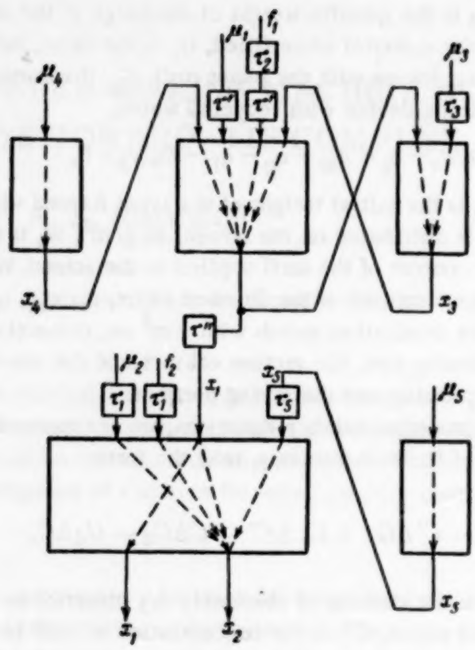

Fig. 2.

the mixing pump to the head box, the controlling organs in the electric drive system and in the system of press roller clamps; f_1 and f_2 are the relative values of the disturbing actions related, respectively, to changes in the parameters of the stuff and the steam; τ', τ'', τ_1 and τ_2 are the "pure" lags, respectively, from the mixing pump to the "slice" of the head box, from the slice to the calender machine, in the steam trunks, and from the machine stuff chest to the mixing pump; $s = d/dt$ is the differentiation operator.

The coefficients of Eqs. (5) – A_{111}, A_{110}, A_{131}, A_{130}, A_{141}, A_{310}, A_{331}, A_{330}, A_{440}, A_{550}, B_{110} – are expressed in simple ways in terms of the known parameters of the technological process. There has been no success in obtaining analogous expressions for the remaining coefficients, since, for this, one must use regularities of the processes in the drying portion. As has already been stated, it is difficult to make use of these latter.

Data for the exhaustive description of the object of control gives its temporal characteristics, obtained experimentally. In taking off the characteristics cited, it should be taken into account that Eqs. (5) define the paper-making machine as an object with interrelated controlled quantities. With this, part of such quantities (the weight of 1 m² and the moisture content of the paper) constitute the object's output quantities, while the other part (concentration and level of stuff in the head box, speed of motion of the wire screen, the vacuum in the suction devices, pressure of the press roller clamps, etc.)

Form of channel	Structure of channel	Sign of controlling action	Object's transfer function
Principal internal channel	$\mu_1 \to x_1$	$-\mu_1$	$\frac{k}{(s+a)} e^{-(\tau'+\tau'')s}$
	$\mu_2 \to x_2$	$+\mu_2$	$-\frac{k_1}{(s+a_1)(s+b_1)} e^{-\tau_1 s}$
	$\mu_2 \to x_2$	$-\mu_2$	$-\frac{k_2}{(s+a_2)(s+b_2)} e^{-\tau_1 s}$
One-sided internal channel	$\mu_4 \to x_4 \to x_1$	$-\mu_4$	$-ke^{-\tau'' s}$;
	$\mu_4 \to x_4 \to x_2$	$-\mu_4$	$-\frac{k}{(s+a)(s+b)} e^{-\tau'' s}$
	$\mu_4 \to x_4 \to x_1$	$+\mu_4$	$-\frac{k_1 s}{(s+a)(s+b)} - k_2 e^{-\tau'' s}$
	$\mu_4 \to x_4 \to x_2$	$+\mu_4$	$-\frac{k_1 s}{(s+a_1)(s+b_1)} - \frac{k_2}{(s+a_2)(s+b_2)} e^{-\tau'' s}$
	$\mu_3 \to x_3 \to x_2$	$+\mu_3$	$\frac{k}{(s+a)(s+b)} e^{-\tau'' s}$
Two-sided (cross-over) internal channel	$\mu_1 \to x_1 \to x_2$	$+\mu_1$	$\frac{k}{(s+a)(s+b)} e^{-(\tau'+\tau'')s}$
	$\mu_2 \to x_2 \to x_1$	$+\mu_2$	$\frac{k_1}{(s+a_1)(s+b_1)} e^{-\tau_1 s}$
	$\mu_2 \to x_2 \to x_1$	$-\mu_2$	$\frac{k_2}{(s+a_2)(s+b_2)} e^{-\tau_1 s}$
	$\mu_3 \to x_3 \to x_1$	$+\mu_3$	$\frac{ks}{(s+a)(s+b)} e^{-\tau'' s}$

relate to the individual portions of the process and, for the object as a whole, can be considered as input stimuli. The structural schematic of the object of control is shown on Fig. 2. In correspondence with such a scheme, the temporal characteristics (run curves) were obtained by applying disturbances in the form of unit steps from each of the controlling organs. The object's reaction was determined as its output by a simultaneous measurement of the weight of 1 m^2 and the moisture content of the finished paper. Some of the characteristics obtained are shown on Fig. 3. On the basis of the results obtained, the transfer functions of the various channels of the object of control were established. These data are given in the table.

A comparison of Eqs. (5) with the tabulated data attests to the coincidence of the qualitative features of the process obtained by a theoretical analysis and those established experimentally. This is also attested to by the quantitative estimates of object dynamics from Eqs. (5). Figs. 3a, c and d show, together with the experimental run curves, the temporal characteristics obtained by calculation.*

3. Determination of Object's Dynamic Characteristics from Data on Normal Use

The methods considered for the experimental investigation of the object of control are inapplicable to high-speed paper-making machines. In this case, the applica-

*The characteristics were obtained for paper-making machine No. 3 of the Krasnyi Kursant (Red Student) Plant.

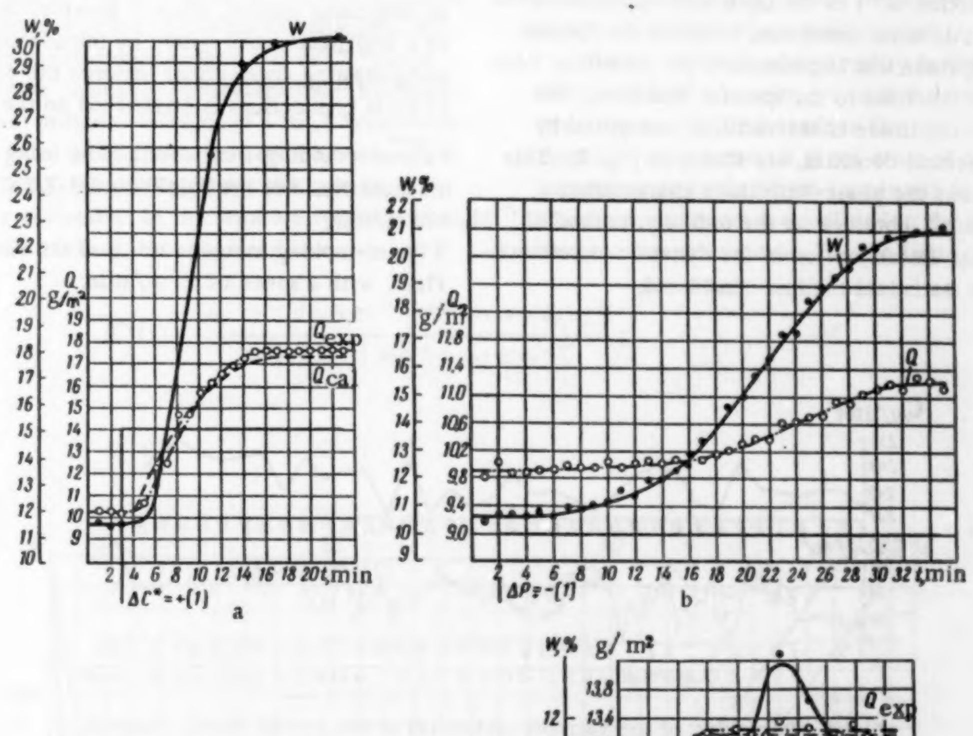

Fig. 3. Curves of a run of the object of control. Q is the weight of 1 m^2 of paper, W is the paper's moisture content, ΔC^* is the unit step of concentration, ΔP is the unit step of drying steam pressure, ΔH is the unit step of level in the head box, ΔV is the unit step of velocity of wire screen motion.

tion of disturbing stimuli to the object's input leads to a significant disruption of the process. Moreover, random uncontrolled disturbances, whose effect is tremendously difficult to eliminate, essentially limit the accuracy of the measurements of object reaction.

In view of this, the possibilities of determining the dynamic characteristics of a paper-making machine by the statistical processing of its normal use data were studied. In correspondence with the theory of the method expounded in [2, 3], we obtained simultaneous recordings of the variable process quantities by the channel "concentration of stuff in the machine stuff chest – weight of 1 m^2 of finished paper" (Fig. 4).† In this case, the paper-making machine is considered as a linear object with one input and one output. After processing the curves obtained on a mechanical correlator, we obtained the autocorrelation function $R(\tau)$ of the input signal (Fig. 5a) and the cross-correlation function $C(\tau)$ of the input and output processes (Fig. 5b). On this same correlator, based on the Fourier transformation, there was implemented the transition from the correlation functions to the spectral densities. The object's phase-amplitude characteristics, computed by dividing the spectral densities, are shown on Fig. 6. This same figure shows the phase-amplitude characteristics of another object,‡ obtained by the ordinary method of an experimental determination of the dynamic characteristics and by the statistical method considered.

4. Certain Principles of Complex Automation of Paper-Making Machines

Results of the investigation of the dynamic properties define the paper-making machine as an object of control with interrelated controlled quantities. For the automatic control of such objects, one should create the conditions for which it is possible to control each controlled quantity independently of the others. It is known that such a problem is solvable on the basis of the principle of autonomous control. The realization of this principle is possible by two routes:

1) dissection of a multiconnected control system into a series of dynamically independent systems;

2) provision of high control quality of the individual quantities by constructing optimal processes in the separate systems.

The first of these methods is known as the method of connected control, since it provides dynamic dissection of a multiconnected system by introducing artificial compensating connections between the controllers. Design of these connections is carried out on the basis of a dia-

† These recordings were obtained by using paper-making machine No. 5 of the Balakhninskii TsBK at a speed of 400 m/min.

‡ Paper-making machine No. 3 of the Krasnyi Kursant Plant, with a speed of 50 m/min.

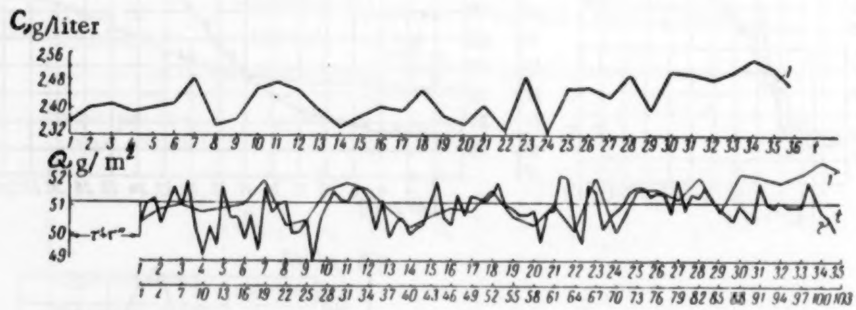

Fig. 4. Recording of the variable quantities of the process in the channel "concentration in machine stuff chest – weight of 1 m^2 of paper": 1 is concentration, 2 is weight of 1 m^2 of paper.

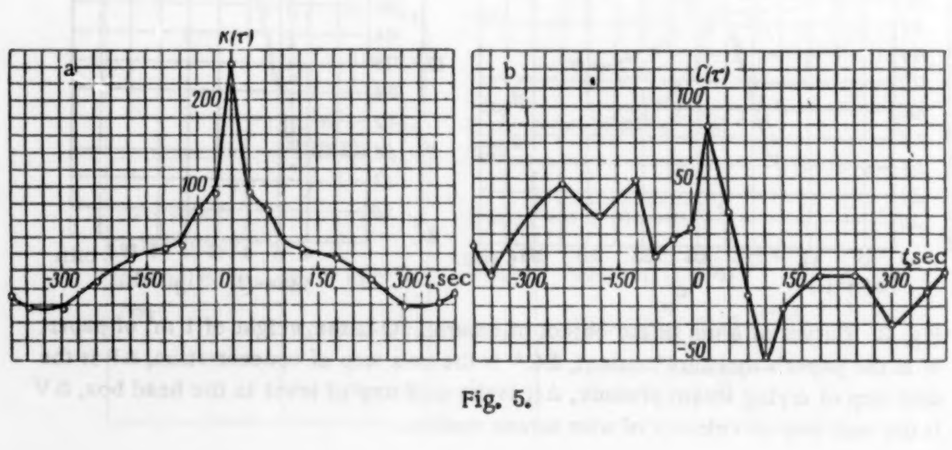

Fig. 5.

gonalization of the basic matrix of the transfer functions of the closed-loop control systems of the interrelated quantities [4, 5].

For the compensating connections for the automation of a paper-making machine, the following relationships were obtained:

$$D_{12}(s) = -\frac{M_{11}(s)}{M_{12}(s)} D_{22}(s), \quad (6)$$

$$D_{21}(s) = -\frac{M_{22}(s)}{M_{21}(s)} D_{11}(s), \quad (7)$$

$$D_{13}(s) = \frac{M_{13}(s)\, M_{22}(s) - M_{12}(s)\, M_{32}(s)}{M_{11}(s)\, M_{22}(s) - M_{12}(s)\, M_{21}(s)} D_{33}(s), \quad (8)$$

$$D_{23}(s) = \frac{M_{11}(s)\, M_{23}(s) - M_{21}(s)\, M_{13}(s)}{M_{11}(s)\, M_{22}(s) - M_{12}(s)\, M_{21}(s)} D_{33}(s), \quad (9)$$

where $M_{11}(s)$, $M_{22}(s)$, $M_{12}(s)$, $M_{21}(s)$, $M_{13}(s)$ and $M_{23}(s)$ are the transfer functions of the object for the channels, respectively: "concentration – weight of 1 m^2," "steam pressure – moisture content," "concentration – moisture content," "steam pressure – weight of 1 m^2," "concentration – level in head box," "level in head box – moisture content"; $D_{11}(s)$, $D_{22}(s)$ and $D_{33}(s)$ are the transfer functions of the controllers in the separate systems – weight of 1 m^2, moisture content, level in head box.

The second of these methods cited is based on the unbounded increase in the gains of the separate systems with the condition that their stability be retained [6]. Although this method provides absolute autonomy of the processes in certain ideal cases, its use is possible, since a large part of the separate systems of the complex automated paper-making machine can possess partial autonomy.

In the study of the dynamic properties of the object of control, it was established that, of all the technological connections of the controlled quantities, the most powerful connections were those in which the weight of 1 m^2 of paper participated. It was on the basis of this that there arose the possibility of solving the problem under consideration by constructing an optimal process only in the system for the automatic control of the weight of 1 m^2 of paper sheet. The difficulties which ensue with this in view of the object's high inertia, and the significant delays in passing signals through it, are overcome in the case of sampled-data (discontinuous) control.

As the criterion of optimality in the control of the weight of 1 m^2 of paper, we took the minimum of the dynamic errors. This leads to the requirement that the control system possess a given order of astatism and have a transient response which terminates in minimal time. These requirements are realized in the system whose discrete transfer function has the form

$$K^*(z) = \frac{b_0 z^{-(l-l_1)} + b_1 z^{-(l-l_1+1)} + \cdots + b_{l_1} z^{-l_1}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_l z^{-l}}; \quad l_1 \leqslant l, \quad z = e^{sT}, \quad (10)$$

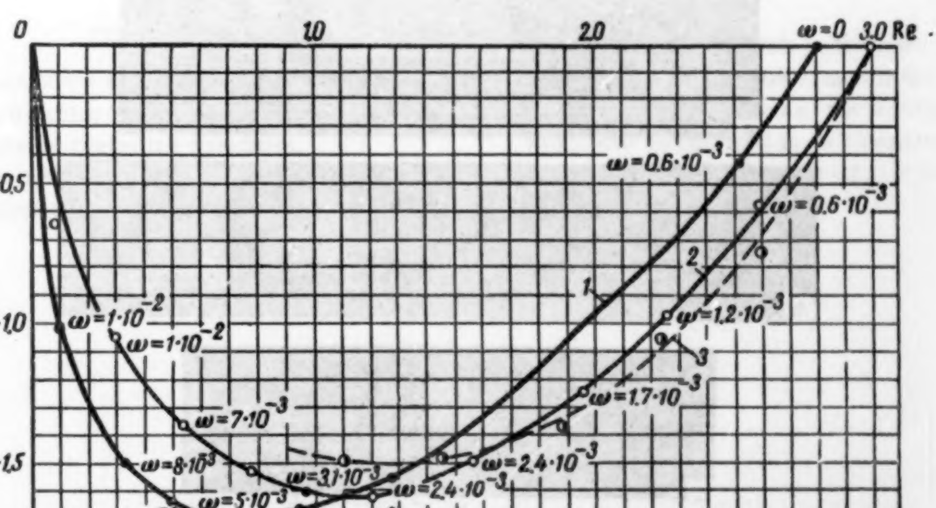

Fig. 6. Phase-amplitude characteristics of the object of control for the channel "concentration in machine stuff chest – weight of 1 m^2 of paper" (without discounting lags): 1 is from the correlation functions for paper-making machine No. 5 of the Balakhninskii TsBK, 2 is from the run curves for machine No. 3 of the Krasnyi Kursant Plant, 3 is from the correlation functions for the same machine.

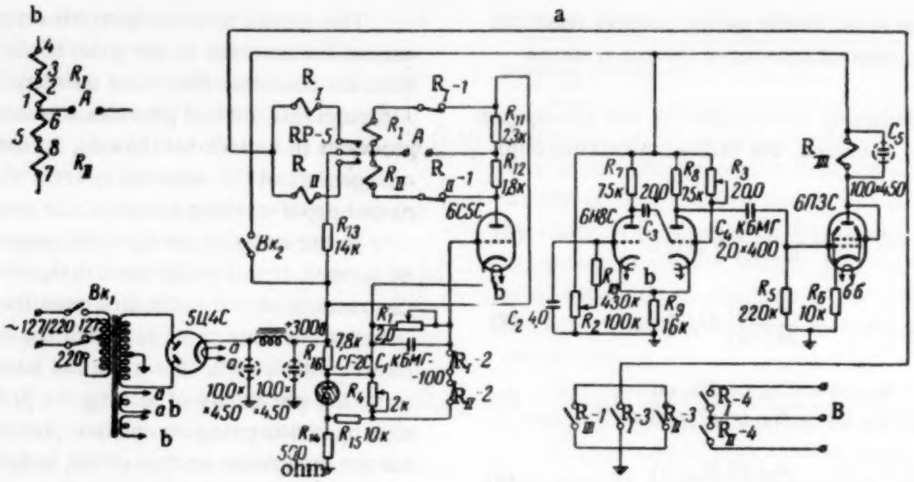

Fig. 7. a) Scheme of the sampled-data element. A is the controlling signal, B is the correction pulse, R_I, R_{II} and R_{III} are type RPN (U1720213) relays; b) scheme for connecting the windings of relay RP-5 (U1722025).

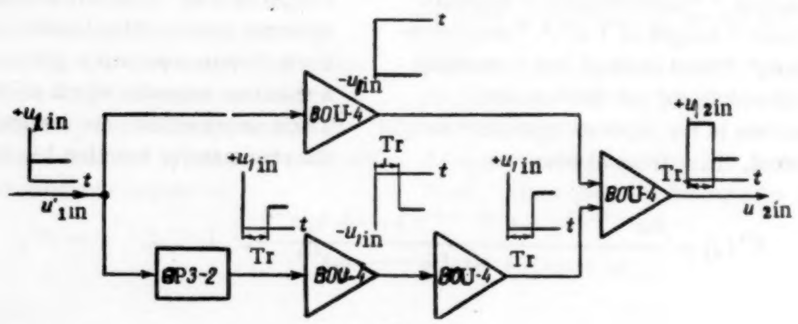

Fig. 8.

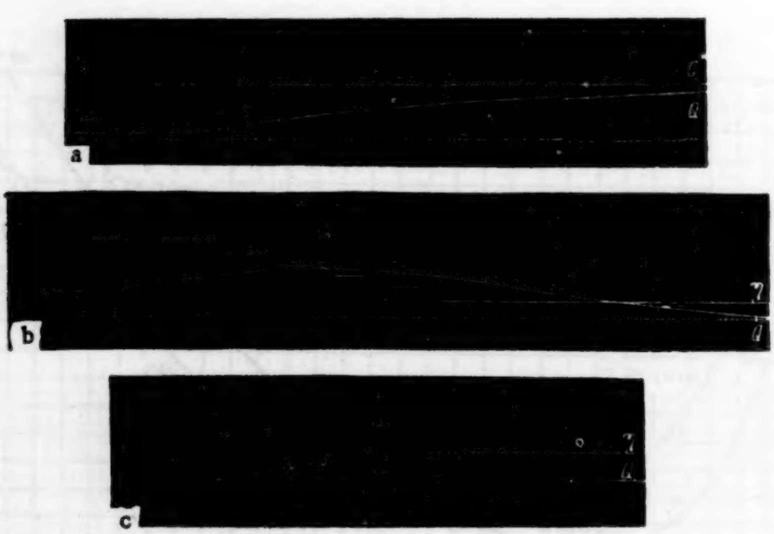

Fig. 9.

if the following conditions** hold:

$$a_0 + a_1 + a_2 + \cdots + a_l = b_0 + b_1 + b_2 + \cdots + b_{l_1}$$
$$b_0 + b_1 + b_2 + \cdots + b_{l_1} = 1 \qquad (11)$$

The practical realization of conditions (11), as was shown in work [7], requires the introduction of a correcting circuit containing a system of sampled-data filters or a digital computing device.

In conclusion, we consider one of the variants of the solution proposed for the problem.

5. Optimal System for the Automatic Control of the Weight of 1 m² of Paper Sheet

In the system for the automatic control of the weight of 1 m² of paper, there was used a discrete controller. This controller contains a sampled-data element providing width modulation of the sequence of error signals [8]. The schematic of such a sampled-data element is shown in Fig. 7. The correction circuit, connected in series, was assembled from blocks of analog computer MPT-9 and constant lag block BPZ-2 (Fig. 8). The MPT-9 simulator also provided the realization of a model of the object (the channel "concentration of stuff – weight of 1 m² of finished paper"). The results of an experiment performed on the model thus obtained for the control system are shown on Fig. 9, where one can see the object's run curve (a), the control process in the system without the correction circuit (b), and the process in the corrected system (c). As is clear, the speed of response and the accuracy are close to their limiting values,†† which allows us to consider the automatic control system under consideration for controlling the weight of 1 m² of paper as an optimal system.

SUMMARY

1. The data of the theoretical analysis described a paper-making machine as an object of control with several interrelated controlled quantities.

2. To determine the dynamic properties of such an object, it is required to use the theoretical method proposed, in conjunction with the well-known methods of experimental investigation.

3. Consideration of a paper-making machine as a complex to be automated is related to the solution of the autonomous control problem. In a first approximation, this problem can be solved by the optimization of the process in the system for the automatic control of the weight of 1 m² of paper.

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** For controlling signals in the form of unit steps.

†† We have in mind the values of the maximum deviation (y_{max}) and the duration of the transient response (t_{tra}) which can be obtained in an automatic control system of the type considered with infinite gain.

‡‡ See English translation.

AUTOMATION AND CENTRALIZATION OF A COPPER MINE

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Moscow

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 867-876, June, 1960

The paper describes the set of measures adopted for the automation and centralization of the technological processes executed in the pits of one of the large mines in the USSR with subterranean extraction of the ore, the Degtyarskii Copper Mine.

The Degtyarskii Copper Mine is the oldest mining enterprise, comprised of several underground pits, thoroughly reconstructed during the years of the first Five-Year Plans. The copper ore of the Degtyarskii mine is processed on the nearby large concentration plants and copper smelters.

The set of measures adopted, and the experimental design and research work implemented, and to be implemented, at this mine aroused the justified interest of many other mining enterprises. The experience here was sucesssfully extended to analogous shafts and mines of nonferrous and ferrous metallurgy.

Employment was found at the mine for the devices of a developed modern centralization of the pit, a signalling, centralization and blocking system (SCB) with high-frequency communications of the transport dispatcher with the electric locomotive engineers, devices for high-frequency and low-frequency (conductor) production communications between the general works dispatcher and the work divisions (the so-called system of loudspeaker announcements, calls and conversations), devices for the centralized control of electric locomotives at the sites of loading and unloading ore. Also automated were the skip hoist machines, a device for hoisting empty rock dumps, control for the shaft top cager mechanism, transfer of trucks in the cages, compressors, furnace, ventilation, pumping stands, and many other shaft devices.

Devices were developed for the automatic weighing of trucks with ore in transit, inspection of the state of the steel cables of the hoisting machines, various systems of remote control devices, while sharply decreasing the requirements on the cable shops.

The mine pits were equipped with television in the devices for centralization and control of truck hoist machines, instruments using radioactive isotopes for the inspection of ore level in ore chutes and bins, temperature inspection devices using semiconductors, etc.

Parallel with the solution of the questions of automation and centralization of the mine, there was executed significant work on the mechanization and dissemination of new technological processes of extracting and transporting ore.

Pit Centralization

A modern mining enterprise is equipped with complicated machines, mechanisms and devices, the majority of which are included in a unitary technological process – the extraction and transportation of minerals – so that on their concordant operation depends the production of the entire mine as a whole. It is in connection with this that centralized control of pit mines has particular importance today.

With the introduction of the centralized system, the dispatcher of the Degtyarskii Copper Mine pits obtained the capability of continuously inspecting:

1) number of ore-laden skips,

2) number of trucks unloaded on dumpers, by level;

3) ore level in bins, by level

4) operation of the machines for transportation crushing conversion,

5) quantity of rocks taken from the pits,

6) number of trucks let down into the pits from the scaffolding,

7) operation of the cage hoists,

8) expenditure of compressed air and water, by level,

9) magnitude of insulation impedance in the 380 volt lines,

10) pressure of compressed air and water in one of the most typical levels,

11) lowering of pit top-cager temperature below +2°C,

12) operation of the automated water-draining equipment.

At the dispatcher's disposal there are various means of control and, in particular, a widely ramified communication system with all the productive divisions.

One of the basic devices of centralized communication is the announcement call and conversation system, implementable by means of high-frequency apparatus using the trolley lines of the electric locomotives' contact network. The given communications system is intended for powerful loudspeaker announcements at the various production divisions as to preparations for use of explosives, possible accidents, and also for inquiry by the dispatcher

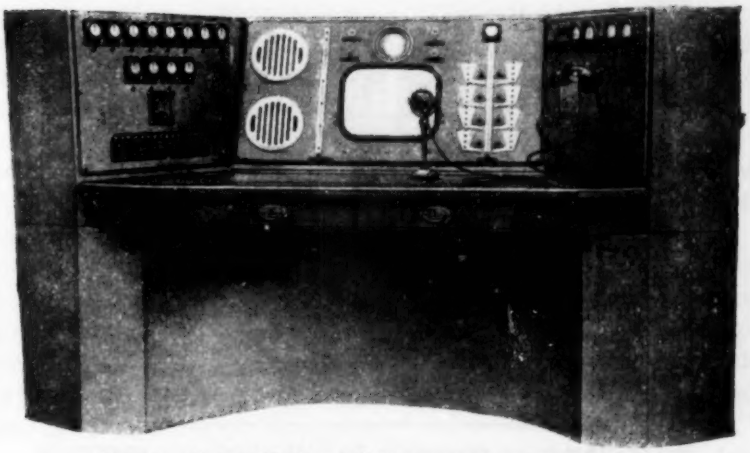

Fig. 1. Over-all view of the central control desk.

as to individual workers located in the production divisions. The apparatus allows the dispatcher to be called from any division where he may be and for conversations to be held with him.

The loudspeakers are used for announcements or for calling workers to the apparatus. After the call, the conversation with the dispatcher is conducted by the usual microtelephone, and the loudspeakers are automatically switched off.

With workers serving in the hoist cages, the dispatcher is connected by means of semiconductor apparatus of the production loudspeaker communications system.

For observation of the mine yards and the receiving areas there is provided a multicamera industrial television setup with five transmitting cameras. By means of a special commutator, the dispatcher can switch any of the five cameras to his viewing screen.

At the objects of observation there are established the transmitting cameras which permit the implementation of remote aiming, optimal focusing, diaphragming and change of objects.

Figure 1 shows an over-all view of the central control desk of the pit "Kapital'naya 2". The central control desk is located in a special room on one of the pit levels. On the desk's left-hand panel there are counters for the numbers of trucks unloaded in bins, by level, quantity of rock extracted, etc., and the equipment for the dispatcher's high-frequency announcement, call and conversation communications system.

On the front panel there is marked a mnemonic scheme of the hoist cages, controls for inspecting bin loading, by level, and instruments for inspecting the number of loaded skips. On the same panel is the closed-circuit television viewing screen.

On the desk's right-hand panel there are instruments for inspecting the magnitude of insulation of the low-voltage 380 volt lines, for inspecting pressure and expenditure of water and compressed air. Here is installed a four-party telephone intercom device, by means of which the dispatcher is connected with the works' switchboard and also with the works' superintendent.

A special cabinet was installed for accomodating the dispatcher's television system receiver, the electric power supply, the relay circuits and the recording instruments.

All the information at the dispatcher's desk is transmitted by means of a contactless remote control system.

Two types of remote control and signalling systems were used.

1. A contactless cyclical teletransmission system based on magnetic elements with rectangular hysteresis loops and on filamentless thyratrons, with time separation of the channels. This system was developed by the Institute of Automation and Remote Control of the Academy of Sciences of the USSR.

The system permits a virtually unlimited increase in the number of channels, requires no special power supplies, can be used not only for inspection but also for controlling mechanisms. The system is sufficiently fast-acting.

2. A system of ferrite-transistor cells, also with time separation of channels, based on circuit elements developed by the Central Research Institute for Complex Automation.

Part of the over-all centralization system is the centralization of underground transportation. The transportation dispatcher is near the pit dispatcher's room and, with his own desk and by means of an SCB system, can inspect movements of the rolling stock, by mining operation, can change signal arms by remote control, can establish routes and can inspect the positions of signal arms and the states of signal lights.

High-Frequency Communications with Electric Locomotives

The transportation dispatcher has at his disposal high-frequency (HF) communications with the moving electric locomotives. These communications are implemented by the electric locomotive contact network.

The introduction of dispatcher communication with the electric locomotive engineers immediately improved the transportation operations, significantly increased the safety of motion and use of the rolling stock. High-fre-

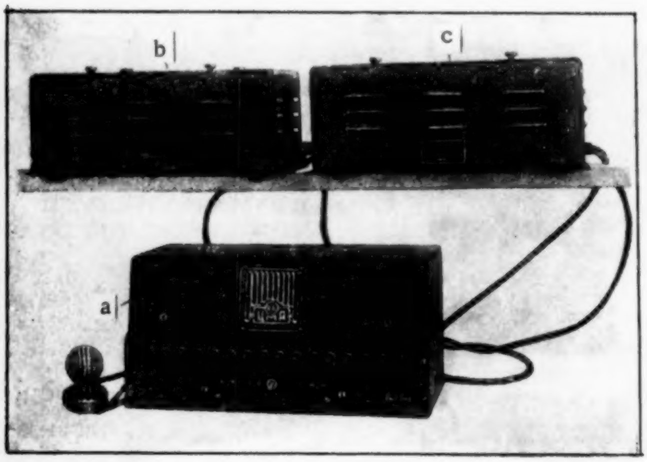

Fig. 2. Dispatcher equipment for high-frequency communication with the electric locomotives. a is the calling block, b is the receiver-transmitter, c is the power supply.

quency communication of the transportation dispatcher with the moving electric locomotives is firmly entrenched in the pit regimen. Electric locomotives without high-frequency communications are considered dangerous, and cannot be placed on the line.

HF communications with the electric locomotives consists of the dispatcher's and the "subscriber's" assemblies. The dispatcher's assembly is a receiver-transmitter HF set with selective calling of the out-stations. The assembly (Fig. 2) consists of three blocks: the calling block a, the receiver-transmitter b and the power supply c.

The calling block, with the microphone, is installed at the dispatcher's desk. All the controls are placed on the calling block's front panel. Here are located a loudspeaker, 15 calling buttons, the stop button, a button for switching in the local telephone network and the lamp signals. To call any out-station, the dispatcher pushes the corresponding call button; this switches in the receiver-transmitter and the audio-frequency call generator. Depending on the ordinal number of the button pressed, a voltage of one of 15 fixed calling frequencies is generated in the line, modulating the transmitter's carrier frequency. After the signal denoting that the called out-station has switched in, the sending of the call is automatically stopped, and the communication line is set up for the ensuing conversation.

Obtaining a call signal, the out-station worker converses with the dispatcher without himself doing any switching. After termination of the conversation, the dispatcher presses the "stop" button, thus switching his transmitter, his assembly and that of the out-station to the "waiting" mode.

The out-station assembly (Fig. 3) consists of a receiver-transmitter with a built-in loudspeaker, a power supply and a microphone with a signal lamp and a call button. The signal lamp is lit when the dispatcher's transmitter is switched in, and shows that the dispatcher is busy. The given communications system is connected into the local telephone network, and the dispatcher can authorize conversations from any electric locomotive with subscribers of the local telephone network.

Remote Control by Electric Locomotives at the Ore Loading and Unloading Sites

An urgent problem was the introduction of a system of remote control by the mine's electric locomotives at the sites of ore loading and unloading. Indeed, the transfer of the functions of chute drawer and tipper to the electric locomotive engineers permitted an increase in the productivity of work, a shortening of the time to load an ore truck, an increase in the effective capacity of the switching yard and the freeing of a large number of workers who had been occupied at the chutes and dumpers.

The system of remote control from the electric locomotives makes it possible for the electric locomotive's engineer to carry out all the operations for loading and unloading directly at the chutes and dumpers and to control the movement of the rolling stock from a stationary control desk.

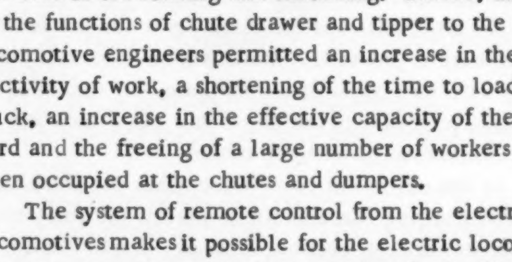

Fig. 3. Out-station assembly of the high-frequency electric locomotive communications system. a is the receiver-transmitter, b is the power supply, c is the microphone.

For the implementation of remote control by the electric locomotives, the corresponding portions of the trolleys are insulated from the remainder of the contact network and form the zone of remote control. The voltage on the contact conductors of this zone are applied via the contactor of the command block. In the power circuit on the electric locomotive are connected the contactors of the receiving device's reverser. One of the contactors, "forward", with ordinary control, is mechanically held in the switched-in position, maintaining the normal supply scheme of the electric locomotive's motor. In the remote control mode, the mechanical holding of the electric locomotive with the contactor is removed.

Upon the command "forward", the full voltage of the contact network is applied to the insulated trolley segment, providing inclusion of the "forward" contactor in the electric locomotive's receiving device.

Upon the command "backward", the voltage on the insulated trolley segment is momentarily applied (during 0.2 seconds) through a current-limiting resistor, which is thereafter shunted, and the insulated trolley segment is immediately connected into the contact network. This provides inclusion of the "backward" contactor in the electric locomotive's receiving device, and a change in the direction of motion.

The command "stop" switches the insulated segment of the trolley out of the contact network.

The scheme of remote control from the electric locomotives provides:

a) the capability of the electric locomotive's engineer to switch, from his cab, the electric locomotive onto remote control, which is implemented from a stationary control desk;

b) remote control of the electric locomotive's motion "forward" and "backward";

c) automatic cessation of electric locomotive remote control upon transmission of the command "stop";

d) stopping of the electric locomotive if it accidentally leaves the controllable segment;

e) voice and light signalling as to the motion of the remotely controlled electric locomotive;

f) automatic repetition of the transmitted command upon accidental momentary separation of the pantograph from the trolley conductor;

g) uninterrupted motion in the loading or unloading zone of a normally controlled electric locomotive;

h) impossibility of passing control signals to neighboring portions.

The following devices enter into the set of apparatus for remote control from electric locomotives at the loci of ore loading and unloading:

Control apparatus installed in the hauling gear right at the chutes or dumpers, and serving for the application of the commands "forward", "backward" and "stop" by means of a special lever, inserted by the engineer in the apparatus frame.

Command blocks, installed at the beginning of the cross-over or at the chutes, and serving for the transmission of the commands obtained from the control apparatus to the electric locomotives' receiving blocks, and for switching in the power voltage to the insulated trolley sections.

Receiving blocks, installed in the electric locomotives and intended for the connecting of its motor in accordance with the received commands, and also for stopping the electric locomotive if it accidentally leaves the remote control zone.

Auxiliary electromechanical braking devices, installed on the electric locomotive motors, and serving for the automatic braking of the electric locomotives upon receipt of the command "stop" and also for accidental departures of the electric locomotives from the remote control zone. The braking device consists of a brake disk on the motor shaft, a braking tape and a braking magnet.

The remote control scheme can also embody other means of automatic braking: electrohydraulic or electropneumatic.

Automation of Shaft Ventilation Doors

To improve ventilation of the mining works, and also to increase the capacity of the haulage lines in the mine, the shaft ventilation doors were automated.

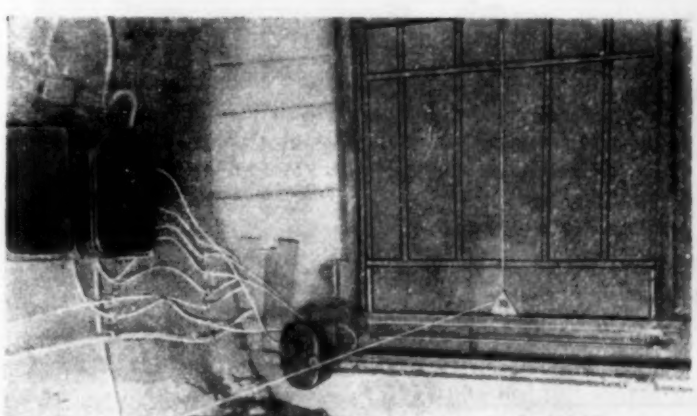

Fig. 4. Over-all view of the shaft's automatic ventilation door.

The automation of the process of opening and closing the doors permitted the freeing of a significant number of workers who had been occupied in the maintenance of the ventilation doors.

In addition to the economy realized by freeing the service personnel, the use of automatic shaft doors significantly improved the operating conditions of underground transportation and provided safe operation in their servicing. The ventilation doors were provided with electric drives, as being the most reliable, economical and most readily amenable to automation. The over-all view of a door is given in Fig. 4.

The ventilation door automation scheme provides for automatic opening of the door as an electric locomotive or a truck approaches, and the automatic closing of the door after passage of all the rolling stock, signalling of door position to the dispatcher's SCB desk, elimination of collisions of electric locomotives with closed doors by taking voltage off the trolley segments on either side of the door, the impossibility of opening the door when rolling stock approaches from both sides simultaneously, switching of a yellow signal light to red upon approach of rolling stock in the contrary direction, acoustical announcement to people of the contrary rolling stock as to the closed door, impossibility of closing the door while rolling stock is situated in it.

Automatic Weighing of Ore Trucks in Transit

For the inspection of the amount of rock extracted, equipment for the automatic weighing of trucks in transit was developed. With this, the weighing of trucks does not require uncoupling of the rolling stock or any other operations connected with the use of manual labor. Such a weighing scheme significantly increases the productivity of the electric locomotives, the turnover of the truck yards, and frees the majority of the service personnel. Moreover, it eliminates the possibility of erroneous recording.

To automate the process of weighing ore trucks in transit, use was made of the method of weighing by means of measuring the bending of a rail from the action of the truck's weight on it. Foil tension impedances were used as the transducers by means of which the rail bending is measured.

The equipment for weighing an ore truck in transit consists of three basic assemblies: a) a weighing portion with tension impedance weight transducers; b) path transducers; c) recording instrument.

For weighing, the weighing portion is cut out from the rail path. The weight transducer with foil tension impedances is welded to the rails. Switching in of the electrical scheme is implemented by pedal-type path transducers, installed before and after the weighing segment.

The recording instrument (Fig. 5) is an electronic self-recording bridge with a built-in mechanism for adding the weights of all the ore passing over the segment, or for weighing the empty and loaded trucks separately, and a device for automatically setting the measuring scheme to zero before the weighing of each set of rolling stock.

The weighing block schematic is shown in Fig. 6. The weight transducers are welded to the ends and bases of the measurement rails and operate on expansion and compression. The tension impedances installed in these transducers are connected in a bridge circuit. The use of all four arms of the bridge for measurement increases the sensitivity of the measuring scheme and provides temperature compensation for it. In the diagonals of the measuring scheme two slide wires are connected: the first one operational, the second one designed for automatic zero adjustment before the weighing of each set of rolling stock.

The equipment for the automatic weighing of pit trucks in transit provides: a) weighing of trucks with an accuracy of ± 2.5% when moving with speeds up to

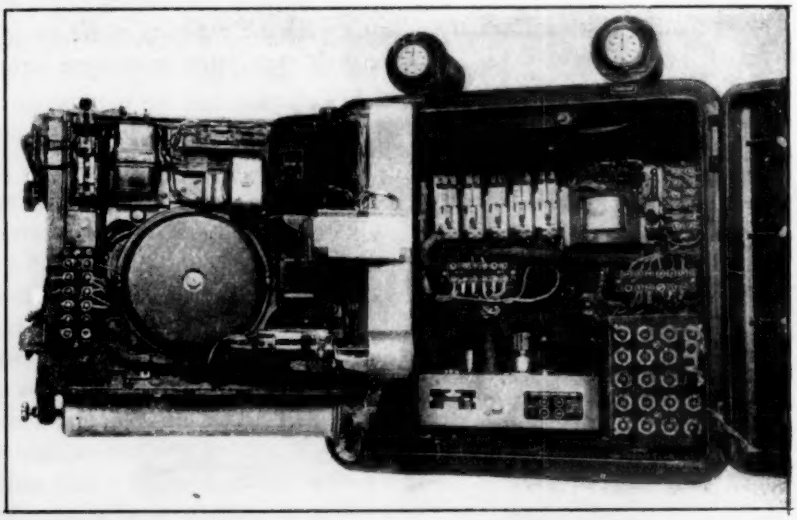

Fig. 5. Recording instrument for automatic truck weighing (with opened hood).

5 km/hour; b) automatic elimination from the total weight of a truck of an average tare weight; c) weighing of empty and loaded trucks separately; d) automatic summation of the weight of all ore passing over the weighing segment; e) recording on a strip diagram of the weight of each truck individually; f) weighing of the rolling stock without including the weight of the electric locomotive.

The given equipment was designed for heavy-duty operation in mine conditions.

Automatic and Remote Control of the Mechanisms of Shaft Top Cagers and Truck Exchange at the Loading Platforms

Control of the shaft top cager mechanisms at all the levels is exercised by one person, the so-called "cager", since complex automation and remote control of the shaft top cager mechanisms was carried out at the mine. With this, all the top cager mechanisms – hammers, detents, grids – are outfitted with pneumatic drives and are controlled either remotely, by the engineer of the hoist machine, or from a stationary desk on the level, or automatically. Control of operation of these mechanisms, is executed by the cager who, as needed, moves from one level to another, and carries out all the work for the receipt and output of loads.

As means of control, the cager is given a new system of shaft top cager signalling with loudspeaker communications and with high-frequency connections with the moving cages. All the technological processes from loading the trucks with rock to unloading them at the receiving platforms have been automated. For this end, the whole complex of mechanisms was developed and modernized.

An over-all view of the equipment is shown in Fig. 7. The operation of the equipment occurs in the following fashion. A truck with rock is pushed from the cage by means of a pneumatic plunger, after which it is clamped by the plunger and fed to the dumper. In the dumper it is automatically stopped and then unloaded. After halting of the dumper, the stopper is removed and the truck is pushed by a pneumatic plunger and moves along a slope to the top cager again.

At the mine, the following measures have also been taken:

Complete automation of the skip hoists, operating by a motor-generator system. The starting impulse for the skip hoist is given from the measuring hopper, after which all operation of the hoist proceeds automatically.

Automation of the principle water draining apparatus, which provides reliability of pump operation, and frees the service personnel.

On the surface, there has been complex automation of the compressor stands, the furnace stands, the rock dumps. The principal ventilation equipment has been subjected to remote control.

The automated mine equipment is supplied with built-in temperature protection of electric motors from winding-overheating and of mechanisms from bearing-overheating.

For the temperature protection of motors and mechanism bearings, use was made of thermal transducers in the form of semiconducting thermal impedances (thermistors), satisfactorily solving the problem of protection of unattended aggregates from overheating.

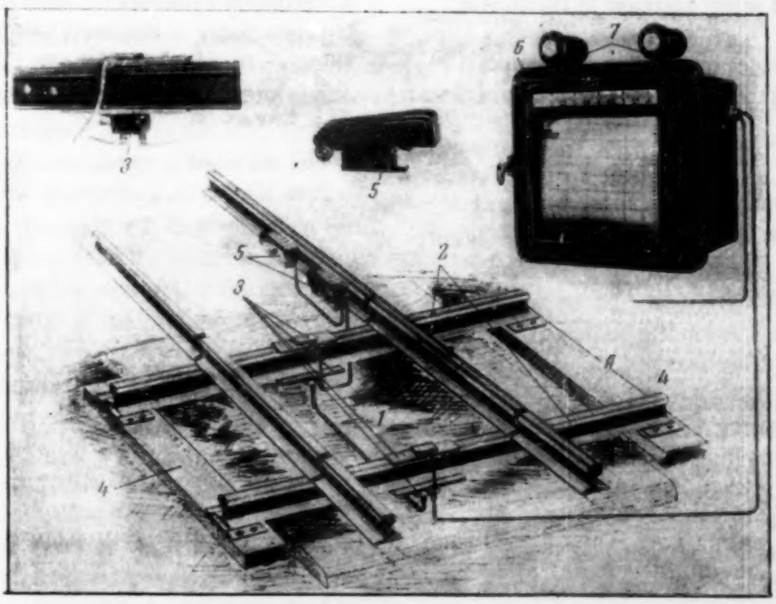

Fig. 6. Block schematic of automatic truck weighing in transit: 1) weighing section of track rail; 2) measuring rails; 3) weight transducer; 4) concrete foundations; 5) path transducers; 6) recording instrument; 7) counter for adding weights of all ore passing over segment.

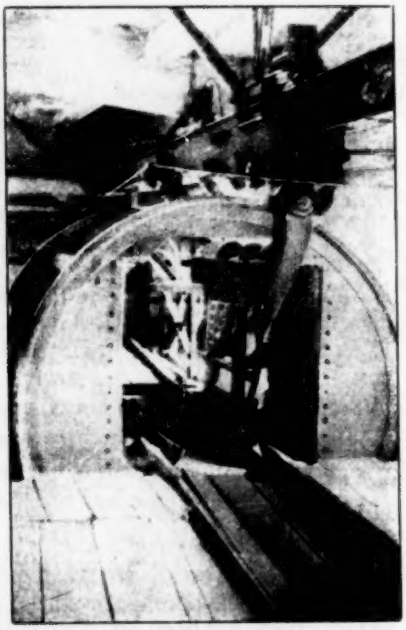

Fig. 7. Over-all view of the automated equipment for exchanging trucks and loading at the dumper.

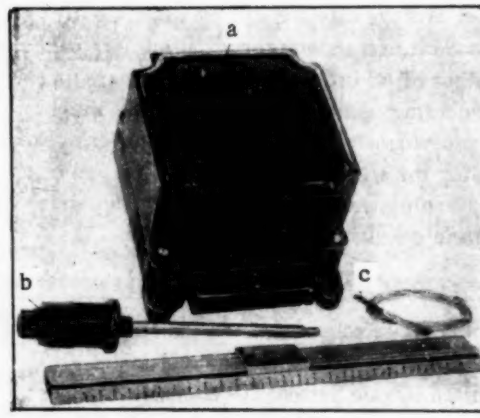

Fig. 8. Apparatus for temperature protection of motors. a is the relay block, b is the transducer for motor protection, c is the transducer for bearing protection.

The action of the apparatus is based on the capability of certain types of thermistors to sharply, by a factor of one hundred, decrease their resistance when the ambient temperature rises above a definite limit. This limit can be regulated by the magnitude of the voltage applied to the thermistors. Miniature thermistors, properly encased (Fig. 8), are inserted in the windings of electric motors, or in mechanism bearings, and a relay winding is connected in series with the thermistor. The relay operates for a sharp decrease in thermistor resistance.

The thermal transducer for bearing protection is a metallic sleeve, screwed into the body of the bearing, with a built-in thermistor, while the thermal transducer for insertion in electric motor windings is executed in the form of a flexible plastic tube with a thermistor fixed in its end.

The relay has a switch setting which allows the temperature of relay operation to be established between the limits of 80 to 110°C in steps of 10°C.

The totality of all the work in this direction permitted a significant reduction in service personnel, an increase in productivity of the mechanisms and in the general productivity of labor, and also the guarantee of a high degree of safety of work in the mine.

USE OF ELECTRONIC COMPUTERS FOR BESSEMER PROCESS AUTOMATION

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Kiev

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1. Introduction

Automatic inspection and control of the Bessemer process makes it possible to obtain steel of a given carbon content with a significantly smaller number and duration of blows.

Implementation of automatic inspection and control of steel was possible after a great deal of experimental work, carried out by the Dneprodzerzhinskii Evening Metallurgical Institute and by the Dzerzhinskii works on the analysis of existing nonautomatic methods of inspecting Bessemer smelting processes and on the development of new methods, amenable to automation on the basis of the employment of high-speed electronic digital computers.

The complexity in the development of inspection methods useful for automation results from the following factors.

The speed at which the process flows virtually eliminates the possibility of carrying out chemical analysis of the metal during the course of the smelting, analysis which would give the most valuable and accurate data for establishing the time for tipping the converter with a given carbon content. At the same time, the same speed of process flow poses very narrow limits within which the converter's tipping (pouring) time can deviate.

Photoelectric transducers not equipped with special cooling devices are positioned, due to the high flame temperatures, at a significant distance from the converter, which is the cause of the appearance of a great deal of noise which distorts the transducer indications. To this is conjoined the great nonuniformity of flame luminosity at different points, and at different times at the same point, which also lowers the value of the information obtained from photoelectric transducers.

The enumerated factors make it difficult to obtain the original experimental data for determining the time of converter pouring. The difficulties in principle are consequences of the circumstance that very many physical and chemical factors influence the speed at which the process flows and, consequently, the time of converter pouring, factors such as, for example, the initial content of carbon and silicon in the pig iron, the number of smelts on a given bottom, the velocity of air blowing, the temperature regimen of the smelter and other factors. To take the influence of the foregoing factors into consideration is difficult for the following two reasons.

1. The physical and chemical processes which specify the dependence of the time of pouring on the factors mentioned have not been completely studied, and we have this dependence only in the form of experimental (very numerous, to be sure) tables and graphs, containing a quite significant number of errors.

2. Taking the aforementioned factors into account can only be done at the time of, or immediately before, the beginning of the smelting run itself, since only at this time are the necessary initial data at hand, for example, information as to the consumption of air, the serial number of the smelting run, etc.

Thus, all the difficulties mentioned can be generalized as follows: For an accurate determination of the time for ending a blow, it is required to process and take into account very rapidly a large quantity of data, each component of which individually can be obtained only with significant random errors.

2. Posing of the Problem. Information Sources

The problem is thus posed: It is required to develop a system which permits cessation of blowing of rail steel with carbon content within the limits of 0.48 to 0.58%.

If we take into account the speed of carbon burn-out, equal to 0.007 to 0.008% per second, we find that the moment of converter tilting must be predicted with an accuracy of ± 5 seconds.

As the result of a theoretical and practical investigation of many methods of inspecting the Bessemer process and determining the moment τ_t of tilting the converter, methods suggested both here and abroad, we adopted the following methods, which are maximally satisfactory due to their relatively high reliability and practicality.

1. Determination of τ_t from the amount of air blown through the converter. This method is based on the fact that the quantity of carbon "burned-out" of the pig iron depends directly on the quantity of air blown through the converter. Thus, τ_t can be predicted from the initial carbon content of the pig iron and the integral expenditure of air through the converter. The reliability of this method is lowered because not all the oxygen in the air enters into reaction with the pig iron's carbon, and part of it is lost with the exhaust gases.

2. Determination of τ_t from a W-diagram. Empirical investigations showed that, if one establishes transducers

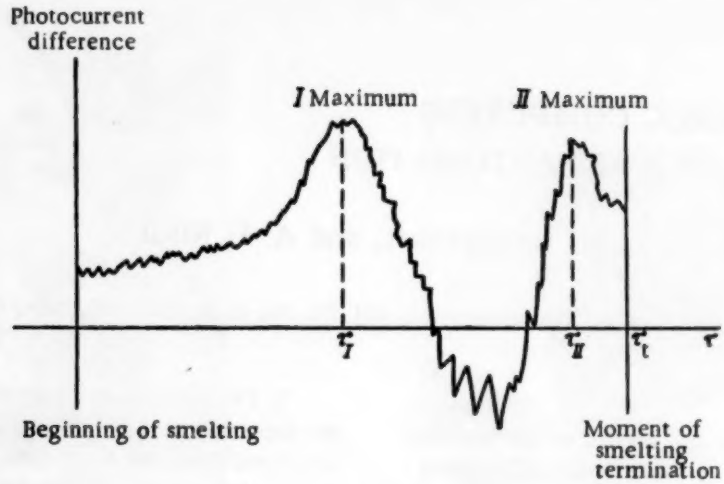

Fig. 1.

which record the converter flame's illumination in two different nonintersecting spectral regions (for example, infrared and yellow), then the difference of currents of these transducers from one smelting operation to another gives a characteristic curve recalling an inverted letter W (Fig. 1). With this, the termination of the smelting operation coincides in time with the dropping off of the curve from maximum II, and can be computed, to a good approximation, from the formulas:

$$\tau_t = \tau_{II} + 50 \text{ sec or } \tau_t = \tau_I + \tau'.$$

Here, τ_I and τ_{II} are the moments at which the maxima appear on the W-diagram, and τ' is some time (8 to 10 minutes) which depends on the temperature of the metal. Physically, the method is based on the circumstance that the time until maximum I, the so-called "blow period" has a large dispersion and is not subject to any accurate estimate, while the processes in the converter after the appearance of maximum I, coinciding with the beginning of intensive combustion of CO, are more determinate, and the time of smelting termination can be predicted.

The reliability of the method is lowered because of the strong noise superimposed on the photoelectric transducer signals.

3. Determination of τ_t from the metal's temperature. The temperature of the metal directly influences the speed at which the chemical processes in the converter proceed, and taking it into account gives both direct and indirect data as to the time of smelting termination.

4. Determination of τ_t from the transparency of the flame. It has been empirically established that, a short time before smelting termination, a powerful formation of powder begins on the surface of the metal, thanks to which the flame's transparency drops sharply, after which, with the cessation of the dust formation, it again increases. The moment of minimum transparency is related to the moment of smelting termination, and gives still another "reference point" for determining the latter. The reliability of the method also drops because of the strong noise superimposed on the signal for determination of the flame's transparency.

5. Determination of τ_t on the basis of statistical data for a large number of smelting runs. With this, one takes into account the large amount of attendant factors such as, for example, ordinal number of the lining, original manganese content of the pig iron, as well as original content of silicon, carbon, etc.

3. Method of Processing Information

Prediction of the moment τ_t of smelting termination on the basis of the information obtained from all the previously described sources is implemented by using the methods of mathematical statistics. There is today no exact analytic description of the process.

The reliability of each of the aforementioned methods, while all are of the same order of magnitude, still differs in amount from one to the next. With this there arise two possibilites of using the information obtained from the transducers: a) choose the most reliable method and compute τ_t on its basis; b) attempt to take into account the information given by all the aforementioned methods.

We found it advantageous to take the second path. With this, we start from the natural assumption that the errors of physically independent methods are so distributed that they partially compensate one another, i.e., that the mean square error in determining τ_t as the weighted average of the times determined by each of the individual methods is less than the mean square error in predicting τ_t on the basis of one of the methods, even the most reliable.

Thus, it is proposed to compute τ_t by the formula

$$\tau_t = \frac{\sum n_i^2 \tau_i}{\sum n_i^2},$$

where the τ_i are the times computed on the basis of each of the individual methods and the n_i are the weights, which depend on the reliability of each method.

With this, the reliability of the method is understood in two senses. First, as the general reliability, inversely

proportional to the mean square deviation of the values of τ_t determined by this method from the true value of τ_t. Second, as the particular reliability, depending on the signal-to-noise ratio, and varying from one smelting opera tion to the next. The fact of the matter is that, as shown by a preliminary investigation by means of self-recorders, the "quality" curves (signal-to-noise ratio) obtained from signals of one and the same transducer changed for different reasons, where the presence of heightened noise is clear even at the beginning of a smelting run, and may be discounted by a corresponding change of the weight coefficients n_i.

It is clear that a huge increase in the number of methods for determining τ_t, while not giving a significant decrease in the mean square error, significantly increases the complexity of the apparatus, and is thus inefficient.

4. Stages of the Work

With all that has been stated taken into account, the work on automating the Bessemer process was divided into two stages.

The first stage was the development of an information (data-gathering) device and its establishment at the Dzerzhinskii Works and its use in the mill conditions with automatic attachment to the transducers and with automatic starting and stopping.

The second stage was the development of a controlling digital machine.

It was proposed that the following ensue from the use of the information device.

1. A definitive determination of the basic parameters of the controlling machine (memory size, speed of action, type of input device).

2. Determination of the coefficients for the statistical relationships, to be subsequently introduced into a passive memory of the controlling machine.

3. Determination of the weights n_i characterizing the reliability of the individual methods of determining τ_t.

4. Definitive determination of the methodology of smoothing the transducer signals so as to isolate the useful information.

5. Verification of the reliability of the automatic operation of the electronic digital computer in factory conditions, and specification of the design pecularities of such a machine (heat protection, screening, etc.).

5. Digital Recording Device

In accordance with the stated plan of work, a digital recording device (DRD) was developed at the Computing Center of the AN USSR.

We now consider its scheme of operation.

During converter blowing, the DRD takes in, and records, signals from four photoelectric transducers and an air consumption meter.

Recording of the photoelectric transducer signals is implemented serially, with the recording of the signal from one transducer taking 0.1 seconds. Thus,receipt of a signal from each photoelectric transducer occurs once in each 0.4 seconds. The signals from each photoelectric transducer are transformed to a digital code in the memory of the DRD. Provision is made in the DRD for the primordial processing of the data obtained as the result of transformation of the transducer signals.

If one considers a transducer signal as the realization of a nonstationary random process consisting of the superposition of a determinate process $F(\tau)$ and a random stationary process $\Phi(\tau)$, then the processing carried out in the DRD corresponds to the smoothing of such a process by the method of moving averages. It amounts to this, that what is printed out is not each value obtained as the result of transforming the transducer signal, but the arithmetric average of the values over some interval (the averaging interval).

This method is the most suitable in the given case since the carrier curves $F(\tau)$ assume the most diverse forms from smelting run to smelting run, and their approximation by means of some analytic expression or another is very troublesome. The possibility is not excluded that, as a result of processing the data obtained from use of the DRD, there will be recommended other methods of smoothing random nonstationary processes.

As a result of preliminary investigations, it was found, that, for the existing amplitudes and correlation function of $\Phi(\tau)$, the optimal value of the averaging interval lies between 4 and 12 seconds (optimal in the sense of obtaining the least mean square error in determining the maximum of $F(\tau)$). In accordance with this, the capability of changing the interval of averaging within these limits was provided in the DRD. The definitive determination of the optimal size of the interval of averaging is one of the basic problems which must be solved as a result of using the DRD.

The signals from the air expenditure meter are taken in by the DRD as they are sent and transformed in the machine's memory to a code corresponding to the numerical value of the quantity of air blown through the converter from the beginning of the smelting run up to the given moment.

Printing of these values is carried out simultaneously with the printing of the averaged values of the parameters obtained from the photoelectric transducers.

The operation of the DRD is strictly cyclic. With the value of the averaging interval $\bar{\tau}$ established prior to the beginning of blowing, each printing (not counting the values of the total expenditure of air) corresponds to the averaged value of some parameter, assigned to the midpoint of this interval.

Thus, to determine the time τ to which some printed parameter value appertains, we can use the formula

$$\tau = (n-1)\bar{\tau} + \frac{\bar{\tau}}{2} = \left(n - \frac{1}{2}\right)\bar{\tau},$$

where $\underline{n}$ is the ordinal number of the print-out from the beginning of the smelting run.

With this, the absolute error in the determination of τ does not exceed ± 0.2 seconds for a duration of the smelting run not greater than 20 minutes.

In the lacunae between blowings (for several seconds until converter charging), the DRD records the signals from the individual photoelectric transducer, * averages the digital values of the parameter thus obtained during one averaging interval, and prints this averaged value. The averaging interval chosen for DRD operation during blowing can differ from the interval chosen for DRD operation in the periods between blows.

The magnitudes of the input signals from the photoelectric transducers are transformed in the DRD to binary code with a relative error of ± 0.5% of the maximum value of the transformed signals. This corresponds to obtaining codes with seven correct, and one doubtful, bits.

Printing of the averaged parameter values of the DRD is executed in the decimal system, with two correct decimal places and a five or a zero in the least significant place, which corresponds to the same relative error of ± 0.5%.

The capability has been built into the DRD of introduction of signal scale factors, the aim being to obtain numerical output values in the ordinarily accepted units of brightness or temperature.

The indication of quantity of blown air is printed in the form of a four-digit number, where the maximum value of the most significant digit is 3. This corresponds to a relative error not exceeding ± 0.1% of the upper limit of the scale, whereby the expenditure transducer can be so constructed that the results are obtained in normal cubic meters.

Conditions of DRD use. The DRD is supplied from a three-phase ac line of 380/220 volts. Preliminary stabilization of the line voltage is provided by means of ferroresonant stabilizers.

The power drawn from the line by the DRD does not exceed 1.5 kw. The DRD is designed to operate with forced internal ventilation. The DRD's ventilation stream must have the following parameters: rate of expenditure is 900 cubic meters per hour, temperature is 10 to 15°C, pressure is 60 mm of water. With internal ventilation which meets these specifications, the DRD can operate reliably when situated in the normal temperature regimen of Bessemer operation.

The DRD is not hermetically sealed against dust.

After completion of the complex adjustment of the DRD in conjunction with the transducers, it is intended that it operate normally, with switching in and out of devices connected to the converter mechanism, without attendance by the DRD maintenance personnel during a shift. Between shifts, cleaning and checking of the DRD by engineers will be necessary.

Physical design and basic elements of the DRD. Physically, the DRD is a metal cabinet measuring 1.8 × × 1.4 × 0.5 meters, comprised of the electronic part of the DRD and the power supply. The cabinet is attached to cables with metallic pedestals measuring 0.1 × 0.5 × × 0.5 meters for a numeric printing device.

The control console is placed on the cabinet's front panel.

For the basic standard elements in the DRD we used flip-flops, amplifiers and cathode followers, developed at the Computing Center of the AN USSR on the basis of the analogous elements used in the high-speed electronic computer "Kiev". These elements provide high reliability of DRD operation. These standard elements were built as metal plug-in units, measuring 150 × 90 × 35 mm.

Standardization and rapid replacement of the blocks provided for easy use and repair of the DRD.

Type RP, RSM, and RS electromagnetic relays were used for switching transducer channels and for controlling the numeric printing device.

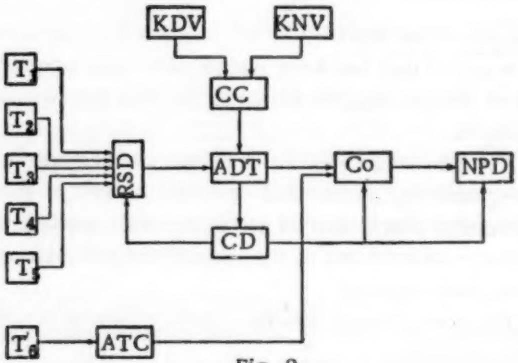

Fig. 2.

The block schematic of the DRD is shown in Fig. 2. Signals from the five photoelectric transducers T_1-T_5 are applied to relay switching device RSD which connects them in turn to the input of analog-to-digital transformer ADT. The codes formed in the analog-to-digital transformer are transmitted to counting device Co which forms the arithmetric average of the numerical values of the signals received during one averaging interval.

The counting device is thus the DRD's flip-flop memory, storing information from all transducers and transmitting it, at the end of the averaging interval, to numeric printing device NPD.

Transformation of the transducers' analog signals is implemented in the ADT by eight cycle pulses supplied by the central control block CC. The cycle pulses are the basic time markers, synchronizing directly or indirectly the entire operation of the machine, including the definition of duration of the averaging interval, the duration of the printing period, etc.

The signal of transformation conclusion is sent by the analog-to-digital transformer to control device CD which, after each transformation, signals switching of the RSD relays. The control device keeps count of the recorded points, and sends a signal when the averaging interval terminates.

* For example, a transducer recording the lining's temperature.

Signals from air expenditure meter T_6, via air transducer control unit ATC, are applied to the counting device. Signals arriving from T_6 during the printing period are stored in the ATC and, at the termination of printing, are then added into the counting device.

The DRD block schematic just described, as well as the devices used in the DRD together with the flip-flops and the relay switching gear, correspond fully to the specific purpose of the DRD: the recording of weak signals with a comparatively slow speed of action.

The DRD was installed in the Dzerzhinskii Works in March, 1960.

Today, the data obtained from the DRD, together with data obtained on the basis of chemical analyses of pig iron and steel specimens, is being subjected to mathematical processing, and the program for the controlling machine is being written.

NEW METHOD AND UNIVERSAL MEASURING DEVICE FOR ACTIVE INSPECTION ON METAL-CUTTING MACHINES

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The use of active inspection is one of the most important directions in the automation of control of high-accuracy technological processes in machine building. Broad use of active inspection is limited by the lack of industrial output of active control instruments which are sufficiently universal to satisfy the very diverse requirements of their installation on machine stands which differ in purpose, type and physical design.

An urgent, and the most pressing, problem today is the development of a unified design of a measuring device, or, at least, its basic and common components, and the organization of their production by the instrument building industry. The principal such component is that for reading off the dimensions of a piece during the course of its technological processing. It must transform the measured quantity to an electrical output quantity. The physical design of such a component (assembly) must be sufficiently universal so that it might be used in diverse machine stands.

The electrical parameter generated by such an assembly must be proportional only to the changes in dimension of the piece, and must not depend on the speed and direction of the piece's motion.

The production of a unified measuring assembly of such a type by the instrument building industry would provide the most rapid and massive use of active inspection of piece dimensions. The price of the instruments produced would be incomparably lower, and the quality higher, as compared with the price and quality of the instruments designed and produced today for the individual needs of different factories and organizations which are instrument users. The assemblies for attaching the measuring device, and the assemblies for mounting it on different machine stands, can also be mass-produced by the instrument building factories, but in view of their heterogeneity and relative simplicity, it would be most efficient to develop standard plans and to print albums of blueprints for their manufacture in the instrument shops of the user factories.

This paper presents the results of work carried out by the author in the S. M. Kirov Ural Polytechnic Institute on the creation of a universal measuring assembly for active inspection, namely, a vibrating-contact transducer-dimension-recorder.

Existing methods and instruments for active inspection of measuring pieces in the process of their fabrication on machine stands do not provide the necessary accuracy and reliability of inspection, require frequent adjustments and are insufficiently universal, which inhibits their widespread introduction into production automation.

The principal feature of existing devices is the use of the method of following the dimensional changes of a piece by means of a probe sliding along its surface with large contact forces and with a lever transfer mechanism to the indicating instrument. Replacement of the mechanical indicating instrument (minimeter, pointer-type indicator) by high-sensitivity electric, pneumatic and other transducers complicates the physical design of the measuring device, but cannot eliminate the inadequacies inherent in mechanical transmission and sliding probes or the measurment errors which thus arise.

Attempts to employ, for active inspection, measuring devices without mechanical probing but based on contactless methods of transforming variations in piece dimensions to some parameter or other (air pressure, reluctance, capacitance, photoelectric or radiation effects) have not found use. This is explained by the specific features of the inspection conditions on machine stands where pieces of different metals are processed with different speeds in different conditions of the ambient medium, with cuttings of different dimensions where access to the cutting zone is difficult and where there are other factors which prevent one from obtaining a single-valued change of the enumerated output parameters of the measuring device for a dimensional change of the pieces.

It is obvious that a universal instrument for active inspection of dimensions must be constructed on the basis of other, new, measurement principles, without the transmission of the measured translation by an inertial level mechanism.

Such a new solution, different in principle from the older ones, is the method of measuring by means of a vibrating probe. In measurements by this method, the surface of the piece to be worked on the stand is contacted by a vibrating tip. The tip is placed at the end of a flat spring, the other end of which is fastened to the frame of a special transducer. The spring's free end is given a harmonic oscillatory motion by an electromagnet.

The vibrating probe is tuned to a natural frequency of 100 cps. The electromagnet, supplied by current at the industrial frequency, synchronously vibrates the probe and maintains its oscillation.

The amplitude of the tip's oscillation is always kept significantly greater than the surplus on the piece, the magnitude of whose removal it is desired to measure during the working process. The transducer's frame is so installed that the oscillations of the measuring tip are limited by its contiguity with the piece's surface.

At the beginning of processing, when the surplus has still not been taken off, the amplitude of the tip's oscillations is minimal. As the surplus is successively removed, the amplitude will increase in proportion to the magnitude of change of the piece's dimensions.

Consequently, the magnitude of the change in amplitude of the probe's oscillations will, in this case, define the amount of surplus removed from the piece.

There exist many methods of remote measurement of amplitude magnitude. The simplest,and at the same time very accurate,of these is electrical, when the oscillatory system is rigidly connected with the armature of a miniature magnetoelectric vibrogenerator.

If the armature oscillations occur with constant frequency, then the magnitude of the voltage of the induced current will be proportional only to the magnitude of the amplitude. By measuring the vibrogenerator's voltage by a voltmeter, the scale of which is calibrated in millimeters, one can determine remotely the magnitude of change of the piece's dimensions in the process of its working.

The vibrating probe eliminates sliding of the measuring tip along the surface of the moving piece, since it touches the piece for only very short intervals of time (of the order of microseconds), approaching it each time in a direction perpendicular to the surface to be measured.

Since the piece is moved, each contact with it is made at a new point of its surface. In completing 100 oscillations a second, the tip makes the same number of measurements at different points on the piece. The quantity obtained as the result of each measurement will be transformed to the corresponding amplitude of the induced ac voltage. By means of an electrical scheme and the choice of a measuring instrument with the necessary frequency of natural oscillation of its moving system, one can average, within wide limits, the results of measurements over time. For a vibrating probe, the speed and direction of the piece's motion in the plane perpendicular to the line of measurement are indifferent, and do not affect the results of the measurements. Oscillations of the tip with amplitudes greater than the surplus allow it to get over sharp humps on the surface of the piece and irregularities, even for comparatively large speeds of motion, make it possible to measure pieces with surface discontinuities (bushings and shafts with key and splined grooves), and introduces the instrument into apertures to be ground together with the abrasive wheel, all without having recourse to special devices which are required by almost all the active inspection devices in existence.

The flexible connection of the transducer with the surface of the piece protects the transducer from disruptions of calibration from random mechanical shocks experienced by the tip.

The electrical connection of the transducer with the sensitive measuring instrument allows the implementation of remote measurement, permits any increase in magnitude of the translation which depends only on the relationship of transducer power and measuring device sensitivity, allows the measurement results to be averaged, and, finally, makes it easy to obtain a current pulse for introducing commands of the executive mechanism controlling the machine stand. The generator principle of transforming mechanical parameters to electrical ones is the most reliable, simple and stable, since it requires no auxiliary apparatus in the form of stabilizers, electronic amplifiers, etc. Use of this principle simplifies the installation and use of the instrument. The transducer, hooked up to the indicating instrument, is a complete aggregate, always prepared for action and not requiring a new calibration when the object to be measured is changed.

The magnitude of the contact force of the vibrating tip at the moment when it touches the surface of the piece does not exceed 1 to 3 grams and, for a narrow range of measurement (for example, for measuring microirregularities), may be reduced to tenths of a gram. The smallness of the contact force is explained by the fact that each successive contact, after the first, of the probe with the piece occurs at the moment when the speed of the probe's motion is minimal and, in its turn, is determined only by the magnitude of the impulse obtained by the probe from the synchronous swinging electromagnet during one half-period.

After a great deal of design and investigative work, and testing of transducers on various machine stands, there was developed a type of universal transducer, whose dimensions were satisfactory for its mounting on different stands, with high sensitivity, indication stability, uniformity of scale for requirements of piece inspection during the working process for the operations of external and internal grinding of cylindrical pieces, plane grinding, turning, boring, honing.

Figure 1 shows the over-all view of the transducer, and Fig. 2 shows its kinematic and electric scheme. In the frame of the transducer, which is a cylinder of diameter about 30 mm and length about 100 mm, two electromagnets are placed. On one end of their face surfaces one end of the plane spring is rigidly fastened. The other end of the spring is attached to the transducer's moving system which has two extensions, one of which is the probe with a tip which makes contact with the piece, the other of which passes through the center of the cylindrical frame and is attached to the armatures of the two electromagnets. The electromagnet positioned closer to the plane

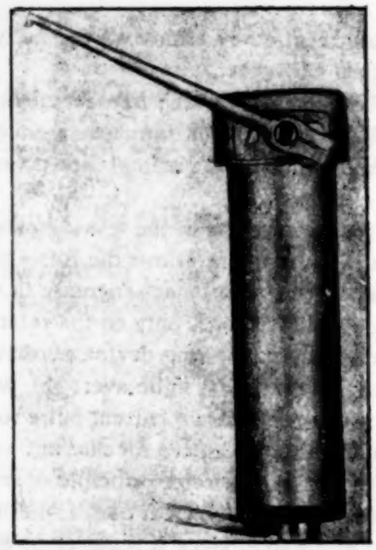

Fig. 1. Exterior view of the universal vibrocontact transducer with rotatable probe.

spring serves to swing the probe. It is supplied from the industrial ac line.

The second electromagnet is the winding of the generator, positioned on the casing attached to the transducer frame, wtih an aperture in the central portion. In this aperture, the armature, consisting of a lamina of transformer steel, can oscillate freely.

At a very small distance from the armature end are placed two constant Alnico magnets, a closed magnetic circuit surrounding the generator winding. Oscillations of the armature before the poles of different polarity induce changes in the magnetic field strength in the armature and induce an electric current in its winding whose voltage is proportional to the speed of these changes.

The speed of the armature's pole piece during its free oscillations varies sinusoidally. Passing through the interpolar distance, the armature has its maximum velocity, while at the limiting positions this velocity equals zero. The ac current induced in the winding has the same correct sinusoidal form. With the oscillations of the transducer's moving system limited by contact of the tip with the piece at the moment of its boundary position, the sinusoidal character of the induced emf is not changed. This allows one to obtain a scale with uniform and stable scale divisions.

Large frictional losses, excessive power of the swinging electromagnet, a very large range of limitation of the probe amplitude in comparison with the amplitude of its free oscillations, all these lead to a distortion of the curve's form and, correspondingly, to nonuniformity in the scale divisions and instability of instrument graduation.

Figure 3 shows an oscillogram taken off for the operation of a normally adjusted transducer. The two traces here, with a common null position, are the voltage curves of the generator and of the 50 cps supply for the swinging electromagnet. On the oscillogram has been photographed the processes of switching in and out of electromagnet swing. The swing of the transducer's moving system up to its maximum amplitude (0.7 mm) occurs in the course of 8 to 10 periods, i.e., during a time of about 0.2 seconds. After switching off of the electromagnet swings, the oscillations are smoothly damped at the same frequency, which shows the correct-

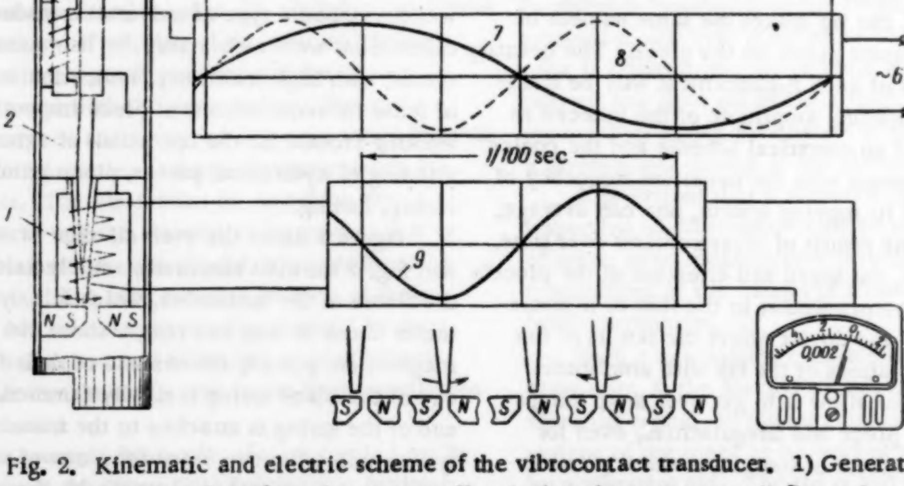

Fig. 2. Kinematic and electric scheme of the vibrocontact transducer. 1) Generator; 2) central connection of moving system; 3) swinging electromagnet; 4) flat spring; 5) probe with a tip; 6) curve of tip's position; 7) curve of electromagnet's voltage; 8) curve of the tip's speed of motion; 9) curve of generator's emf.

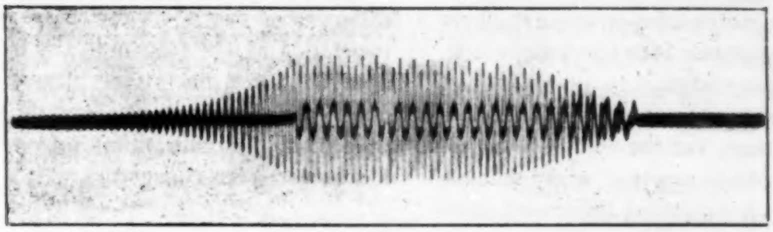

Fig. 3.

ness of the tuning of the moving system's natural frequency, and the smallness of the frictional losses.

All that has been said earlier is necessary to take into account only in the development of a new physical design of the transducer. A correctly designed and adjusted transducer, in conjunction with its measuring instrument, provides stability of graduation under any conditions, and requires no surveillance or new graduation during its use. As measuring instruments, one can use magnetoelectric millivoltmeters or microammeters of any type. The choice of some measuring instrument or another is determined by its dimensions, required class of accuracy, convenience, reliability and conditions of use. As a function of the chosen type of measuring instrument, one designs the winding of the generator in which the emf is induced. The best operation of the measuring instrument is provided in the case when the generator's internal impedance equals the impedance of the external circuit of rectifiers and measuring instrument chassis. The use of a magneto-electric instrument stems from the requirements of high sensitivity, uniformity of scale divisions and low power drain. The most appropriate type of instrument for factory conditions, as concerns electrical data, is a type M-24 microammeter with a scale up to 100 microamperes.

Figure 4 shows a measuring device assembly. The scale of the instrument has been replaced by a new one, calibrated from the transducer, allowing one to read off directly changes in the dimensions in fractions of a millimeter.

The limited size of the electric measuring instrument's scale does not permit full use to be made of the broad range of dimension measurement possessed by the transducer when the maximum amplitude of its tip's oscillations is 0.7 mm, or to obtain the micron accuracy thus provided. Shunting of the measuring instrument, or introducing an additional resistance increases the range of measurement, but with a simultaneous increase in the scale division. For active inspection it is most efficient to use the limited scale of the measuring instrument for reading with the greatest accuracy and clarity only at the limits of the approach of the piece's dimensions to the given ones. For this purpose, the measuring instrument's movable system is clamped, by a stretched spiral filament, to an arrester of pointer motion which is positioned at the zero scale division, and is so designed that its deviations from the rest position occur for three- or four-fold values of the maximum current stated on the scale. The measuring instrument's winding can bear such an increase in current without danger.

By changing the magnitude of the stretch on the pointer filament at the zero rest position, one succeeds, while still retaining high sensitivity, in changing the absolute magnitude of the range of measurement. When taking off a large surplus with the vibration transducer, this allows one to obtain higher sensitivity of the measuring instrument only when the piece's dimensions approximate to the nominal ones. The rest of the time, the instrument's pointer will press against the stop to the left of the zero division.

Experience showed that, for an assembled vibrocontact transducer, it is desirable to produce microammeters with adjustable preliminary stressing of the movable system, allowing one most simply to use the entire range of transducer measurement in the limits of 0.1 to 0.7 mm, with the smallest scale division being 2 microns. The transducer supply is taken from the ac line via a small-power transformer which steps the voltage down to 6 volts. The transducer draws about 0.2 watts of power. Oscillations of the line voltage up to 30% do not affect the results of measurement.

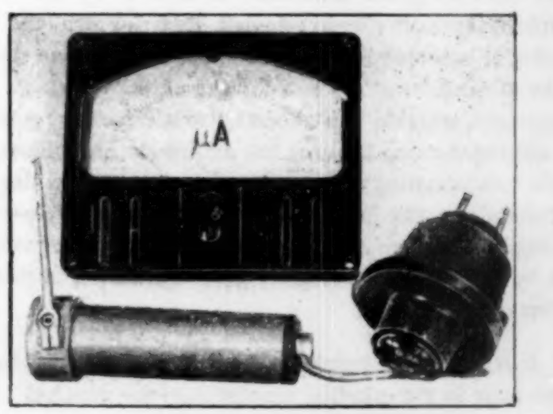

Fig. 4. Measuring device assembly, consisting of a universal vibrocontact transducer and a microammeter with rectifiers mounted in the chassis and with a transformer for the transducer supply from the 220 volt line.

In addition to the basic, most simple, scheme of connecting a transducer to a measuring instrument, there was developed a measuring device with two transducers, connected in a compensation circuit.

The nub of this device is that one transducer directly measures the piece dimensions, and the voltage generated by it is compared with a voltage supplied by the second, compensating, transducer. A measuring instrument whose zero scale division is in the center of the scale is so connected that, for equality of the voltages of both transducers, its pointer is established at the zero division. The amplitude of oscillation of the compensating transducer's probe is limited by a micrometric device. The adjustment of the measuring device is implemented by a preliminary processing of the first workpiece. By regulating the position of the working transducer with respect to the piece, one establishes the amplitude of oscillation of the working transducer at a somewhat higher quantity than the surplus on the piece. Thereafter, by means of the micrometric devices, the amplitudes of the compensating transducer are regulated until the zero position of the measuring instrument's pointer is reached, indicating that, in the given case, the nominal dimension has been reached. After this, a new piece, clamped in the stand, is worked until its dimensions reach the nominal value or lie within the admissible limits stated on the measuring instrument's scale.

The compensation scheme makes it possible to carry out remote electrical tuning of the measuring instrument, to use the measuring instrument without preliminary stressing of the movable system on the stop, to permit the working transducer to operate with any amplitudes, to switch the measuring instrument to different scale divisions by shunting it by different resistances and, thus, to follow the dimensions from the beginning of surplus removal. By connecting, instead of the measuring instrument, a high-sensitivity relay or an electronic device with an analogous action, one can send impulses to control the stand when the piece dimensions established by the micrometric devices have been attained. One can also send any number of successive pulses to control the machine stand as the dimensions of the piece get closer to the given dimensions, wherein it is necessary to use the first pulse for shifting the stop limiting the amplitude of oscillation of the compensating transducer's probe to the next degree of dimension. The use of such a measuring device permits the implementation of program control by servo drives, both by time and by previously given dimension degrees (orders of magnitude).

In measuring apertures by the single-contact scheme, errors arise in the grinding process from the mandrel being squeezed out of the workpiece by the cutting forces. Installation of a second, compensation, transducer in which the amplitude of probe oscillations is accurately limited by the ground surface of the face plate, as shown in Fig. 5, allows these errors to be compensated automatically. Squeezing out of the mandrel from the ring to be ground engenders a decrease in the amplitude of oscillation of both transducers by one and the same amount. Since the voltages tapped off from the transducers are connected, after rectification, in series opposition, the resulting indication of the measuring instrument is not changed.

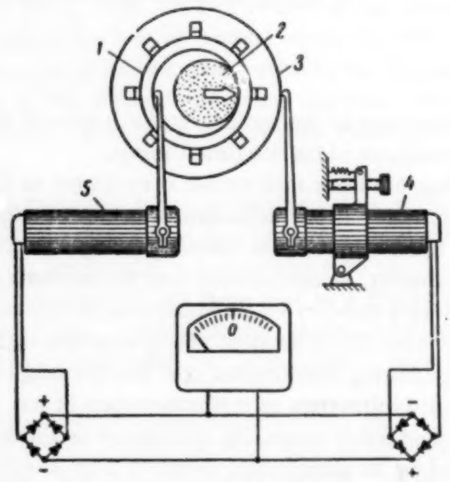

Fig. 5. Scheme of a measuring device with two transducers for measuring aperture diameters with automatic compensation of errors from the mandrel being squeezed out of the workpiece by the cutting forces. 1) Ring to be ground; 2) abrasive wheel; 3) face plate; 4) compensating transducer with the controlling device; 5) working transducer.

When the hole to be ground has been ground to the given dimension, the voltages of both transducers are compared and the pointer of the measuring instrument is established at the zero division at the center of the scale. Adjustment for the given dimension of the hole is implemented by shifting the compensation transducer by means of a screw, changing the transducers's position with respect to the face plate's cylindrical surface.

On the basis of a vibrocontact transducer, one easily solves the problem of building an automatically compensated measuring and controlling instrument with a large angular scale. The compensating voltage is taken off from the transducer whose amplitude of probe oscillation is limited by the profile of a cam positioned on the axis bearing the measuring instrument's pointer. The cam and the pointer are turned by a servo motor supplied by the amplified error emf of the working and compensation transducers (i.e., by the amplified voltage difference).

The advantages of such an auto-compensated device over the analogous devices with wire potentiometers are the absence of dc supply sources and normal elements,

the decrease of the servo motor power to a very low magnitude, since the rotation of a profile cam requires virtually no force apart from that of overcoming the friction in its supporting miniature bearings.

The low power of the servo motor allows the number of amplification stages in the mismatch amplifier to be decreased, and decreases the dimensions of the entire instrument. On the basis of a universal vibrocontact transducer, there were designed, built and tested instruments for inspecting hole diameters in a grinding operation in one section, inspecting deep, large-diameter holes along the entire length, for inspecting hole diameters in discontinuous surfaces in one section, diameters of smooth and ground shafts over the entire length, trunnion diameters from 3 to 8 mm with tolerances of 4 microns, diameters of shafts on universal precision lathes, for the inspection of deep holes for honing over the entire length.

In all these experiments, the universal vibrocontact transducer devices operated without failures, providing stability of results, high sensitivity and lack of temperament with respect to operating conditions in strong jets of cooling fluid (water and kerosene). The transducer operated reliably in difficult dusty conditions with dry grinding and with various size chips on the lathe.

The vibrocontact transducer satisfactorily solved the problem posed, namely, to create a unified design for the basic assembly for the measuring devices of active inspection.

The physical design of the transducer is quite compact and is completely sealed hermetically. The probe, free to rotate by 360 degrees is provided with a hard-fused tip and makes it possible to implement measurement of opened surfaces and holes. The transducer's sensitivity and small contact force allow measurements to be made with micron accuracy.

Adjustment of the transducers during their assembly provided identity of their sensitivities, allowing interchangeability of transducers and measuring instruments. The transducer can be used for visual inspection in the working process, for active inspection in automated inspection, for implementing feedback of actual dimensions in machine stands with program control, for feeding dimension information to computing devices and as the controlling organ in automatic following and copying systems.

QUESTIONS OF STABILITY OF OPERATION OF CLOSED (OR LENGTHY) SCHEMES CONSTRUCTED OF CERTAIN TYPES OF LOGICAL ELEMENTS

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Prague, Moscow

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The requirements on the "input-output" characteristics of logical elements are developed for providing stable operation of rings or lengthy circuits of these elements. Conditions are derived which are necessary for obtaining characteristics of this nature for schemes consisting of identical logical repeaters or identical inverters.

1. Stability Conditions for Closed Circuit Operation

It is well known [1] that, to obtain stable operation of ring circuits consisting of identical logical elements (for example, ring counters, shift registers, or dynamic memory flip-flops), it is necessary that the "input-output" characteristic of the open-loop ring have the form shown on Fig. 1, i.e., for input signals less than some critical magnitude the characteristic should fall below the line 0-I and, for signals larger than the critical value, should lie above the line 0-I. Thus, the characteristic must have three segments:

first segment, lying below the line 0-I, on which $du_{out}/du_{in} < 1$;

second segment on which $du_{out}/du_{in} > 1$;

third segment, lying above the line 0-I, on which $du_{out}/du_{in} < 1$.

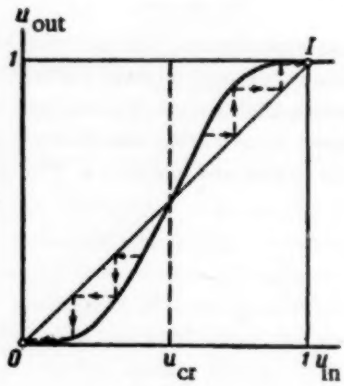


Fig. 1.

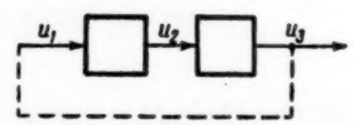


Fig. 2.

A characteristic of this type will always have two stable points of intersection (near 0 and I) with the line 0-I representing the geometric locus of points on which the input equals the output, i.e., with the characteristic of the closed system. These points are the two stable states of the closed system for zero and for complete outputs.

If the circuit is a long open circuit of logical elements, then, for proper operation of such a circuit it is also necessary that signals less than some critioal signal u_{cr} be damped in it, and that signals greater than this critical value pass without damping.

Obviously, the conditions for proper operation of such a scheme coincide with the stability conditions for a ring circuit which, in this sense, is equivalent to an open-loop scheme with an infinite number of elements.

For ring circuits constructed of asynchronous or single-cycle elements, it is necessary that each element of the circuit have a characteristic such as that shown in Fig. 1, since the minimal number of elements forming a ring circuit equals one.

In two-cycle schemes, the minimal number of elements forming a closed ring equals two. Therefore, to obtain stable operation of a closed scheme consisting of identical elements, it is mandatory that each element have the input-output characteristic such as that of Fig. 1, but that the total input-output characteristic of two series-connected elements have the form defined above.

Below we shall consider only two-cycle systems of elements, in which a closed-loop scheme must always contain an even number of elements.

We consider what requirements must be placed on the characteristics of each element in order that two series-connected elements (Fig. 2) in a closed-loop circuit operate stably.

Ring circuits are ordinarily constructed either of repeaters or of inverters.

Repeaters. Fig. 3 shows three types of repeater characteristics (curves 1, 2 and 3). Series-connection of

two identical repeaters with characteristics of this type does not permit one to obtain an over-all input-output characteristic with two stable states as on Fig. 1. This is easily explained by refering to Fig. 4. A series-connection of two repeaters with characteristics of different types, such as curves 2 and 3 of Fig. 3 allows one to obtain an over-all input-output characteristic with two stable states (Fig. 5). But, to construct a ring circuit of such elements, it is necessary to have two types of repeaters, which is usually inconvenient.

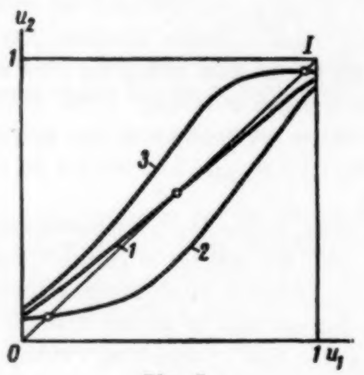


Fig. 3.

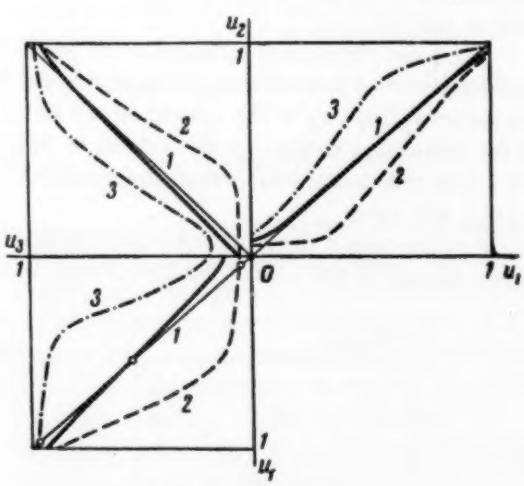


Fig. 4.

Thus, if the ring circuit must be constructed of identical repeaters, it is necessary that each repeater have an input-output characteristic of a form similar to the form of the characteristic of Fig. 1.

Inverters. To obtain stable operation of a closed-loop scheme consisting of inverters, it is sufficient if the characteristic of each inverter element has the form shown in Fig. 6 a and b by the solid lines. Then, with a series connection of two elements, the over-all input-output characteristic will possess two stable states for closed-loop operation (the solid lines in Fig. 6 c).

If the inverter has a characteristic similar to that shown by the dashed lines in Fig. 6 a, then a closed-loop scheme of such inverters will not be stable (Fig. 6c, the dashed line).

2. Analysis of Repeater Operation

We consider, from the point of view of stability of operation, logical elements constructed on the principle of single-core fast-acting Ramey amplifiers.

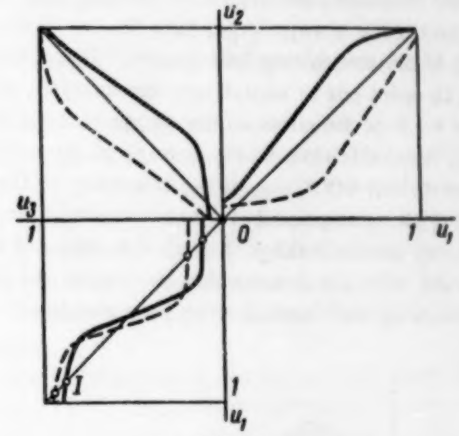


Fig. 5.

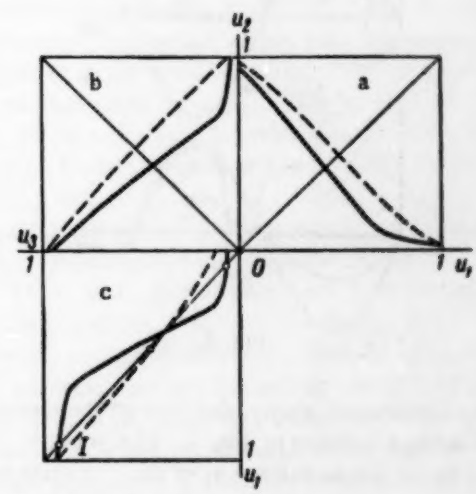


Fig. 6.

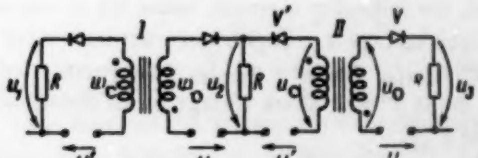


Fig. 7.

The scheme of a circuit segment of two such logical elements is shown in Fig. 7. The voltages are so chosen that

$$u_A = -u_B = \frac{w_0}{w_C} u'_A = \frac{w_0}{w_C} u'_B.$$

Conditions for stable operation for signals close to the maximum. In cores made of material with a rectangu-

lar hysteresis loop, the actual ratio B_r/B_s is always less than unity. This leads to a small phase shift between currents and voltages, both in the working and in the controlling circuits of the cores. However, for those parameter relationships which occur for normal operation of logical elements, the lag of the leading edge for the output voltage is always larger than the lag of the beginning of the controlling half-period. Therefore, in the sequel, in order not to complicate the analysis, we assume that the form of the curve of the output voltage of each element is a half-wave supply voltage of the working circuit with cut-off α_{2min} at the beginning of the half-period. This assumption is very close to the actual facts since, due to leakage through the diodes of the working circuit, the core is somewhat demagnetized during the controlling half-period, even for a maximum controlling signal.

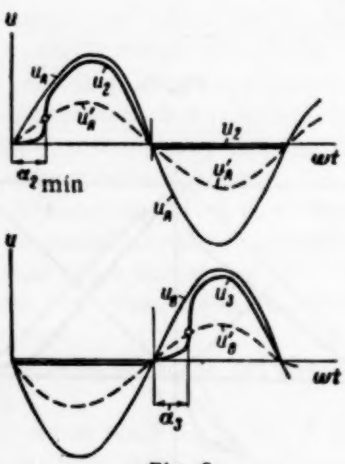


Fig. 8.

For a sinusoidal supply, the form of the element's output voltage is shown in Fig. 8. The presence of a cut-off in the output voltage u_2 of the first element leads to a further increase in the cut-off angle of the output voltage u_3 of the following element. This is explained by the fact that, at the beginning of the controlling half-period, the following element, under the action of reference voltage u_A, begins to be demagnetized until "time" α_{2min}. For $\omega t = \alpha_{2min}$, demagnetization ceases, since diode V' will block voltage u_2 for the condition that $|u_2| \geq |u'_A|$.

After this, the magnetic flux in the core during this half-period still cannot return to the original saturated state, since the diode remains cut-off.

Therefore, at the beginning of the working half-period of the second element, the magnetic induction of its core is less than B_r by an amount corresponding to the demagnetizing action of voltage u'_A in the interval $0 < \omega t < \alpha_{2min}$.

Consequently, output voltage u_3 will have a cut-off angle α_3 with $\alpha_3 > \alpha_{2min}$.

Thanks to this, the element's input-output characteristic, controlled from the output of the same element independently of the ratio of the numbers of turns, w_c and w_o, will lie below the line 0-1 (Fig. 1).

It is impossible to eliminate this phenomenon by increasing the amplitude of controlling voltage u_2, since an increase in the amplitude of the controlling voltage only increases the voltage drop, cutting off diode V'. Therefore, an increase of the ratio of the voltage supplying the operating circuit u_A to the voltage u'_A supplying the controlling circuit, which can be achieved by increasing the ratio w_o/w_c, gives nothing in this sense.

The presence of leakage in the controlling circuit diode V', of the second element can dilute this phenomenon somewhat, since it creates the possibility of returning the flux to the initial state during the time when $u_2 > u'_A$.

The presence of leakage in the operating (working) circuit, as was previously said, can only strengthen the inequality $\alpha_3 > \alpha_{2min}$, i.e., worsen the characteristic (Fig. 9).

However, improving the input-output characteristic by increasing the leakage of current through diode V' of the control circuit is disadvantageous, since the diode's leakage current must be greater than the magnetization current of the core's controlling winding, and this sharply decreases the number of admissible input diodes of the circuit, which are used to obtain logical functions of one kind or another.

The most correct way to improve the input-output characteristics of such elements is to reduce the form of the supply voltage u'_A of the control circuit to the form of the controlling voltage for full output, so that, during the entire controlling half-period, the condition $|u_2| \geq |u'_A|$ will hold.

For this, with a sinusoidal half-wave source, the supply voltage in the controlling circuit must have a

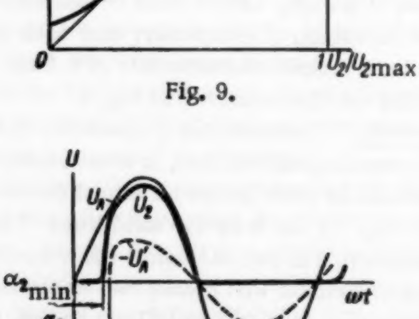


Fig. 9.

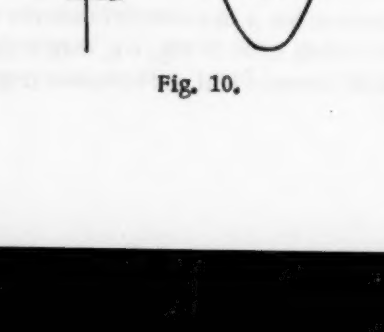


Fig. 10.

cutoff α_0 in the controlling half-period equal to, or greater than, the quantity α_{2min}, as shown in Fig. 10.

For such a form of supply voltage of the control circuit, the element's input-output characteristic for $\alpha_0 = \alpha_{2min}$ will have the form of curve 1 in Fig. 11.

If the cut-off angle α_0 of the control circuit's supply voltage is chosen greater than α_{2min} then, for control signals for which $\alpha_{2min} < \alpha_2 < \alpha_0$, the core will generally not be demagnetized, and the output voltage will be complete (the upper horizontal portion of the characteristic (in Fig. 12). For control signals for which $\alpha_2 > \alpha_0$, the core is partially demagnetized and the output is decreased (the sloping portion of characteristic 1 in Fig. 12).

An increase in the minimum magnitude of the output voltage gives rise to a decrease of the quantity $\int_0^{\pi} u'_A d\omega t$ due to cutoff. This decrease can be compensated by increasing the amplitude of u'_A but, with this, there occurs opening of diode V in the operating circuit during the controlling half-period which loads the control circuit. The core is demagnetized under the action of voltage u_c, applied to the control winding, this voltage being less than voltage u'_A by the amount of the voltage drop across resistance R induced by the demagnetizing current and the transformation from the working circuit.

If the voltage u'_A is chosen so that

$$\frac{1}{\omega w_y}\int_{\alpha_0}^{\pi} u_y\, d\omega t = 2\Phi_s,$$

then, in the controlling half-period without a signal, the core's magnetic polarity will be completely reversed and its free-running voltage will be determined only by the magnitude of magnetization current in the operating half-period. In this case, the input-output characteristic will have the form of curve 2 in Fig. 11, for $\alpha_0 = \alpha_{2min}$, and of curve 2 on Fig. 12, for $\alpha_0 > \alpha_{2min}$.

Conditions for stable operation with small signals. Further increase of the amplitude of the control circuit's supply voltage leads to the consequence that, in the controlling half-period, the second core is saturated for $\omega t < \pi$.

With the previously considered conditions in the second core's controlling half-period – the first core's working half-period – after the demagnetizing voltage u'_A begins to act, i.e., for $\omega t > \alpha_0$, the second core is demagnetized more rapidly than the first is magnetized, since the first core is magnetized under the action of voltage $u_A - R[i_{\mu o} + w_o(i_{\mu o} + i_T)/w_c]$, and the second core is demagnetized under the action of voltage $u_B - i_T R = -u_A - i_T R$, where $i_{\mu o}$ is the magnetization current of the operating circuit, and i_T is the transformation current.

After negative saturation of the second core, at time $\omega t = \alpha_M$ (for signals close to zero), magnetization of the first core continues under the action of voltage $u_A - u'_A$. It can be shown that, at the end of the half-period, the first core has returned to its original state, i.e., that the saturation of the second core in its controlling half-period, for $\alpha_M < \pi$, does not affect the stability of the circuit's operation.

Demagnetization of the second core until negative saturation in the controlling half-period occurs for signals for which the cut-off angle $\alpha_2 \geq \alpha_M$ since, under the action of such signals, the second core's control circuit is disconnected after its saturation. Thus, the second core's output voltage will equal zero independently of the magnitude of the cutoff angle α_2 of the first core, with the condition that $\alpha_2 \geq \alpha_M$. This leads to the consequence that, in the element's characteristic, there appears a lower horizontal segment (curve 3 in Fig. 11 for $\alpha_0 = \alpha_{2min}$ and curve 3 in Fig. 12 for $\alpha_0 > \alpha_{2min}$). This characteristic possesses all the properties necessary for stable element operation in closed circuits (Cf., Fig. 1).

Thus, for an element executed in accordance with the scheme of Fig. 7 to have the characteristic necessary for stable operation in closed or lengthy open circuits, it is necessary that the controlling circuit's supply voltage have cut-off α_0 whose magnitude must exceed the minimum possible cutoff α_{2min} of the element's output voltage induced by incomplete rectangularity of the core material. The amplitude of this voltage must be chosen from the condition

$$\frac{1}{\omega w_c}\int_{\alpha_0}^{\alpha_M} u_c\, d\omega t = 2\Phi_s,$$

where $\alpha_M < \pi$.

The choice of the voltage u'_A from this condition always entails that the voltage transformed from the

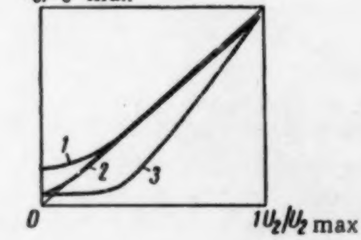


Fig. 11.

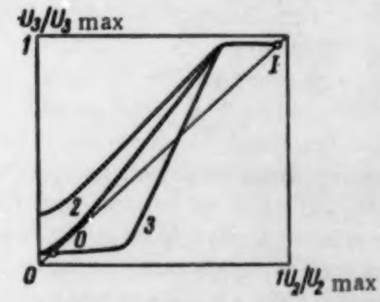


Fig. 12.

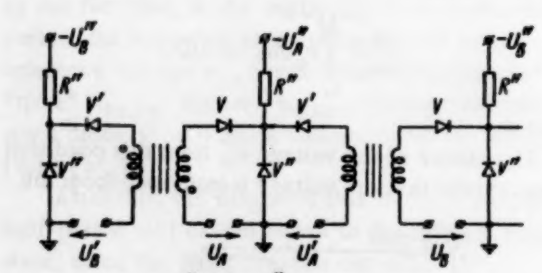
element's controlling circuit to its working circuit is higher than the working circuit's supply voltage u_B and opens diode V of the working circuit.

In order to avoid this, one can either open the element's working circuit in the controlling half-period or increase the supply voltage of the element's working circuit in the controlling half-period. The first is achieved by connecting in the element's circuit, instead of resistance R, a nonlinear resistance in the form of a diode, opened in the working circuit only during the working half-period (Fig. 13) by means of auxiliary voltages u''_A and u''_B, properly phased.

The second can be achieved by using a special supply source [2] or by the circuit of Fig. 14 which allows one to obtain a larger half-wave supply voltage for the working circuit in the controlling half-period than in the working half-period, so that the following relationship holds during the course of the controlling half-period

$$u_B = -u_A \geqslant u'_A \frac{w_0}{w_c}.$$

Returning now to the logical elements of the scheme of Fig. 13, we note that the operation of the elements of the scheme of Fig. 13 differs from the operation of the elements of Fig. 7's scheme only in that, instead of resistor R, there is a nonlinear resistance (a diode, controlled by voltage u''_A or u''_B) which is close to zero during magnetic polarity reversal of the core and, after saturation of the cores, increases sharply and limits the current. Thanks to this, the elements of the scheme of Fig. 13 have lower losses in the active impedances of the windings and the forward impedances of the diodes which, in our previous considerations, we neglected. This decrease of losses in the elements permits an increase in the number of elements which can be connected at the output of each element, which is very important in complicated logical schemes. An increase in the operating voltage in the controlling half-periods in the scheme of Fig. 7 leads to the same advantage.



$-U''_B(\omega t) = -U''_A(\omega t + \pi)$

Fig. 13.

However, the use of nonlinear impedance V'' (the scheme of Fig. 13) instead of resistor R (scheme of Fig. 7) or an increase in the working voltage of the scheme of Fig. 7 do not change the character of the element's operation from the point of view of obtaining an input-output characteristic necessary for stable element operation in closed circuits. Therefore, the conditions for stable operation remain the same as those obtained for the scheme of Fig. 7.

3. Analysis of Repeater Operation with a Simplified Source of Supply

We consider still another form of logical element based on the Ramey scheme (Fig. 15). This scheme differs from that of Fig. 7 in that it has one and the same source of supply for the working and the controlling circuits, which would correspond to a choice of $u_A = u'_A$ in the scheme of Fig. 7.

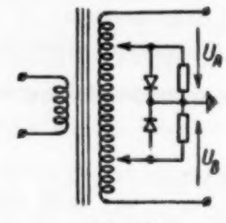


Fig. 14.

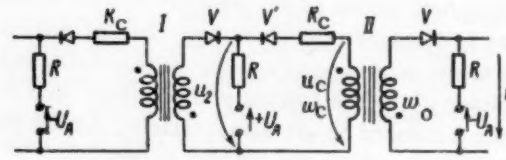


Fig. 15.

We now show that, to obtain a stable characteristic for the scheme of Fig. 15, it is necessary to connect a resistance R_C in the control circuit, while simultaneously choosing $w_C < w_O$.

We consider the operation of element II in the absence of an output from element I (i.e., for a null signal). We assume that the voltage drop across resistor R from the magnetization current $i_{\mu O}$ of the operating circuit of element I is small in comparison with voltage u_A.

Then, magnetization of element I will occur almost from the very beginning, of the working half-period, i.e., as soon as u_A becomes greater than zero (Fig. 16), under the action of voltage

$$u_2 = (u_A - i_{\mu O}R)\frac{R_C}{R + R_C} \approx u_A \frac{R_C}{R + R_C}.$$

With $\omega t = \alpha_0$, when the current flowing through the controlling winding of core II attains the magnitude of the demagnetization current $i_{\mu C}$ of core II, i.e., the quantity $u_A/(R + R_C)$ becomes equal to $i_{\mu C}$, core II begins to be demagnetized under the action of voltage $u_C = -u_2 + i_{\mu C}R_C$, where $u_2 = u_A - R(i_{\mu O} + i_{\mu C})$.

The diode of the second element's operating circuit remains cut off until the inequality $|u_C| w_O/w_C < |u_A|$ no longer holds. After the advent of the inequaltiy $|u_C| w_O/w_C \geq |u_A|$ (for $\omega t > \alpha_1$), this diode is opened and the transformation current appears which is added to the

current $i_{\mu C}$, and demagnetization occurs under the action of the voltage $u_C = -u_2 + (i_{\mu C} + i_T)R_C$, where $u_2 = u_A - R(i_{\mu O} + i_{\mu C} + i_T)$.

After voltage u_A passes through its maximum, all these phenomena repeat in the reverse order: For $\omega t = \alpha_1'$ diode V is cut off and transformation ceases, then demagnetization of core II ceases for $\omega t = \alpha_M$.

To meet the conditions for obtaining a stable input-output element characteristic, it is necessary, first, to choose the magnitudes of R and R_C so that the current of the second element's control circuit, approximately equal, until the beginning of its magnetic polarity reversal, to the quantity $u_A/(R + R_C)$, attain the magnitude $i_{\mu O}$ for $\omega t = \alpha_0$, where $\alpha_0 > \alpha_{2min}$ (α_{2min} is the minimum cut-off angle of total output voltage, defined by the nonrectangularity of the core material's hysteresis loop and the working circuit diode's current leakage). Second, the ratio of number of turns w_O/w_C must be so chosen that the equality

$$\frac{1}{w_O}\int_0^{\pi} u_2\, d\omega t = \frac{1}{w_C}\int_{\alpha_2}^{\alpha_M} u_C\, d\omega t,$$

holds, i.e., that the core be demagnetized during the controlling half-period by the same amount as the core in the working half-period is magnetized.

From Fig. 16, it is easily explained that, for small control signals for which $\alpha_2 > \alpha_M$, the output voltage of element II will be determined only by the voltage drop from current $i_{\mu O}$, i.e., the element's input-output characteristic will have a lower horizontal portion (Fig. 17). Analogously,

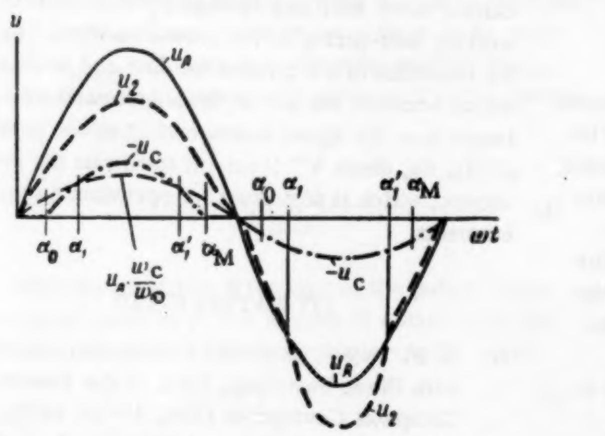


Fig. 16.

for control signals for which $0 < \alpha_2 < \alpha_0$, i.e., for signals less than the full output signal, the output voltage of element II will equal the maximum, i.e., the input-output characteristic will have an upper horizontal segment (Fig. 17).

Thus, the scheme of Fig. 15 possesses a stable input-output characteristic for correctly chosen parameters when it operates from an ordinary ac supply source without use of a device creating a cut-off in the controlling half-period of controlling supply voltage. This is the virtue of the scheme of Fig. 15. The presence of transformation current in the element's controlling half-period is a disadvantage of the scheme, since it increases losses in the element and decreases the number of elements it is possible to connect to the output of each element. To eliminate this disadvantage, one can increase the amplitude of the working voltage in the controlling half-period if one uses a special asymmetric supply source (Fig. 14) with two output voltages and a common current.

When a supply source with one output voltage is used, this is not possible.

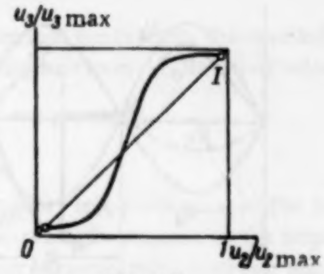


Fig. 17.

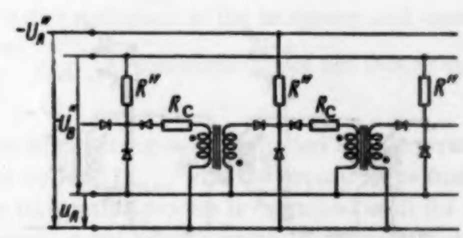


Fig. 18.

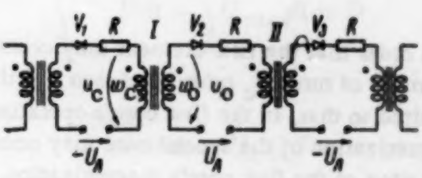


Fig. 19.

In that case, in the circuit of Fig. 15, instead of resistor R, one can use the same diode with an auxiliary voltage as in the scheme of Fig. 13. The scheme thus obtained (Fig. 18) has a stable characteristic without a special cut-off in the supply circuit of the element's control winding and has sufficiently small losses, i.e., permits the control of a large number of logical elements connected to the output.

We note that schemes of the type of Figs. 10 and 13 possess stable characteristics only for such forms of supply voltage curves which have leading edges with finite slopes (as, for example, sinusoids or trapezoids with sloping leading edges). Only in this case is automatic cut-off obtained. With a rectangular supply, the angle α_0 will equal zero, and automatic holding of the condition $0 < \alpha_2 < \alpha_0$ is impossible.

4. Analysis of Inverter Operation

Figure 19 shows a circuit segment consisting of two logical elements, namely inverters.

It was shown earlier that, for stable operation of closed inverter circuits, it suffices that each element have an input-output characteristic of the form shown by the solid line in Fig. 6a, i.e., the output voltage must be close to zero for signals which do not attain the maximum value.

We now consider the conditions which must hold in the circuit of Fig. 19 in order to obtain such a characteristic.

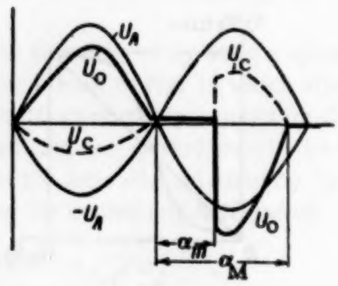


Fig. 20.

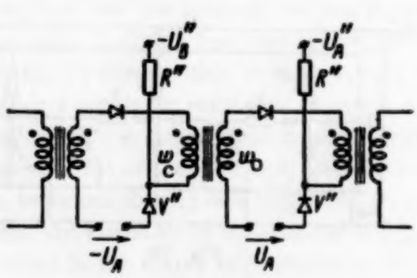


Fig. 21.

In order that the first element may control the second, the number of turns w_c must be chosen less than w_0. This is required so that, in the first core's operating half-period, demagnetization of the second core may occur only after termination of the first core's magnetization. For a magnetized core, its control circuit is always open in the working half-period, since the voltage transformed in the control circuit for a magnetized core is always less than the supply voltage (since $w_c < w_0$).

For demagnetization of the second element's core in its controlling half-period, i.e., after saturation of the first element's core, the voltage transformed in the second element's working circuit can exceed the element's supply voltage u_A. In this case, diode V_3 will be opened, and a transformation current arises. However, the ampere-turns induced in the third element by this current will not suffice for demagnetizing this element's core.

In order that the second element's output voltage reach zero for control by an incomplete signal from the first element, it is necessary that demagnetization of the second element's core in its controlling half-period occur more rapidly than its magnetization in the operating half-period (Fig. 20). This can be achieved by choosing the ratio of the number of turns by starting from the condition

$$\frac{1}{\omega w_c}\int_{\alpha_m}^{\alpha_M} u_c\, d\omega t = \frac{1}{\omega w_0}\int_{0}^{\pi} u_0\, d\omega t = 2\Phi_s,$$

where $u_c = u_A - (i_{\mu c} + i_T)R$ and $\alpha_m < \alpha_{2min}$.

For equal speeds of magnetic polarity reversal in the controlling and operating half-periods, an input-output characteristic of the form of that shown in Fig. 6a, cannot be obtained.

The presence of transformation current for core demagnetization increases the element's power requirements, i.e., decreases the number of logical schemes that can be hung on the element's output. Transformation may be obviated by using an aysmmetric supply source (Fig. 14).

Figure 21 shows an inverter scheme not requiring an asymmetric voltage supply, in which resistor R is replaced by a controllable nonlinear impedance, namely, a diode. The diode prevents the formation of transformation current since auxiliary voltage u_A'' cuts it off in the nonworking half-period of the second element. In addition, for saturation of the cores of the first and second elements, which occurs at the end of demagnetization for signals larger than the signal corresponding to the point of inflection of Fig. 6a, diode V'' is cut-off and limits the elements' current, which is important for operation on several logical elements.

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ON ONE METHOD OF CONSTRUCTING ANALOG-TO-DIGITAL TRANSFORMERS

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A method is suggested for constructing a transformer of voltage to its numerical equivalent, the method being based on a preliminary transformation of the input quantity to proportionate time intervals by integration.

A block schematic of the transformer and an analysis of its operation are provided.

For the transformation of voltages to their digital (numerical) equivalents, wide use has been made of devices in which the continuous (analog) quantity is first transformed to a time interval proportionate to it. During this interval, pulses with a constant repetition rate pass through a concidence scheme and are accumulated in a counter. At the end of the interval, the number staticized in the counter unambiguously characterizes the value of the transformed voltage.

The time intervals of the proper lengths are ordinarily obtained as the result of a comparison of the voltage to be transformed with a sawtooth voltage. The interval commences when the sawtooth passes through zero. The interval ends when the values of both quantities coincide. Thus, if the maximum value of the voltage to be transformed, U_{max}, corresponds to the time interval T_{max}, then the following interval will correspond to the current value of voltage U

$$T = \frac{T_{\max}}{U_{\max}} U. \tag{1}$$

With the repetition frequency of the pulses filling the counter equal to f, the number N staticized in the counter as the result of a transformation cycle equals

$$N = fT \tag{2}$$

or, by taking Eq. (1) into account,

$$N = \frac{fT_{\max}}{U_{\max}} U. \tag{3}$$

It is obvious that, with such a method of transformation the numerical equivalent is proportional to the instantaneou value possessed by the voltage to be transformed at the end of the interval.

Processing of the time interval by means of the method just described is now the generally accepted procedure. However, another method, with no less advantageous conditions, is possible for the implementation of the preliminary transformation of the analog quantity to a time interval. It amounts to the implementation of the preliminary transformation in two steps by an integrator. Initially, the voltage U is integrated during a strictly defined time T_{max} such that, at the end of the first cycle, the value staticized in the integrator will equal $\int_0^{T_{\max}} U dt$. At the beginning of the second cycle, the input of the integrator is supplied by a generator of reference voltage U_{max} with the opposite sign from that of U. The integration process is continued until the total quantity becomes equal to zero which is characterized by the expression

$$\int_0^{T\max} U dt - \int_0^{T} U_{\max} dt = 0 \tag{4}$$

or

$$\int_0^{T\max} U dt = U_{\max} T, \tag{5}$$

where T is the time of the second cycle.

The left member of (5) can be expressed in terms of the average value U_{av} of the voltage during time T_{max}, namely,

$$\int_0^{T\max} U dt = U_{\text{av}} T_{\max}. \tag{6}$$

From (5) and (6) we can define

$$T = \frac{T_{\max}}{U_{\max}} U_{\text{av}} \tag{7}$$

If, during the duration of the second cycle, we pass pulses through a coincidence scheme at the counter's input then, on the basis of (2) and (7), we obtain the

following numerical equivalent, staticized in the counter

$$N = \frac{fT_{max}}{U_{max}} U_{av} \qquad (8)$$

It is clear from this last expression that, with this method of developing the time interval, the numerical equivalent corresponds, not to the instantaneous value of the voltage at the end of the interval (as occurs when a sawtooth voltage generator is used), but to its average value over the fixed time interval T_{max}.

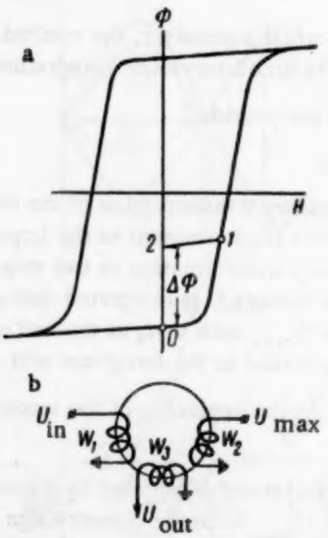

Fig. 1. Magnetic element and its hysteresis loop.

A voltage transformer constructed on the basis of the method just described can be obtained quite simply if, as the integrator, one uses a magnetic element whose core material has a rectangular hysteresis loop. As is well known, when a voltage pulse u(t) is applied to the magnetic element's input, the magnetic flux in the core changes by the amount $\Delta\Phi$. This means that, if the core's magnetic state corresponds to point 0 (Cf., Fig. 1a) then, under the stimulus of the pulse, the core transfers to the state characterized by point 2. Such a transition is executed along the portion 0-1-2 of the hysteresis loop. With this, the magnetic flux changes by the amount

$$\Delta\Phi = \int_0^t u(t)\, dt. \qquad (9)$$

In the particular case when u(t) is a rectangular pulse of height U_{in} and duration T_{max}, the change in flux will equal

$$\Delta\Phi = U_{in} T_{max} \qquad (10)$$

We now consider the inverse problem. Let it be known beforehand that the core is found in state 2. Some rectangular pulse of height U_{max} is applied to the magnetic element's winding, taking the core to the zero state. It is required to determine the time T during which the core makes the transition to this state. With (10) taken into account, the solution of this problem leads to the expression

$$T = U_{in} \frac{T_{max}}{U_{max}}. \qquad (11)$$

In the more common case, when the magnetic element has two windings with number of turns equal to W_1 and W_2 (Cf., Fig. 1b), Eq. (11) takes the form

$$T = U_{in} \frac{W_2 T_{max}}{W_1 U_{max}}. \qquad (12)$$

It follows from expressions (9)-(12) that, if there is applied to winding W_1 of the magnetic element, which has already been established in the zero state, a rectangular voltage pulse of fixed duration T_{max} and unknown amplitude U_{in}, and if, during a certain time, a voltage pulse of opposite polarity of sufficient duration and with accurately fixed amplitude U_{max} is applied to winding W_2 then, at output winding W_3, there will appear a rectangular pulse U_{out} whose duration is proportional to the magnitude of the input voltage U_{in}. This has been shown quite well experimentally. In particular, for the practical implementation of the transformer, the integrating elements were subjected to a special investigation, these elements having cores which were formed of five Perminvar rings (strip thickness 10 mm, number of turns 16, height of one ring 6 mm, mean diameter 8 mm). With this, the maximum duration of the output pulse was 1000 microseconds. The output pulse duration was measured with an error not exceeding 2 microseconds, and the error in measuring the input voltage did not exceed 0.2%. As a result of the experimental investigation, it was established that the deviation of the relationship T = F(U_{in}) from linearity did not exceed the errors in measurement.

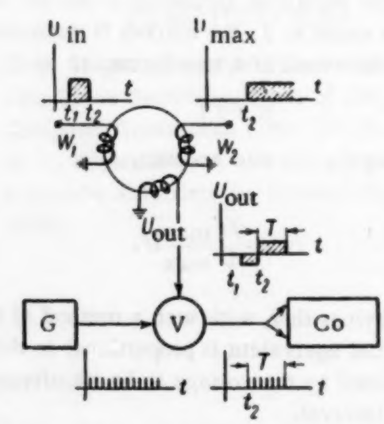

Fig. 2. Block schematic for explaining the transformer's principle of operation.

If the signal U_{out} is used to control the valve V (Fig. 2) through which the pulses from generator G pass, to be staticized in counter Co, then the number accumulated in the counter during the time of application of the output pulse will be proportional to the magnitude of the voltage U_{in}. The principle of operation of such a device is clear from Fig. 2, and requires no further explanation.

If one relates the magnitude of the interval T_{max} to the repetition rate of the pulses applied to the counter input, then one can automatically compensate errors arising because of instability of the pulse frequency or instability of the value of T_{max}. It should be mentioned that such compensation is difficult to implement in transformers based on the comparison of the voltage to transformed with a sawtooth voltage.

In the transformer under consideration, these errors can be completely compensated by means of matching shift registers, as follows from an analysis of the block schematic shown in Fig. 3 and, in its general features, explaining the device's principle of operation.

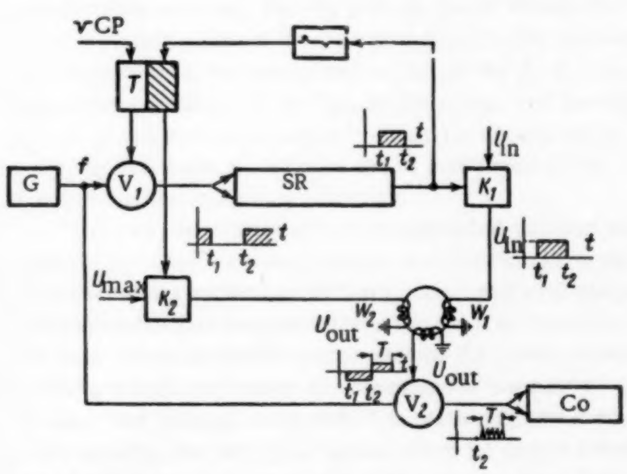

Fig. 3. Block schematic of a transformer with two counters.

At the beginning of operation, trigger (flip-flop) T is established in such a state that valve V_1 is closed and key K_2 is open. A current sufficient to establish the core in the zero position flows through winding W_2 of the magnetic element. After the advent of a clearing pulse (CP) the trigger transfers to its second stable state, opening V_1 and closing K_2. Staticizing of pulses of frequency f transpires in the shift register (SR). At time t_1, the final flip-flop of the shift register operates, opening key K_1. This key will remain open until time t_2, when the last flip-flop of the shift register again transfers to the zero state. During the interval $T_{max} = t_2 - t_1$, a current, induced by voltage U_{in} and changing the core's magnetic state, passes through key K_1 and winding W_1. It is obvious with this that

$$T_{max} = \frac{2^{m-1}}{f}, \tag{13}$$

where m is the number of stages of the shift register.

At time t_2, key K_1 is closed and trigger T is transferred to its initial state, closing V_1 and opening K_2, by means of a pulse obtained after differentiation of the trailing edge of the shift register's output signal (the differentiating circuit is shown by the rectangles with the representations of the two heteropolar pulses; the hatched pulses are not used). The core is transferred to the zero state by the current induced by the voltage U_{max}. With this, a rectangular signal U_{out} of duration T is put out from the output winding, opening valve V_2. Pulses of frequency f pass through V_2 and are accumulated in the counter (Co). The relationship between the number N staticized in the counter and the magnitude of the voltage U_{in} can be determined by solving Eqs. (2), (12), and (13) simultaneously, giving the expression

$$N = U_{in} \frac{2^{m-1} W_2}{U_{max} W_1}. \tag{14}$$

It is clear from this last equation that the value of the numerical equivalent is proportional to the voltage U_{in} to be transformed, and depends neither on the pulse repetition frequency f nor on the absolute value of the time interval T.

A more detailed analysis of the transformer just considered shows that the shift register and the accumulating counter can be combined. This is illustrated by the block schematic of Fig. 4. Its difference from the block schematic of Fig. 3 is that valve V_1, via a discrimination scheme, shown conventionally by the diodes D_1 and D_2, is controlled both by trigger T and by output pulse U_{out}. With this, the requirement for a special counter disappears.

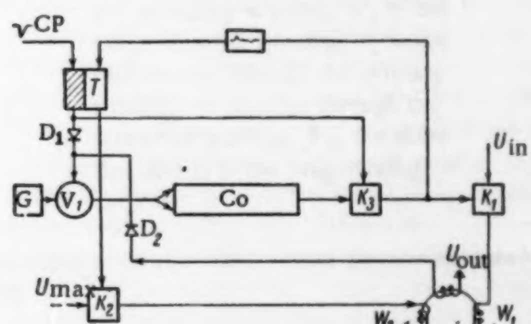

Fig. 4. Block schematic of a transformer with one counter.

The operating cycle of a device constructed in accordance with the scheme of Fig. 4 is executed in two stages. In the first stage, there occurs a change in the core's magnetic state under the action of voltage U_{in} which is completely analogous to the first stage of the scheme described above. It is easily seen that, at the moment when the first stage terminates (at the moment when a negative pulse is applied to trigger T), all the flip-flops of the shift register are in the zero state. At the termination of the

first stage, key K_3 is closed and K_2 is opened. The magnetic element is returned to the zero state, thanks to which an output pulse U_{out} of the proper polarity is put out, opening valve V_1. Through V_1 there pass pulses to be staticized in the flip-flops of the shift register, acting now as an accumulating counter. Thanks to lock-out key K_3 (which can be opened only after the advent of a clear pulse CP), for any number written in the counter, key K_1 will be closed during the course of the entire second stage, which is a necessary condition for correct operation.

The principle cited was verified experimentally on practical devices, which were tubeless transformers to the proper binary equivalents. Among its virtues are the simplicity of the practical schemes, ease of implementation completely by semiconducting elements, absence of rigid requirements on stability of pulse repetition frequency f and transformation interval T.

NEW ELECTROMAGNETIC EXECUTIVE ORGANS FOR AUTOMATIC CONTROL SYSTEMS

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Gor'kii

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Brief descriptions are given of new electromagnetic executive organs—original transformers controlled by shunt magnetization. The operating principles of such transformers with feedback are given. A new group of three-phase transformers is described, as well as original automatic devices for transforming single-phase current to three- or two-phase.

There is a very pressing problem in the creation, for automatic control systems, of executive organs without transferrable contacts, moving parts or power electronic-ionic elements. One of the new directions in the solution of this problem is the work, carried out in the A. A. Zhdanov Polytechnic Institute of Gor'kii, in the design and investigation of original transformers* which are controlled by changing the magnetization of shunts positioned in the apertures of the secondary windings.

The new shunt magnetization controlled transformers (SMCT) are more economical than transformers [1] with power windings located on different cores and with magnetic shunts divided into two parts (Fig. 1). In these transformers, the poor electromagnetic connection of the power windings leads to a high utilization of copper and a low power factor. The leakage in an SMCT is relatively lower and, consequently, the weight of active material is also lower. An SMCT is more economical, more compact and lighter than a set-up with power magnetic amplifiers — with saturable reactors — in those cases when a transformer is necessary to connect the supply line with the load. Today, SMCTs are used by a number of enterprises in the USSR in various automatic devices, for example, in voltage and current stabilizers, in controlled rectifier machines, in transformers of numbers of phases, in program control apparatus, etc. The SMCTs in use have powers of 0.1 to 150 kv-a; a single-phase SMCT is being developed with a power of 5600 kva-a.

The domain of application of the SMCTs can still not be accurately defined, due to the short span of time elapsed since their development. They will probably find further application as executive organs of automatic voltage controllers in those cases when reliability and compactness are decisive in the choice of the power elements.

This paper basically contains information on various SMCT systems. A significant part of it reflects work carried out in 1959 and not yet published; data on work executed somewhat earlier have already appeared in print [2].

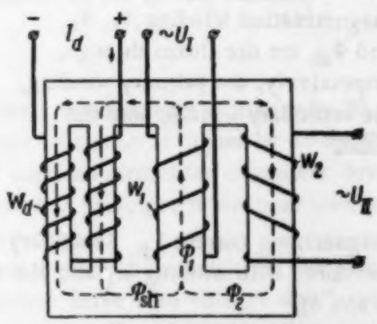

Fig. 1. Scheme of a transformer with smooth voltage control. W_1 is the primary winding , W_2 is the secondary winding, W_d is the magnetization winding, Φ_1 is the flux running through the primary winding, Φ_2 the flux through the secondary winding, Φ_{sh} the shunt flux and I_d is the magnetizing current.

1. Single-Phase SMCT

We consider two basic physical designs of single-phase SMCTs. The first was developed for controllers with thorough control, the second is advantageous, principally, for executive elements of various stabilizers.

In the first design (Fig. 2), core 1, on which primary winding W_1 is placed, and core 2, together with the corresponding portions of the frame, constitute the basic magnetic circuit. Cores 3 and 4 are the magnetic shunt, divided into two portions. On these cores are placed identical windings W_d, connected in opposition and sup-

*** A. M. Bamdas and V. A. Somov, Patent application No. 110770, Committee on Patents, USSR.**

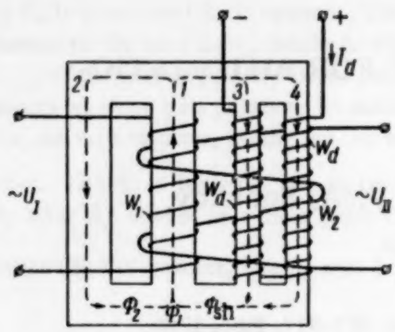

Fig. 2. Scheme of a transformer with a magnetized shunt positioned in the secondary winding's aperture: 1 and 2 are the cores of the basic magnetic circuit, 3 and 4 are the cores of the magnetic shunt, W_1 is the primary winding, W_2 is the secondary winding, W_d is the magnetization winding, Φ_1, Φ_2 and Φ_{sh} are the fluxes through, respectively, the primary winding, the secondary winding and the shunt.

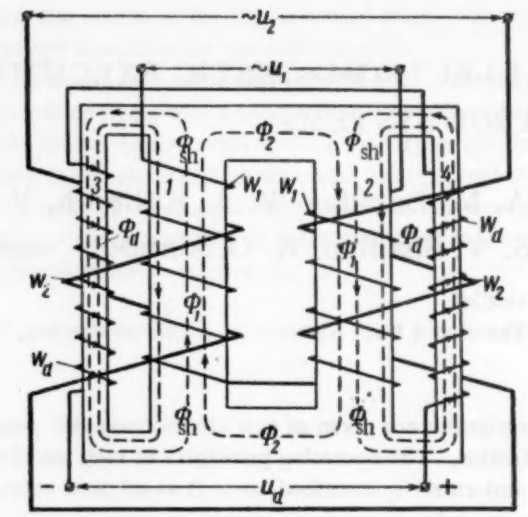

Fig. 3. SMCT scheme for voltage control within narrow limits. 1 and 2 are the cores of the basic magnetic circuit, 3 and 4 are the cores of the magnetic shunt, W_1, W_2 and W_d are the primary, secondary and magnetization windings respectively, Φ_2 is the basic flux, Φ_{sh} is the shunt flux (variable) and Φ_d is the shunt flux (constant).

plied by dc magnetizing current I_d. Secondary winding W_2 encompasses core 1 with winding W_1 and the magnetic shunt (cores 3 and 4).

The variable magnetic fluxes Φ_{sh} and Φ_1 have contrary directions. By taking this into account, we obtain the magnitude of the flux Φ_2 through the secondary winding from the equation (in symbolic form)

$$\dot{\Phi}_2 = \dot{\Phi}_1 - \dot{\Phi}_{sh}.$$

Obviously, control of current I_d and the variations of flux Φ_{sh} connected with this permit the implementation of smooth control of secondary voltage U_{II}. For an invariant primary voltage, the greatest value of U_{II} is obtained for the greatest current I_d; in this operating mode, there is almost complete expulsion of variable flux from the shunt.

To a certain extent, the SMCT is a development of a well-known system of a controlled transformer with a movable magnetic shunt [3] in the aperture of the power winding. Indeed, the magnetization shunt in the SMCT is analogous in its effect to a displacement of part of itself (i.e., to an increase of the air gap between the shunt and the basic magnetic circuit).

In the second design, shown in Fig. 3, both basic cores 1 and 2 are used for the locus of winding W_1. In comparison with the first design, the expenditure of copper for the magnetization windings is considerably less here.

Cores 3 and 4, symmetrically positioned at the sides of the basic cores are, in conjunction with the corresponding portions of the frame, the magnetic shunts. On these cores are placed the magnetization windings, connected in series opposition. The secondary circuit consists of the two W_2 windings.

The constant magnetic flux Φ_d, induced by current I_d, is completed by cores 1 and 2 of the basic magnetic circuit.

We now characterize certain of the SMCT parameters.

1. The power factor of the SMCT primary circuit during the control process for a constant active load varies within approximately the same limits as the power factor for machines consisting of transformers with series-connected saturable reactors.

2. For an invariable I_d, the SMCT load characteristic is a falling one. Automatic control of current I_d permits an artificial characteristic of any form to be obtained.

3. The power dissipated in the dc circuit of an SMCT without feedback is approximately twice the value of the loss in the primary winding's copper.

4. With account taken of the magnetization losses, the efficiency of the prepared specimens of SMCTs, with power from 1 to 50 kv-a, lies between the limits of 0.85 to 0.95 for a normal load.

5. The form of the SMCT's secondary voltage curve changes during the control process. For a nominal I_d, the curve is practically a sinusoid; for the least value of I_d, the distortion coefficient for the prepared specimens amounted to 30 to 35%.

6. The gains and time constants of SMCTs have values of the same orders of magnitude as the saturable reactors of the corresponding power.

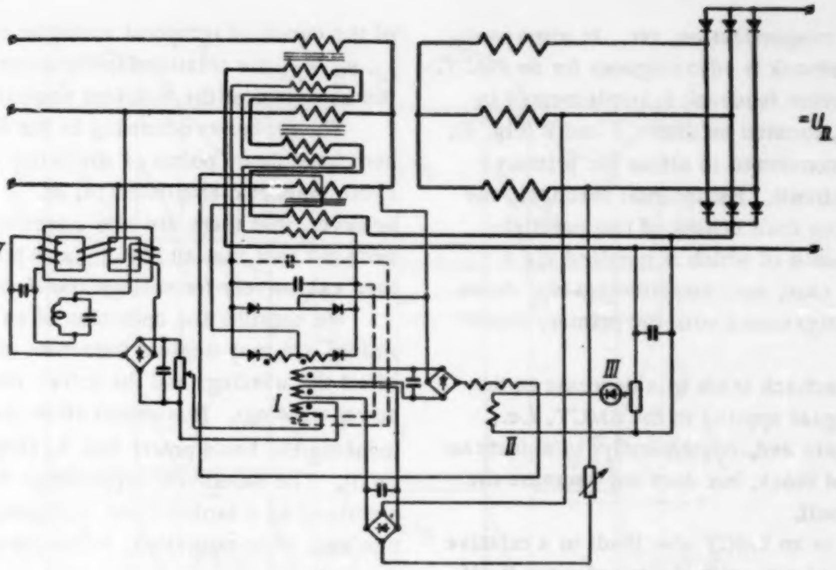

Fig. 4. Automatic control device for an electric drive with voltage stabilization and with current cutoff. I) and II) saturable reactors of the control scheme, III) stabilitron, IV) ferroresonant stabilizer.

7. Oscillations of the line frequency, just as for saturable reactors, have an insignificant effect on the SMCT properties.

8. The expenditure of active materials for an SMCT depends on its physical design, the depth of control and the character of the load. For an SMCT (Fig. 2) with control of U_{II} from zero to the nominal value, approximately, the expenditure of steel is greater than for an ordinary transformer by a factor of 1.8 to 1.9, while the expenditure of copper is greater by a factor of 2.2 to 2.5. In an SMCT (Fig. 3) intended for use as the executive organ of a voltage stabilizer, operating for deviations of the primary voltage within the limits of ± 12% from the nominal value, the expenditure of steel, in comparison with ordinary transformers, is higher by 30 to 35% and the expenditure of copper is higher by 45 to 50%.

The control schemes for automatic devices with SMCTs do not differ in principle from those used in devices with saturable reactors.

Figure 4 gives the scheme for the automatic control of an electric drive with voltage stabilization and with a sharp current cutoff; with overloading, a special device, cutting in a stabilitron, sharply lowers the secondary voltage.

2. Single-Phase SMCT With Feedback

In SMCTs, as in saturable reactors, the magnetization shunt can also be implemented by several individual circuits [4, 5, 6], and here also it is possible to have feed-

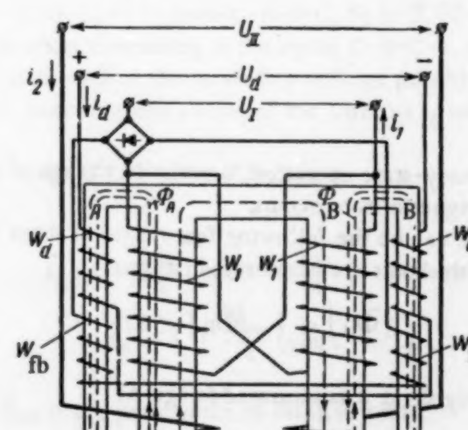

Fig. 5. SMCT scheme with external feedback.

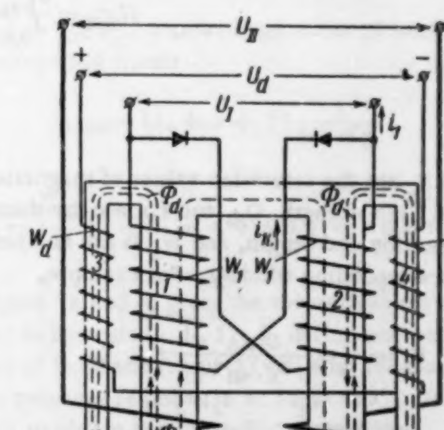

Fig. 6. SMCT scheme with internal feedback.

back windings, initial magnetization, etc. In some cases, the introduction of feedback is advantageous for an SMCT.

Here, external current feedback is implemented by auxiliary windings W_{fb}, located on shunts A and B (Fig. 5), and a rectifier bridge connected in either the primary or secondary winding circuit. For internal feedback, the SMCT's primary winding must consist of two parallel-connected halves W_1, each of which is supplied via a diode (Fig. 6). In this case, auxiliary windings W_{fb} shown in Fig. 5 could be amalgamated with the primary windings W_1.

Use of external feedback leads to a decrease in the power of the control signal applied to the SMCT, i.e., to an increase in the gain and, consequently, to a decrease in weight of the control block, but does not decrease the weight of the SMCT itself.

Internal feedback in an SMCT also leads to a relative decrease in the weight of copper in the transformer itself by approximately 15%. The expenditure of copper for the primary circuit of an SMCT with internal feedback increases by approximately 50% due to the fact that current flow in any of the W_1 windings only during a half a period and due to the decrease in the power factor. On the other hand, as was shown by investigations, the weight of the magnetization winding is decreased by a factor of from 5 to 8. We now give several parameters of an SMCT with internal feedback, the transformer power being 1.33 kv-a: weight of winding copper 21 kg, weight of steel 23 kg, dimensions 416 × 152 × 270 mm. The transformer was designed for explosion-proof usage.

3. Certain Elements of the Theory of Operation of a Single-Phase SMCT

We now give a mathematical analysis of the operation of an SMCT which will allow us to determine the form of the curves of temporal variation of the quantities i_1, i_2, i_d, u_2, Φ_A, the relationships between these quantities and the character of the transient response.

The processes occurring in the SMCT designs considered here have many points of similarity with those in single-cycle magnetic amplifiers [4, 5]. It should be mentioned, however, that there are also essential differences stemming from the fact that an SMCT has a powerful magnetic flux used exclusively for voltage transformation.

We consider the operation of an ideal SMCT, in which, one may neglect inductive impedance, dissipation of all the windings and the active impedances of the power windings. It is assumed that the magnetizing force inducing the basic power flux Φ_2 (Fig. 5) is negligibly small. The functional dependence $\Phi_{sh} = f(B_{sh})$ is characterized by a broken line: a sloping line until saturation and, after saturation, a line parallel to the axis of abscissas.

For the scheme of Fig. 5, we have the following basic equations of the ideal SMCT:

$$\bar u_1 = \bar i_1 \bar R_L + \frac{d}{d\tau}(\bar\Phi_A - \bar\Phi_B), \tag{1}$$

$$\bar u_d = \bar i_d \bar R_d + \frac{d}{d\tau}(\bar\Phi_A + \bar\Phi_B), \tag{2}$$

$$\bar i_d + \bar i_1 (1 \pm K_{fb}) = \bar i_A, \tag{3}$$

$$\bar i_d - \bar i_1 (1 \mp K_{fb}) = \bar i_B, \tag{4}$$

where $\tau = \omega t$ and $K_{fb} = W_{fb}/W_1$.

The quantities $\bar u_1, \bar u_d, \bar i_d, \bar R_d, \bar R_L, \bar\Phi_A, \bar\Phi_B, \bar i_1, \bar i_A, \bar i_B$ are given in relative units, the base values adopted being

$$\Phi_{bas} = B_s Q_{sh} \quad U_{bas} = \omega W_k \Phi_{bas}\cdot 10^{-8}, \quad I_{bas} = \frac{H_s l_{sh}}{W_k},$$

$$R_{bas} = \frac{U_{bas}}{I_{bas}} = \omega \frac{B_s}{H_s}\frac{Q_{sh}}{l_{sh}} W_k^2 \cdot 10^{-8},$$

where B_s and H_s are the saturation values of magnetic induction and field strength, Q_{sh} and l_{sh} are the shunt's active cross section and length, and W_k is the number of turns of the corresponding winding. For example,

$$\bar u_1 = \frac{u_1}{\omega W_1 B_s Q_{sh}\cdot 10^{-8}},$$

$$\bar i_d = \frac{i_d}{I_{basd}} = \frac{i_d W_d}{H_s l_{sh}},$$

$$\bar R_L = \frac{R_L}{R_{bas\,2}} = \frac{R_L}{\omega \frac{B_s}{H_s}\frac{Q_{sh}}{l_{sh}} W_2^2 \cdot 10^{-8}},$$

$$\bar\Phi_A = \frac{\Phi_A}{\Phi_{bas}} = \frac{\Phi_A}{B_s Q_{sh}} = \frac{B_A}{B_s}.$$

In steady-state operation, a periodic change of the shunt's magnetic state occurs.

There can be the following four stages of shunt states:

1) both shunts are unsaturated (stage C)

$$|\bar\Phi_A| < 1, \quad |\bar\Phi_B| < 1;$$

2) only shunt A is saturated (stage A)

$$|\bar\Phi_A| = 1, \quad |\bar\Phi_B| < 1;$$

3) only shunt B is saturated (stage B)

$$|\bar\Phi_A| < 1, \quad |\bar\Phi_B| = 1;$$

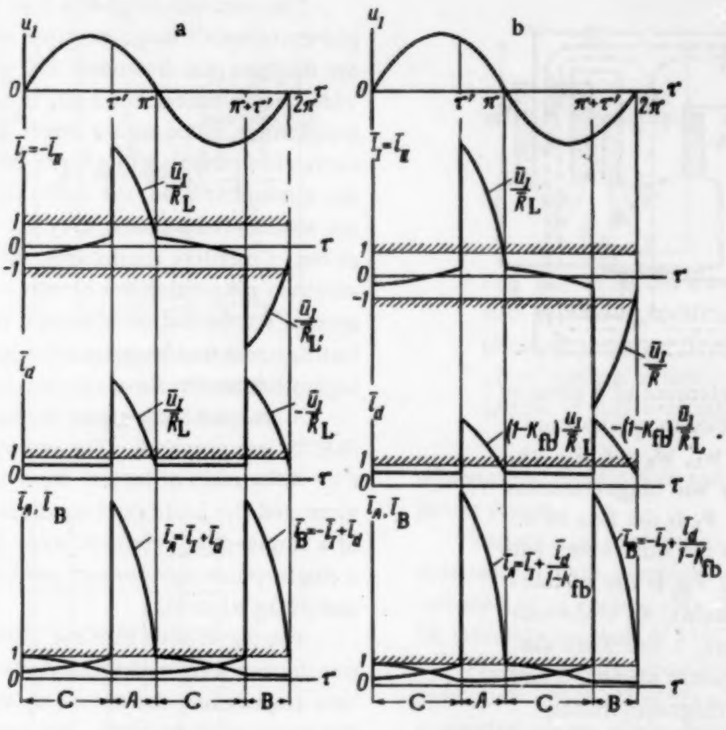

Fig. 7. Curves of the temporal variation of $\bar{i}_1$, $\bar{i}_2$, $\bar{i}_d$, $\bar{i}_A$, $\bar{i}_B$ during one period (the cycle C-A-C-B-C); a) without feedback ($K_{fb} = 0$) and b) with feedback ($K_{fb} = 0.5$).

4) both shunts are saturated (stage H)

$$|\bar{\Phi}_A| = 1, \quad |\bar{\Phi}_B| = 1.$$

As a function of the magnitude of the primary voltage and the magnetizing current, the following cycles of stages are possible during a period: 1) C-C, 2) C-A-C-B-C, 3) C-H-C-H-C, 4) C-H-A-C-H-B-C, 5) H-H.

The most interesting is the cycle C-A-C-B, since only with it is control of the secondary voltage possible. For this, the instantaneous values of the currents $\bar{i}_1$ and $\bar{i}_d$ equal:

for stage C C($0 < \tau < \tau'$)

$$\bar{i}_1 = -\frac{\bar{U}_m}{2}\cos\tau + \frac{\bar{U}_m}{4}(1+\cos\tau'),$$

$$\bar{i}_d = 1 + \frac{\bar{U}_m}{4}(\cos\tau' - 1),$$

where $\bar{U}_m$ is the peak value in relative units;

for stage A ($\tau' < \tau < \pi$)

$$\bar{i}_1 = \frac{\bar{U}_m}{\bar{R}_L}\sin\tau,$$

$$\bar{i}_d = 1 + \frac{\bar{U}_m}{\bar{R}_L}(1-K_{fb})\sin\tau + \frac{\bar{U}_m}{\bar{R}_L}(\cos\tau' - 1),$$

where τ' is the moment when stage C is replaced by stage A. These expressions are obtained from the condition that $\bar{R}_L < 0.1$. It can also be shown that

$$\cos\tau' = \pi\frac{\bar{I}_d - 1}{\bar{U}_m}\bar{R}_L - 1,$$

where $\bar{I}_d = \bar{U}_d/\bar{R}_d$. The average value of load current over one half-period equals

$$\bar{I}_{1,\,av} = (\bar{I}_d + a + 1)\frac{1}{1-K_{fb}},$$

where

$$a = \frac{\bar{U}_m}{2\pi}(1 + \tau' - \pi + \sin\tau' + \pi\cos\tau'). \tag{5}$$

Figure 7a and b, gives the theoretical curves of the changes in currents $\bar{i}_d$, $\bar{i}_1$, $\bar{i}_A$, $\bar{i}_B$ during one period in the absence of feedback ($K_{fb} = 0$) and with feedback ($K_{fb} = 0.5$). For the transient response in an ideal SMCT, it is not difficult to obtain the following expressions:

$$\bar{I}_d = \bar{I}_{d,\,av} + \frac{\pi}{2}\frac{\bar{R}_L}{\bar{R}_d}\frac{1}{1-K_{fb}}\frac{d\bar{I}_{d,\,av}}{d\tau},$$

$$\bar{I}_d\frac{1}{1-K_{fb}} = \bar{I}_{1,av} + \frac{\pi}{2}\frac{\bar{R}_L}{\bar{R}_d}\frac{1}{1-K_{fb}}\frac{d\bar{I}_{1,\,av}}{d\tau}.$$

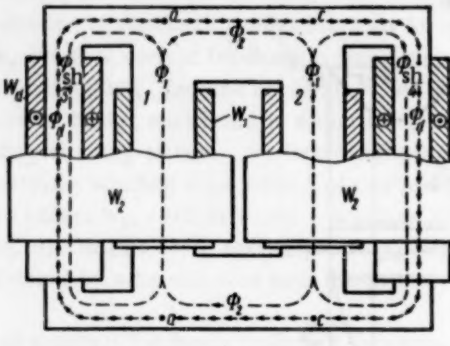

Fig. 8. Single-phase element of a grouped transformer with matched magnetization winding connections. W_1, W_2 and W_d are the primary, secondary and magnetization windings respectively, Φ_1 is the flux in the primary winding, Φ_2 is the flux linked with the secondary winding, Φ_{sh} is the variable flux of the magnetic shunts, Φ_d is the constant magnetization flux, 1 and 2 are the cores of the basic magnetic circuit, 3 and 4 are the cores of the magnetic shunts.

From whence it follows that an ideal SMCT, just as an ideal magnetic amplifier, is a first-order aperiodic link with time constant

$$T = \frac{\pi}{2}\frac{\bar{R}_L}{\bar{R}_d}\frac{1}{1-K_{fb}}. \qquad (6)$$

4. Grouped Three-Phase SMCTs

In previous publications [2] there were described three-phase variants of the SMCT. We now communicate information on a grouped transformer, developed by the authors, with three individual single-phase SMCTs of a new type (Fig. 8). The grouped transformers are intended for the smooth control of three-phase voltage for symmetrical phase loads, for example, for supplying rectifier stands. Voltage control, as in the SMCT of Fig. 3, is implemented by changes in current I_d flowing through the series-connected magnetization windings W_d.

In the new single-phase SMCT, both windings W_d are so connected that the constant magnetization flux Φ_d virtually doesn't flow through cores 1 and 2 of the basic magnetic circuit, but is closed only through the loop formed by the cores of the shunts and the frame. It is necessary for this that the currents I_d be directed in windings W_d as shown in Fig. 8.

It is easily concluded that the basic cores of the new variant of the SMCT must have a smaller cross-sectional area than in the case of the design shown in Fig. 3. Thanks to this, there is also a relative decrease in the expenditure of the active materials – copper and steel.

In designs of SMCT intended for profound voltage control, higher than 50%, the economy in copper weight amounts to 20%.

The variable magnetic flux induces, in the single-phase element's magnetization circuit, a resulting emf at the fundamental frequency and with higher harmonics. This circumstance, however, is admissible for a grouped transformer, since with a series connection of the magnetization circuits of its three component elements and the symmetric load, the emf's of the fundamental harmonics are annihilated because they are equal and 120° out of phase. Complete annihilation of the variable emf in a common magnetization circuit is given by the use of special compensation windings, placed on the shunts and forming a closed loop around which the currents at the higher harmonics flow.

A grouped three-phase transformer of this type with SMCTs was prepared. The test specimen had a power of 6 kv-a, the ratio of lateral cross-sectional areas of the shunt and the basic core equalled 0.83. The dimensions of a single-phase element were 355 × 120 × 335 mm, with a single-phase transformer containing 21 kg of copper and 22 kg of steel.

Testing showed that the characteristics of the grouped transformer were satisfactory. For a constant nominal load impedance, the secondary voltage was varied within the limits of 22 to 100%. For nominal values of load and current I_{d_N}, the value of cos φ was 0.93, and the efficiency in this case, with losses, in the magnetization winding taken into account, was 91%.

The form of the curve of line voltage U_2 depends on the magnitude of the load impedance and on the relative value of the magnetization current. For their nominal values, the distortion in the curve's form was small.

5. Static Transformers of Numbers of Phases with SMCTs

A common fault in the executive organs of static devices serving for the transformation of single-phase current to three-phase or two-phase is that variations in the load parameters, as well as the supply net regimen, lead to the appearance of asymmetries in the multiphase system of voltages obtained. If we take, as the criterion for judging the operation of a transforming device, the asymmetry coefficient of this system of voltages, we obtain the following expressions for the transformer of single-phase voltage to three-phase (Fig. 9a) and to two-phase (Fig. 9b):

$$\dot{\varepsilon}_U = \frac{\dot{U}_2}{\dot{U}_1} = \frac{2g - \frac{1}{\sqrt{3}}(b_L + b_C) - j\,(2b + b_L - b_C)}{2g + \frac{1}{\sqrt{3}}(b_L + b_C) - j\,(2b + b_L - b_C)}, \qquad (7)$$

$$\dot{\varepsilon}_U = \frac{\dot{U}_2}{\dot{U}_1} = \frac{(g - b - b_L) - j\,(g + b - b_C)}{(g - b + b_L) + j\,(g - b + b_C)}, \qquad (8)$$

where g is the active component of conductance of one phase of the load, b is the reactive component of conductance of one phase of the load, b_L and b_C are the inductive and capacitive conductances of the transformer arms.

Figure 10 shows, for a transformer of single-phase to three-phase current, the characteristic $\epsilon = f(S/S_L)$, com-

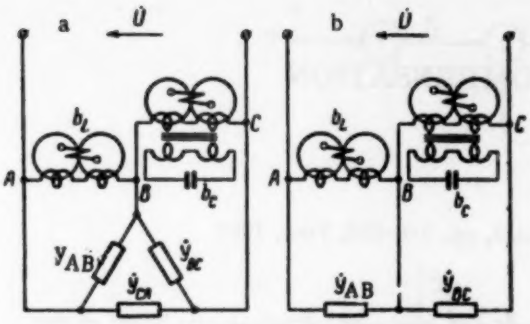

Fig. 9. Basic functional transformer schemes. a) transformer of single-phase current to three-phase, b) transformer of single-phase to two-phase current.

Fig. 10. Transformer device characteristics. 1) Curve of asymmetry coefficient variation; 2) relative change of induction for $\cos\varphi = 1$; 3) relative change of capacitance for $\cos\varphi = 1$; 4) relative change of induction for $\cos\varphi = 0.8$; 5) relative change of capacitance for $\cos\varphi = 0.8$.

puted for the relative variation of apparent power of the active-inductive load with $\cos\varphi = 0.8$, from which it follows that, as the apparent power deviates from its nominal value, the asymmetry of the voltage system rapidly increases (curve 1) and goes beyond the limit of the admissible value of $\epsilon = 5\%$. An analogous phenomenon also occurs in the transformer of single-phase current to two-phase. With variations in the line frequency, there also occurs a rapid increase in asymmetry of the multi-phase voltage systems.

As shown by analysis, to stabilize the symmetry of a multi-phase voltage system, it is required to control the parameters of the elements of the executive organ – the transformer device. Curves 2-5 of Fig. 10 show what relative values must be possessed by the inductance and capacitance of the arms of a transformer of single-phase to three-phase current for variations of the relative values

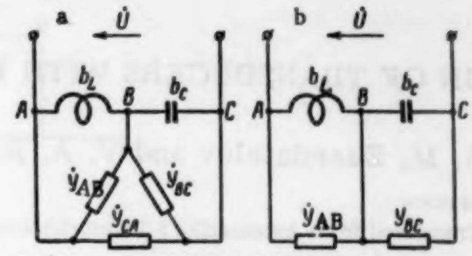

Fig. 11. Modified transformer schemes with symmetry stabilization of the multi-phase systems. a) Transformer of single-phase current to three-phase; b) transformer of single-phase current to two-phase.

of apparent load power and for different values of its power factor.

Variation of parameters can be implemented automatically† if the elements of the executive organ are constructed of controllable devices. Such elements might be saturable reactors or SMCTs.

Figure 11 shows modifications of the circuits given previously which allow them to be transformed to controller executive organs of transformers of numbers of phases with automatic stabilization of secondary voltage symmetry. In the transformer whose circuit is shown on Fig. 11a, a saturable reactor is used as the inductive arm. In the capacitive arm, control of the reactive power of the condenser battery is implemented by varying the voltage on its terminals, thanks to its connection via an SMCT.

The scheme of Fig. 11b, is a modification of the transformer of single-phase current to two-phase.

Test results for a trial device of 1.5 kv-a power showed that the value of the asymmetry coefficient is less than 5% for a fourfold change in the impedance of the load circuit and a change in the power factor between the limits of unity and 0.6. These results were also obtained for changes in voltage and frequency of the supply line within ± 20%.

In conclusion we note that the executive organs – transformers with automatic stabilization of output voltage symmetry – can be constructed for significant output power.

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DESIGN OF TRANSDUCERS WITH FORCE COMPENSATION

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The paper considers questions in the design of pneumatic transducers and dc electrical transducers based on the principle of force compensation. Considerations are adduced as to the choice of the instruments' functional schematics and physical designs. The causes are established for the appearance of instrument errors: nonlinearity errors in the transformation of a measured parameter to an output signal, sensitivity thresholds, temperature errors, and errors from changes in the static pressure of the medium. Computational formulas are provided, and recommendations are given for decreasing the magnitude of the errors.

INTRODUCTION

In the design of industrial instruments, ever greater use is being made of the force compensation method, the essence of which is that an input signal to be measured is transformed by the instrument's sensitive element to a force, and this force is equalized (compensated) by a force developed in the instrument at the cost of some energy from an external source. The equalizing force is proportional to the instrument's output signal. Thus, the input quantity is transformed to an output signal.

In those cases when both the input quantity and the output signal of the instrument are transformed to forces simply and without significant errors, the use of force compensation allows one to obtain small values of error, in particular, those components of the error which are engendered by changes in the ambient conditions and by imperfections and instability of the instrument's individual elements. Therefore, despite some complexity of the instrument's physical design, devices using force compensation have been widely employed for measuring, and transforming to compressed air pressure or to dc force, such quantities as pressure, pressure drop, density, level of liquid, and temperature. Force compensation has also been widely used in pneumatic and electrical transformers, automatic controllers and calculating devices.

In the development of such devices, it should be kept in mind that, in addition to reliability, stability of operation, simplicity of maintenance and low cost – mandatory properties for industrial instruments – the functional scheme and the physical construction must provide the capability of actually attaining the good static and dynamic accuracy which are the basic virtues of force compensation.

Creation of high-accuracy instruments is significantly facilitated if one knows the dependence of the various components of error on the instrument's physical (design) parameters and if one could compute, if only approximately, the probable magnitude of the error. Questions of design and construction of instruments with force compensation have received little treatment in the literature. Here, we make the attempt to consider certain of these questions as they appertain to industrial pneumatic transducers and dc electrical transducers. Formulas are given, without derivations, which define the instrument accuracy as a function of the design parameters. These formulas can be used for instrument design.

1. Basic Errors

The static error of force compensation instruments is made up, basically, of the following components:

1. Errors in the realization of the given functional relationship of the output signal to the quantity to be measured. Since this relationship must be a linear one in the majority of cases, these ordinarily are called nonlinearity errors.

2. The error caused by friction losses in mechanisms and hysteresis losses in elastic elements and magnetic systems. It determines the threshold of sensitivity and the variation.

3. Temperature errors, expressing themselves in a shift of the range of the output signal and in changes in the magnitude of this range as the instrument's temperature changes. The second component of these errors in compensation transducers is ordinarily significantly less than the first.

4. An error from changes in the static pressure of the medium to be measured. It occurs at points where the force arising from the sensitive element is derived from the medium to be measured in the atmosphere via an elastic discriminating element, and when a change in static pressure gives rise to a change in the force on the sensitive element (for example, in the case when fluid level is measured by a float).

5. Errors from changes of the supply source parameters (pressure of the air supplied to the instrument, electrical line voltage).

6. Errors from "aging" of the instrument's elements leading to changes of the instrument's indications with the

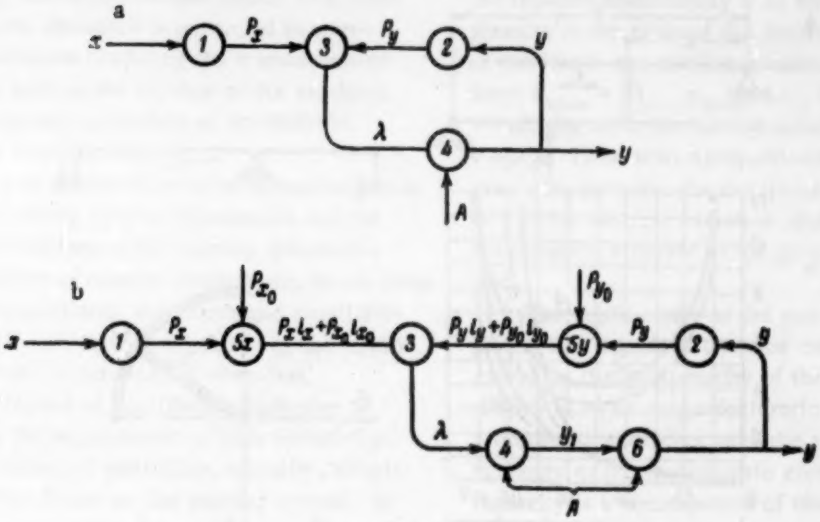

Fig. 1.

passage of time. Although this error component can be eliminated by periodic calibration of the instrument, its appearance, under actual conditions of use, should always be considered possible.

A dynamic error appears in the form of pulsations of the output signal and a displacement of its average magnitude which arise both from external jars and vibrations and from auto-oscillatory modes in the closed compensation system. Moreover, a dynamic error can appear in the form of a lag in changes of the output signal after changes in the quantity to be measured, or in the form of distortions of amplitude, phase or form of the output signal for a periodic change of the quantity to be measured.

2. Choice of Instrument Scheme

Compensation measurement of forces can, in a first approximation, be presented by the following functional schematic (Fig. 1a). The quantity $\underline{x}$ to be measured is transformed by sensitive element 1 to the force P_x which is compared with force P_y of compensation element 2, proportional to the output signal $\underline{y}$.

The difference of these forces disturbs the equilibrium of movable system 3 and creates a translation λ of it. This latter is applied to an equilibrium indicator – transformer 4, which also generates the output quantity $\underline{y}$, obtaining energy A from an external source.

To any range of variation of the quantity to be measured there must correspond a definite range of variation of the output signal. In the scheme being considered, this correspondence is defined only by the parameters of the sensitive and the compensation elements. For this there is frequently used a lever transfer mechanism with a definite transfer ratio $\underline{i}$. On the schematic (Fig. 1b), this mechanism is denoted by 5x and 5y. Since the physical design of such instruments is already relatively complex in itself, it is ordinarily considered worthwhile to introduce some additional complication in the form of a mechanism with a variable (adjustable) transfer ratio, permitting the transducer to be tuned for different ranges of variation of the quantity to be measured for some adopted standard range of the output signal. To the mechanism there are also applied the constant (adjustable) forces P_{x_0} and P_{y_0}, introduced for the formation of the nonzero range of variation of the measured and output quantities.

As a rule, to increase the output signal's power, the output quantity $\underline{y}$ is not taken off directly from the equilibrium indicator – transformer 4, but from power amplifier 6 which, obtaining some quantity y_1 from the indicator, amplifies its power and transforms it to the quantity $\underline{y}$.

The functional schematic and the physical configuration of the transfer mechanism and, in particular, the mechanism for adjusting the transfer ratio, determine to a large extent the physical design of the entire instrument.

In the majority of industrial instruments, a two-lever mechanism (Fig. 2a) is used in which the force P_x to be measured creates a torque on one lever, while the compensating force P_y creates a torque on the other. The transfer ratio is determined by the position of the connection 1, executed in the form of a shaft or a slider, transmitting force from one lever to the other. Curve $\underline{i}$ in Fig. 2b shows the transfer ratio $i = a/(l - a)$ as a function of the connection position. The curve s_i characterizes the so-called tuning sensitivity $s_i = (di/l)/(da/l) = l^2/a(l - a)$ which is relatively small for middle positions of the connection and increases sharply as the connection is shifted toward the bearings. From the curve for i^2 one can determine the complete change of the transfer ratio for identical limiting positions of the slider on both sides of the middle position.

The "one-lever" scheme (Fig. 2c) used in many instruments has the same tuning characteristic as the two-lever one.

The majority of such mechanisms are designed for changes of the transfer ratio by a factor of from 6 to 10. Greater variations of the transfer ratio are rarely employed since they render difficult the tuning of the necessary

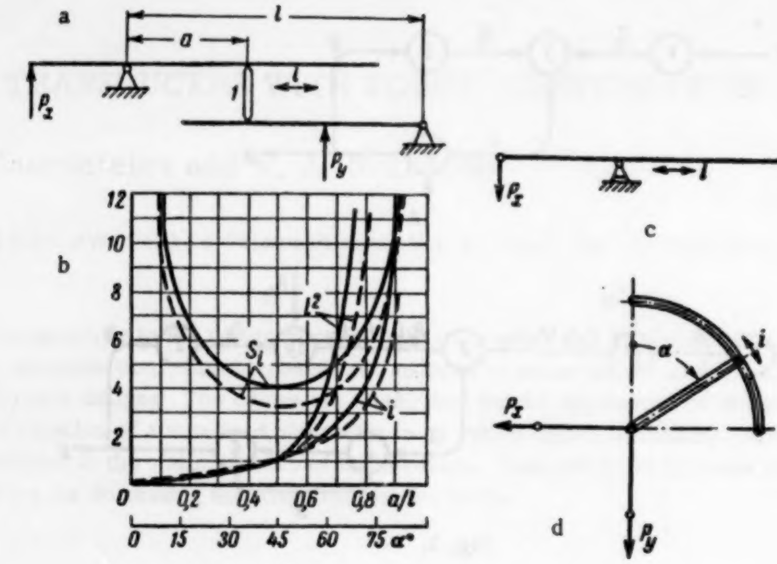

Fig. 2.

range, and lower the stability of instrument operation for slider positions close to the bearings where the sensitivity of the tuning is too great.

Comparatively recently, use began to be made of the so-called "vector" mechanism (Fig. 2d), consisting of three shafts hinged together. Two shafts, transmitting the forces P_x and P_y are at right angles to each other. The end of the third shaft is hinged to the frame of the instrument. Change of the angle α at which this shaft is established is implemented by the tuning of the mechanism's transfer ratio. The sensitivity of tuning of this mechanism, $s_i = \pi/\sin 2\alpha$, in the ordinarily used middle region, is less than in the lever schemes. The curves s_i, $\underline{i}$ and i^2 for this mechanism are given by the dashed lines in Fig. 2b.

Tuning of the transfer ratio is sometimes implemented also by shifting the compensation element, by replacing it or by changing the transfer ratios of auxiliary levers. Such rough (step-wise) tuning, in connection with a smooth tuning, allows one instrument to be used, in wide range of variation of forces, which is an undeniable advantage in use. However, this may decrease the accuracy due to the difficulty of creating a mechanism which is sufficiently sensitive for small measured forces and, at the same time, is strong and reliable in measuring large forces.

In choosing the mechanism scheme, great importance inheres in the positioning of the equilibrium indicator. In existing instruments it is placed either before the tuning mechanism in the sensitive element or after this mechanism in the compensation element. As applied to a two-lever mechanism, these variants of indicator positioning are shown in Fig. 3a and b.

The second variant is more convenient to build, since it allows implementation of the equilibrium indicator, compensation element and other assemblies to be joined in one device with a lever for receiving forces from the transfer mechanism. This device is easily unified and a

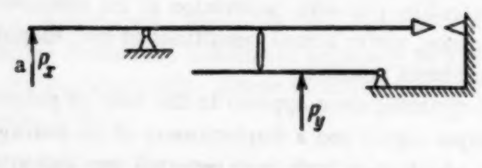

Fig. 3.

single physical realization of it can be used for different instruments: manometers, differential manometers, level meters, hardness meters, thermometers and others, so long as the parameter to be measured is transformed to a force of a definite magnitude. The entire range of forces generated by the sensitive elements of different instruments may be transformed to the output signals of a small number of standard size devices of this type.

The first variant gives a physical design which is less convenient for unification, since here the equilibrium indicator is directly, ordinarily via a lever, connected with the sensitive element. The advantage of this scheme amounts to this, that inreturning the range of forces to be measured, the sensitive element keeps invariant its position which it was given when the instrument was regulated. This manifests itself favorably in the accuracy when there are changes in static pressure and when there are oscillations of the ambient temperature. Moreover, it is easier here to obtain small translations of the sensitive element, which also makes the instrument capable of high accuracy in its operation.

It is necessary to bear in mind that all actual instruments implement force compensation only with some approximation, since only a part of the force to be meas-

ured is equalized by the compensation force. The other part of the force to be measured is expended in overcoming various resistances hindering the translations of the moving system, such as the rigidity of the sensitive and compensation elements, rigidity of the flexible bearings, as well as frictional forces.

Naturally, the part of the force to be measured which is expended in overcoming elastic impedances will be decreased as the translations of the moving system decrease. In the majority of modern instruments, use is made of high-sensitivity equilibrium indicators and amplifiers which provide a total operational excursion of the equilibrium indicator of tens of microns, or even less.

Of course, the choice of equilibrium indicator is subject, not only to the requirement of high sensitivity, but also to considerations of reliability, stability, simplicity, and low reaction forces on the moving system. In penumatic instruments, nozzle-flapper type transformers are used almost exclusively while, in electrical instruments, induction, capacitance, contact and other types of transformers are used.

The choice of the basic dimensions of the transfer mechanism, the sensitive and the compensation elements, is determined by the necessity of obtaining a given accuracy, by the magnitude of the forces to be measured, and by requirements for small dimensions.

From the point of view of accuracy, apparently, it is advantageous to have large forces developed by the sensitive element: the larger these forces, the less will be the deleterious effect of rigidity of the elastic elements, frictional forces and additional forces giving rise to errors with changes in the medium's static pressure.

3. Nonlinearity Error

This error is made up of the nonlinearities of the transformations to forces of the quantity to be measured and the output signal, and the nonlinearities which occur in compensation.

The nonlinearity created by the sensitive elements is small as a rule, since these are hardly translated at all under conditions of force compensation. No nonlinearities at all arise from such devices as buoys, bells, and other forms of sensitive elements of fluid differential manometers. In practice, the nonlinearities created by tubular manometric springs and bellows are negligible.

Noticeable nonlinearities can appear upon changes in the effective areas of membranes, in particular, elastic ones. For example, let the effect area f vary proportionately to the pressure p by the function $f = f_0 (1 + np)$, where f_0 is the effective area of the unloaded membrane and the product of the pressure p by the coefficient n represents the relative change of effective area with changes in pressure, since $np = (f - f_0)/f_0$. Then, the force $P = pf$ on the membrane will depend nonlinearly (quadratically) on changes of pressure. By comparing it with a linear change of force with pressure $P^* = pf_{max}$, where $f_{max} = f_0 (1 + np_{max})$ and defining, as is usually done, the relative nonlinearity η of force as a function of pressure as the ratio of the greatest difference $(P^* - P)_{max}$ of the linear and nonlinear functions to the maximum force $P_{max} = f_{max} p_{max}$, i.e., as $\eta = (P^* - P)_{max}/P_{max}$, we obtain, after the corresponding transformations, $\eta = np/4$. Thus, with a proportionate change of effective area with pressure, the nonlinearity of the force equals 1/4 of the relative variation of the effective area of the manometric element in the given range of pressure variation.

The nonlinearity of the compensation elements of penumatic transducers can be compensated to a certain extent by the nonlinearity of the manometric sensitive elements. The magnetoelectric force mechanisms of electrical transducers have the same character of nonlinearity as the manometric elements. Here, the nonlinearity is a consequence of changes in induction of the magnetic field due to the magnetizing or demagnetizing action of the compensation current, which is particularly marked for large currents. Ferrodynamic compensation elements can also provide a significant nonlinearity.

The nonlinearity in the compensation of forces is a consequence of the conjunction of the nonlinearity of the equilibrium indicators' characteristics and the previously noted incompleteness of compensation when part of the force to be compensated is expended in overcoming the resistance of flexible elements. If we denote this part of the force P_x to be measured by P_z, we can then express the relative nonlinearity $\eta = \eta_{P_x - P_y}$ of the force to be measured to a compensating force as $\eta_{P_x - P_y} = \xi \eta_{\lambda - y}$, where $\xi = P_z/P_x$ is the coefficient of noncompensation and $\eta_{\lambda - y}$ is the nonlinearity of the equilibrium indicator; the nonlinearity of the compensating element is not taken into account here.

The functional relationship of the nonlinearities and the design parameters of the instrument can be considered in greater detail by setting up the equilibrium equation of the moving system. For this, by means of the transfer ratios i_x and i_y, we relate the force P_x to be measured and the compensating force P_y to the equilibrium indicator. We then relate to this same place the total force $T = \sum_n k_n \lambda_n i_n$, generated by the flexible elements with rigidities k_n and translations λ_n. It is also necessary to take into account the additional forces $Q_x = \sum_{n_x} k_{n_x} \delta_{n_x} i_{n_x}$ and $Q_y = \sum_{n_y} k_{n_y} \delta_{n_y} i_{n_y}$ of the flexible elements, from the sensitive element up to the equilibrium indicator and from the equilibrium indicator up to the compensation element, these forces arising due to the deformations δ_n of the instrument mechanism. We then obtain

$$P_x i_x - Q_x = P_y i_y - Q_y + T$$

or

$$P_x i_x - \sum_{n_x} k_{n_x} \delta_{n_x} i_{n_x} =$$

$$P_y i_y - \sum_{n_y} k_{n_y} \delta_{n_y} i_{n_y} + \sum_n k_n \lambda_n i_n.$$

We introduce the relationship between the translations λ and the output signal y for a concrete equilibrium indicator and amplifier, obtained by computations or selected experimentally, in the form, for example,

$$\lambda = \alpha_1 y + \alpha_2 y^2 + \alpha_3 y^3$$

and we then compute the magnitude of the deformations δ of the mechanism. Then, by assuming a linear relation between the output signals and the compensation forces, we can obtain an equation of the form

$$P_x = a_1 P_y + a_2 P_y^2 + a_3 P_y^3.$$

Investigation of the nonlinearity of this functional dependency shows that, to decrease it, one should choose the largest possible linear working segment of the equilibrium indicator's characteristic, should decrease the magnitude of its total excursion λ_{max}, should decrease in every way the rigidity of the flexible elements (sensitive and compensation flexible bearings, elastic output) and, conversely, should increase the rigidity of the levers and connections of the moving system and the instrument frame.

4. Sensitivity Threshold

Friction in the bearing of the moving system can give a significant increase to the sensitivity threshold. In view of the small translations of the mechanism, it is advantageous to use flexible bearings, for example, in the form of cruciform belt supports. Knife and ball bearings, although simpler physically, can be sources of significant friction, particularly when the manufacture and mounting are of inadequate quality.

Large frictional forces arise at the place of contact of the slider of the range regulator and the levers, and these are particularly noticeable in instruments with small forces to be measured. Therefore, from the point of view of frictional losses in the connections between the levers, it is preferable to employ flexible belting, despite the great complexity of its fabrication and of instrument calibration.

Serious attention must be given to the construction of the connection of the mechanism with the sensitive and compensation elements. From the point of view of minimal frictional losses, a rigid attachment is the best. But, in the majority of cases, such a solution leads to a significant increase in rigidity, so that flexible connections are more frequently used. It is also necessary to bear in mind that significant additional translations of the moving system can appear if the belting connection is only a little twisted and there has not been, with this, sufficient preparatory tightening.

In view of the smallness of the working translations of the moving system, hysteresis of the flexible elements is negligible in practice. However, with significant deviations of the moving system from the equilibrium position, for example, for overloaded instruments, the variations can attain significant magnitudes.

If one relates to the equilibrium indicator, and adds, all the forces P^0 which must be applied to each element of the instrument in order to eliminate hysteresis corresponding to the total possible excursion of the moving system, then one can calculate the instrument's sensitivity threshold η^0 as the ratio of this total related hysteresis force $\sum_n P_n^0 i_n$ to the total force $P_{x\,max}$ to be measured, also related to the equilibrium indicator

$$\eta^0 = \frac{\sum_n P_n^0 i_n}{P_{x\,max} i_x}.$$

Ferrodynamic force mechanisms have large values of hysteresis. The most radical measure in the fight to overcome these is to choose materials with small magnetic losses and to prepare the magnetic systems of thin sheet material or sintered powders.

5. Temperature Error

Just as the nonlinearity error, this error is made up of the errors of the sensitive element, the compensation element, and the error of the force-equilizing mechanism.

Temperature changes of the manometric elements lead to changes in their effective areas, which can lead to force changes which are very noticeable when elastic nonmetallic membranes are used. Therefore, serious attention should be given to the temperature stability of the effective area of such membranes.

In differential manometric transducers there have been used complicated physical designs of the sensitive elements, protected from overloads, consisting of two and more membranes with liquid charges. Increase of liquid pressure due to its expansion from temperature rises can be the cause of large temperature errors, and sometimes such devices have special temperature compensators.

Noticeable errors can be generated by magnetoelectric, electromagnetic and ferrodynamic mechanisms due to temperature variations, of magnetic induction and magnetic permeability. Here too, thermal compensators are frequently used.

Temperature variations in the dimensions of the mechanism assembly with rigidity present gives a component η_λ^t of the temperature error which can be computed from the formula

$$\eta_\lambda^t = \xi \frac{\Delta\lambda_t}{\lambda_{max}},$$

where $\Delta\lambda_t$ is the calculated magnitude of the temperature excursion of the equilibrium indicator, λ_{max} is the total working excursion of the equilibrium indicator and ξ is the noncompensation coefficient.

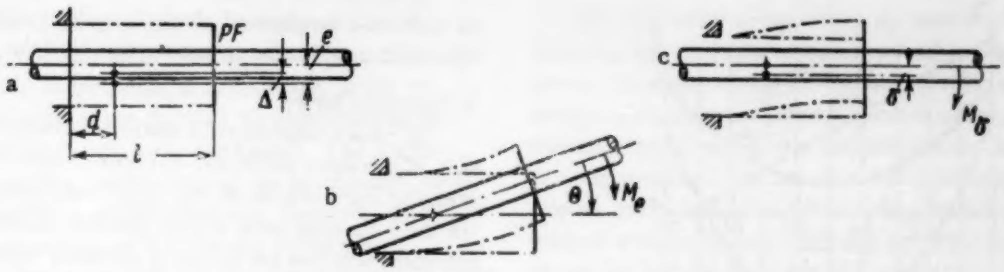

Fig. 4.

Temperature variations of the modulus of elasticity of the flexible elements lead to the appearance of temperature errors due to changes in the strain forces of these elements. The relative error per degree change in temperature is calculated as the ratio of the algebraic sum of the temperature changes of the forces of the elastic elements, referred to the equilibrium indicator, $\sum_n \gamma_n P_n i_n$, to the total force $P_{x_{\max}}$ to be measured, i.e.,

$$\eta_\gamma^t = \frac{\sum_n \gamma_n P_n i_n}{P_{x_{\max}} i_x}.$$

Here, γ_n are the temperature coefficients of the moduli of elasticity of the individual flexible elements.

One should not lose sight of the possibility of temperature errors arising due to nonuniform heating of the different parts of the instrument and to their deformation by, for example, one-sided heating close to a heat source. For protection against this, it is advantageous to provide the instrument casing with a layer of heat insulation and to use coverings for it with low absorption coefficients.

For the first heating-cooling cycles of the instrument there is not infrequently observed an irreversible shift of the range due to relaxation forces and the associated deformations of the various assemblies. The most radical method of combating this is to remove the overstrained and temperature-mechanically-aged assemblies and components of the transducer. This is a most desirable measure to take, since it renders possible the improvement of the instrument's stability of operation in the course of time, and this index of accuracy of instrument operation is one of the most important of its use characteristics.

It is necessary to mention that the temperature error is the most difficult to eliminate because of the incomplete homogeneity of properties of the materials employed. With the necessity of obtaining small temperature errors, adjustable temperature compensation elements have been introduced (for example, in the equilibrium indicator) although temperature calibration of the instrument is entailed by this.

6. Errors from Changes in the Medium's Static Pressure

Derivation of the sensitive element's force from the medium to be measured is implemented, in the majority of compensation differential manometers, level meters and hardness meters, by means of a pivoting lever with compression of a bellows or a membrane. Despite the complexity of its physical design, a bellows derivation device is more frequently used, in view of its significantly lower rigidity.

The error of bellows extraction due to changes in the medium's static pressure is related both to the imperfections of its fabrication (which may be corrected by calibration) and with the unavoidable deformations of the extraction components with instrument operation; this part of the error cannot be eliminated by calibration, but can be lessened by the proper choice of the design parameters of the extraction device.

Among the first group of causes is noncoincidence of the line of action of the resultant pressure force on the bellows head with the center of vibration of the lever and the bellows' torsion axis. With this, a torque acts on the lever which is propotional to the static pressure; it also gives rise to an instrument error.

The various design methods of compensating this torque by instrument calibration must be implemented by at least one of the following kinematic operations:

1. Translation of the lever's center of rotation with respect to its axis by the amount Δ (Fig. 4a), which decreases the initial eccentricity.

2. Rotation of the lever, together with the bellows head, by the angle θ (Fig. 4b) which creates, by virtue of the torsion of the bellows, a torque M_θ, proportional to the static pressure.

3. Displacement of the extraction lever together with the bellows head and the bearings (Fig. 4c). With this, as in the previous case, there appears a torque M_δ proportional to the static pressure. The torques M_θ + + M_δ can compensate the torque giving rise to the instrument error.

Their magnitudes, determined by the methods of strength of materials, can be computed from the formulas

$$M_\theta = pFlm_\theta \theta \text{ and } M_\delta = pFm_\delta \delta.$$

Here, p is the medium's static pressure, F is the effective area, and l is the length of the extraction bellows, and

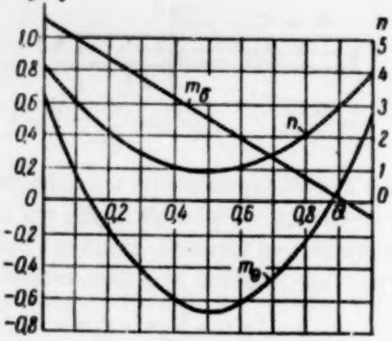

Fig. 5.

the coefficients

$$m_\theta = \frac{2}{15}(1 - 9\alpha + 9\alpha^2)$$

and

$$m_\delta = \frac{11 - 12\alpha}{10}, \text{ where } \alpha = \frac{a}{l}$$

(Fig. 4a). The functional relationship of m_θ and m_δ on the position of the bearings along the length of the bellows is given in Fig. 5, which provides a representation of the effectiveness of these calibration steps.

The second group of causes of errors from static pressure results from a displacement of the extraction lever's center of rotation with respect to the line of action of the pressure's resultant force on the bellows head due to the deformations of the lever, bearings and instrument frame under the action of the static pressure and the force to be measured, and also to changes in the extraction unit's rigidity as the magnitude of the static pressure changes.

The rigidity k of the bellows extractor as a function of the torque necessary to rotate the extraction lever to the angle θ can be determined by the formula

$$k = \frac{M}{\theta} = \frac{B}{l} n = 2pFlm_\theta,$$

where B is the flexion rigidity of the bellows, p is the medium's static pressure (considered to be positive if it acts on the outside of the bellows) and the coefficient $n = 4(1-3\alpha - 3\alpha^2)$, just as the previously introduced coefficient m_θ, depends on the placement of the bearings along the length of the bellows. The graphs of these coefficients show (Fig. 5) that, for $\alpha \approx 0.127$ and $\alpha \approx 0.873$, the extractor rigidity does not depend on the static pressure but, with such positions of the bearings, the rigidity is approximately 2.7 times greater than the minimum rigidity, corresponding to a position of the bearings in the center of the bellows. Therefore, the definitive position of the bearings is established with account being taken of the admissible magnitude of the extractor rigidity.

A membrane extractor has an analogous character of error as a function of changes in the static pressure. This extractor has been less studied than the bellows type.

7. Dynamic Errors

In contradistinction to static accuracy, the computation of dynamic errors and analysis of methods of decreasing them are relatively more complicated, and here we shall consider only several general considerations and recommendations for lowering dynamic errors.

Very small translations of the moving system provide the transducer with faster acting force compensation than in noncompensation instruments. However, a closed compensation system, with significant inertia of the moving system and with high sensitivity of the equilibrium indicator and amplifier, acting practically like a relay for oscillations of the moving system, operates, as a rule, in an auto-oscillatory mode.

To eliminate oscillations, relatively powerful dampers are frequently used (due to the small velocities of motion of the moving system). In many of the most recent differential manometer models, damping is implemented in the sensitive element with a fluid charge serving simultaneously to protect this element from overheating. Since excessive damping lowers the instrument's speed of response, the damper must have a controllable degree of damping. This control is necessary in view of the fact that the frequency of the natural oscillations of the moving system and the natural damping change strongly with changes in the range of the quantity to be measured, and as functions of variations in the density and viscosity of the medium to be measured, the dimensions of the impact conduits, the temperature of the instrument, etc.

Recently, to decrease dynamic errors, there have been used stabilizing devices, permitting the introduction in the output signal and the compensation force of components proportional to the integral and the derivative (with respect to time) of the difference of the measured and the compensating forces. The additional complexity of the instrument occasioned by this is repaid by the capability for a significant improvement of the dynamic characteristics.

In developing the physical design of these instruments, one should take into account, the necessity for equalizing all the moving elements. In addition to decreasing the effect of external shocks and vibrations, this makes the instrument less sensitive to slants, and allows the transducer to be established in an arbitrary position.

In designing the frame, it is desirable to make provisions for mounting the transducer on shock absorbers, which might be necessary in case of particularly heavy vibration conditions.

SUMMARY

The course of development of schemes and physical designs of instruments with force compensation, and the foregoing considerations as to their accuracy, allow the following remarks to be made.

The indices of static accuracy are improved with decreases in the coefficient of noncompensation. Therefore, in designing instruments, it is necessary to try to decrease the rigidity of the flexible elements and to increase the forces (torques) to be measured. The instrument's frame and the components of the mechanism must be as rigid as possible, which must not, however, lead to an increase in weight of the moving elements.

One of the basic directions in the development of force compensation devices must be the improvement of their stability of operation for prolonged use and in conditions of changing ambient temperatures.

The necessity of increasing the instruments' speed of response, will probably produce a further tendency to decrease dimensions and, principally, weight of the moving elements of the mechanism, which requires a corresponding decrease in rigidity of the flexible elements and an increase in their stability. On the other hand, seemingly, one should expect complication of force compensation schemes by various stabilizing and auxiliary devices both for improving dynamics and for widening the possibilites of using these instruments for solving functional dependencies in logical inspection and control schemes.

FORCED PERIODIC MOTION OF A HYDRAULIC ACTUATING MECHANISM WITH A POSITIONAL LOAD

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In this paper we investigate the forced periodic motion of the piston of a hydraulic actuating mechanism, throttle controlled (slide valve), with a positional load. We determine the transient process of the mechanism during sinusoidal slide valve motion. The effect of fluid compression is estimated.

It is well known that throttle (slide valve) controlled hydraulic actuating mechanisms, in the absence of external loads, can to a high degree of accuracy, be approximated by integrating sections. However, if loading is taken into account, excluding dry friction, the motion of the piston is described by a nonlinear equation [1].

In this paper we investigate the forced motion of the piston under positional (spring) loading

$$P = kx, \tag{1}$$

where P is the load (force) applied to the piston, x is the coordinate of motion of the piston relative to its average position and k is the "stiffness" of the load.

A schematic of the mechanism is shown in Fig. 1 where 1 denotes the controlling four-chambered valve, 2 is the piston and 3 is the spring representing the load.

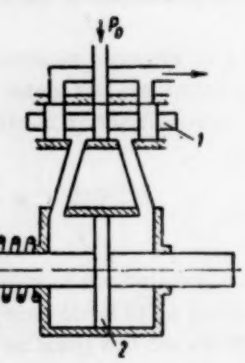

Fig. 1.

In this paper, we have made the following assumptions: there is no leakage, the coefficient of loss of the fluid in the valve chambers and the pressure p_0 in the main pressure chamber are constant. The initial overlaps or gaps in the slide valve are equal to zero and the valve openings are rectangular.

INITIAL EQUATION

Let us consider first the motion of a hydraulic actuating mechanism with an incompressible working fluid. As a result of the presence of a nonholonomic relation between the piston and the slide valve, the motion of the mechanism cannot be determined by only one force equation, for example, in d'Alembert's form. In writing the equation of motion it is necessary to take into account the equality of the forces. We will assume as a basic condition in writing the equation that the flow of the fluid is continuous. For a hydraulically actuated mechanism with a four-chambered sliding valve the condition of continuous flow for the assumptions which we have made can be presented in the form [1]

$$\frac{dx}{dt} = k_v \sqrt{1 - \frac{\Delta p}{p_0} \operatorname{sign} \rho}\, \rho, \tag{2}$$

where k_v is the steepness of the velocity characteristic at no load, ρ is the displacement of the slide valve from the average position, $\Delta \rho$ is the pressure drop in the working cylinders due to the applied load, sign ρ is the sign of the direction of the displacement of the slide valve from the mean position (the plus sign corresponds to the movement of the slide valve to the right, Fig. 1).

Force equation (1) may be written in the form

$$\Delta p = \frac{kx}{F}, \tag{3}$$

where F is the effective area of the piston.

Substituting (3) into (2) we get the equation of motion of a hydraulic actuating mechanism in the open loop condition

$$\frac{dx}{dt} = k_v \sqrt{1 - \frac{kx}{Fp_0} \operatorname{sign} \rho}\, \rho. \tag{4}$$

In this paper we examine the forced motion of the piston due to the sinusoidal motion of the slide valve $\rho = \rho^* \sin \omega t$. The initial equation for the analysis of the mechanism takes the form

$$\frac{dx}{dt} = k_v \sqrt{1 - \frac{kx}{Fp_0} \operatorname{sign} \sin \omega t}\, \rho^* \sin \omega t. \tag{5}$$

THE REACTION OF THE PISTON TO THE SINUSOIDAL MOTION OF THE SLIDE VALVE

The reaction of the unloaded piston of a hydraulic actuating mechanism to the sinusoidal motion of the slide valve is determined by the integral equation (5) for k = 0.

For zero initial conditions

$$x = \frac{k_v \rho^*}{\omega}(1 - \cos \omega t). \tag{6}$$

In order to determine the reaction of the loaded piston ($k \neq 0$) we will write equation (5) in the form

$$\frac{dx}{\sqrt{1 - \frac{kx}{Fp_0}\operatorname{sign}\sin \omega t}} = k_v \rho^* \sin \omega t \, dt. \tag{7}$$

This equation can be integrated over intervals of ωt which are integral multiples of π inasmuch as the sign of the motion of the slide valve, sign sin ωt is constant within each interval.

Integrating equation (7) over the first interval ($0 < \omega t = \varphi_1 < \pi$) for sign sin ωt = +1 we get

$$\frac{2Fp_0}{k}\sqrt{1 - \frac{kx}{Fp_0}} + c_{11} = \frac{k_v \rho^*}{\omega}\cos \omega t. \tag{7a}$$

For zero initial conditions $c_{11} = \frac{k_v \rho^*}{\omega} - \frac{2Fp_0}{k}$,

in this case

$$x = \frac{k_v \rho^*}{\omega}(1 - \cos \varphi_1) - \frac{k_v^2 \rho^{*2} k}{4Fp_0\omega^2}(1 - \cos \varphi_1)^2. \tag{8}$$

Performing successive integrations of equation (7) over the successive intervals, in each of which sign sin ωt is constant and applying the method of matching to determine the initial conditions in each of the successive intervals we can find a general expression for the reaction of the piston

$$x = -\left[x_{i-1} - \frac{k_v \rho^*}{\omega}\sqrt{1 + \frac{kx_{i-1}}{Fp_0}}\right. \tag{9}$$

$$\left.(1 - \cos \varphi_1) + \frac{k_v^2 \rho^{*2} k}{4Fp_0\omega^2}(1 - \cos \varphi_1)^2\right]\operatorname{sign}\sin \varphi_i,$$

where i = 1, 2, 3, . . . , is the interval number for integral values of π.

TRANSITIONAL PROCESS FOR POSITIVE POSITIONAL LOADING

The transitional process for positive positional loading is completely characterized by three parameters: the greatest piston displacement, the amplitude of the oscillations that are set up, and their duration.

The greatest piston displacement for k > 0 may be found from equation (8) if we substitute $\varphi_1 = \pi$:

$$x_1 = \frac{k_v \rho^*}{\omega}\left(2 - \frac{k_v \rho^*}{\omega}\frac{k}{Fp_0}\right). \tag{10}$$

Thus, the greatest displacement of the piston will exceed that at no-load by

$$\Delta x_1 = \left(\frac{k_v \rho^*}{\omega}\right)^2 \frac{k}{Fp_0}. \tag{10a}$$

Let us find the ratio between the amplitude of the slide valve displacement and its oscillation frequency $\kappa = \rho^*/\omega$ during which the pressure developed by the liquid becomes equal to the positional loading during the furthest displacement of the piston. For this condition $x_1 = Fp_0/k$ and equation (10) takes the form

$$\frac{Fp_0}{k} = k_v \kappa\left(2 - k_v \kappa \frac{k}{Fp_0}\right). \tag{10b}$$

Solving this equation for κ, we get

$$\kappa = \frac{\rho^*}{\omega} = \frac{Fp_0}{k_v k}. \tag{10c}$$

Thus, we see that a throttle-controlled hydraulic actuating mechanism with sinusoidal slide valve displacement and positional loading will have the mode of operation of the move only for the condition

$$\frac{\rho^*}{\omega} < \frac{Fp_0}{k_v k}. \tag{10d}$$

We can find the amplitude of the resulting piston oscillations x^* from equation (9) if we let x = 0 for $\cos \varphi_1 = 0$. For this case

$$x^* - \sqrt{1 + \frac{kx^*}{Fp_0}}\,\frac{k_v \rho^*}{\omega} + \frac{k_v^2 \rho^{*2} k}{4Fp_0\omega^2} = 0. \tag{10e}$$

Solving this equation for x^* we find the amplitude of the resulting piston oscillations

$$|x^*| = k_v \frac{\rho^*}{\omega}\left(1 - \frac{kk_v}{4Fp_0}\frac{\rho^*}{\omega}\right).$$

Thus, for the established mode of operation the phase shift between the motion of the piston under positional loading and the motion of the slide valve is $-\pi/2$, this corresponds to the phase shift for the unloaded actuating mechanism. The amplitude characteristic is a function of the ratio of the valve displacement to the frequency of the introduced signal ρ^*/ω and depends upon the parameters of the mechanism: the area F, pressure p_0, steepness of the velocity characteristic at no-load k_v and the load stiffness k.

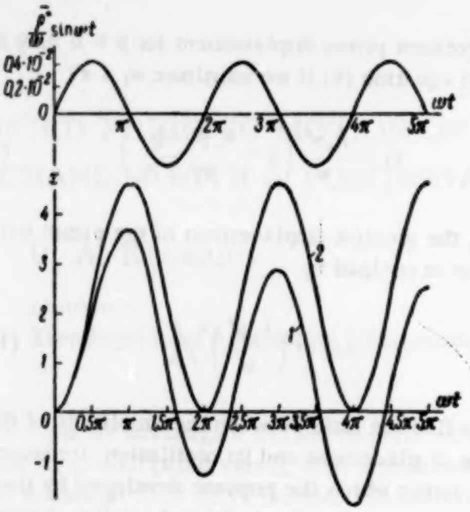

Fig. 2.

The determination of the duration of the transitional process is difficult in the general form. Therefore, it may be found by orientation from the character of the changes in the small deviations x_1 of the piston as computed from equation (9).

As an example of the application of the results which have been obtained, we show in Fig. 2 the transitional process in the motion of the piston of the hydraulic actuating mechanism having the following dimensions and parameters: $F = 44$ cm^2, $\rho_0 = 50$ kg/cm^2, $k_v = 530$ sec^{-1} and $k = 400$ kg/cm. The transitional process (curve 1) has been plotted for the case in which the piston load during the maximum piston displacement is $\frac{2}{3}\rho_0 F$; this corresponds to $\rho^*/\omega = 0.42 \times 10^{-2}$ cm · sec. We also show in Fig. 2 the transitional process for an unloaded piston for the same excitation (curve 2).

ESTIMATION OF THE EFFECT OF FLUID COMPRESSION

Before we evaluate the effect of the compressibility of the fluid enclosed in the force cylinder upon the motion of the piston we will find the dependence of the motion of the cylinder upon changes in the applied force P_0 (Fig. 3a). In the figure Δx denotes the displacement of the cylinder due to the action of the force ΔP and l_0 is the depth of the fluid column in the cylinder.

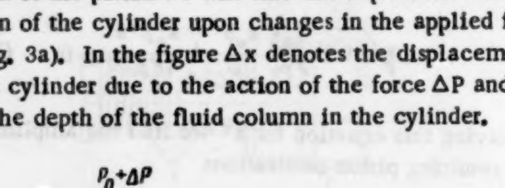

Fig. 3. Diagram of the force cylinder. a) a) one sided, b) two sided

Assuming that the cylinder and the piston have absolute stiffness and that there are no losses, we may write

$$\gamma_0 V_0 = \gamma_1 V_1,$$

where γ_0, V_0 are the specific gravity and initial volume of the fluid under pressure due to the applied force P_0; γ_1, V_1 are the specific gravity and volume of the fluid after the applied force P_0 has been increased by ΔP.

Since $V_1 = V_0$ and $\Delta V = F\Delta x$, we get

$$\Delta x = \frac{V_0(\gamma_1 - \gamma_0)}{F\gamma_1}.$$

We may write the following expression for the specific gravity of the fluid*

$$\gamma_1 = \gamma_0 + k_\gamma \frac{\Delta P}{F},$$

where k_γ is a coefficient of proportionality. We may now write

$$\Delta x = \frac{k_\gamma V_0 \Delta P}{\left(\gamma_0 + k_\gamma \dfrac{\Delta P}{F}\right) F^2}. \tag{11}$$

The above equation shows that the piston displacement is proportional to the applied force and depends upon the change in the specific gravity of the fluid due to this force.

Let us evaluate the influence of the second factor. Comparing equations (11) and (6) in [2] we notice that

$$\frac{\gamma_0 + k_\gamma \dfrac{\Delta P}{F}}{k_\gamma} = G -$$ is the modulus of volume elasticity (bulk modulus).

The modulus of elasticity of the working fluid in the hydraulic system over the range of pressure changes of up to 200-300 kg/cm^2 is practically constant. Then, $k_\gamma = \Delta P/F$ in equation (11) is very close to zero and the specific gravity of the fluid may be considered constant in magnitude, i.e., $\gamma_1 = \gamma_0$. Under these conditions equation (11) may be written in the form

$$\Delta x = \frac{V_0 \Delta p}{FG} = \frac{l_0 \Delta P}{FG}. \tag{11a}$$

The above analysis shows that if we take into account the losses in the compressible fluid passing through the slot in the valve we can make use of the equations for incompressible fluids. With this in mind we will represent the hydraulic actuating mechanism by means of the schematic of Fig. 4. Here 1 is a

* According to [2] this equation is correct for the case where the applied force does not produce a pressure of more than (20-30) × 10^3 kg/cm^3 in the fluid.

spring imitating the stiffness of the applied load, 2 is a spring imitating the stiffness of the fluid, x_1 is the displacement of the fluid cross-section in the vicinity of the boundary walls of the force cylinder (in cross sections I and II).

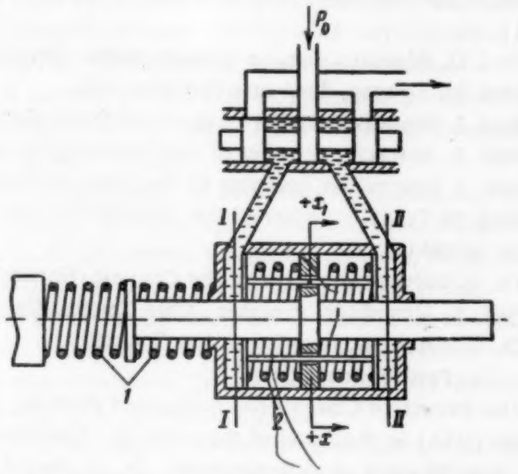

Fig. 4. Equivalent schematic of a hydraulic actuating mechanism which takes into consideration the compression of the working fluid.

We will find the change in stiffness of the fluid enclosed in the force cylinder (Fig. 3b) and the displacement of the piston due to the action of the applied force ΔP. Examining the work of the actuating mechanism only for the actuator we note that the fluid in chambers A and B, Fig. 3b, will always be under excess pressure and the applied load, acting on the piston, does not create a pressure drop exceeding the pressure in the main pressure chamber. Therefore, the pressure in the working cylinder cannot fall below zero (excess). Therefore, the stiffness of the fluid in the cylinders, according to equation (11a) may be represented in the form

$$k_{f\ A,B} = \frac{FG}{l_0 + x}\,.$$

Therefore, the total stiffness k_f is the sum of k_{fA} + + k_{fB}:

$$k_f \ \frac{2l_0GF}{l_0^2 - x^2}\,.$$

The piston displacement under the action of the applied force ΔP is

$$\Delta x = \frac{l_0^2 - x^2}{2l_0GF}\,\Delta P. \tag{11b}$$

Let us now examine the equations of motion of an actuating mechanism, taking into consideration the compressibility of the fluid.

According to the equivalent schematic of the mechanism (Fig. 4) the motion of the piston is determined by three equations:

(1) the force equation (3)

(2) the equation of flow through the slide valve (2), using the new designations this equation takes the form

$$\frac{dx_1}{dt} = k_v \sqrt{1 - \frac{\Delta p}{p_0}\,\mathrm{sign}\,\rho}\ \rho;$$

(3) the equation connecting the pressure drop, due to the load in the working chambers, with the relative piston displacement; this equation according to (11b) is of the form

$$kx = \frac{2l_0GF}{l_0^2 - x^2}\,(x_1 - x). \tag{12}$$

The simultaneous solution of the indicated equations gives us the equation

$$\frac{kl_0^2 + 2l_0GF - 3kx^2}{2l_0GF}\,\frac{dx}{dt} = k_v \sqrt{1 - \frac{kx}{p_0}\,\mathrm{sign}\,\rho}\ \rho. \tag{13}$$

For $\rho = \rho^* \sin \omega t$ equation (13) may be integrated in a similar manner to equation (7).

Neglecting the changes in the stiffness of the liquid during the motion of the piston,† we get

$$\frac{dx}{dt} = \frac{2GF}{kl_0 + 2GF}\,k_v \sqrt{1 - \frac{kx}{p_0}\,\mathrm{sign}\,\rho}\ \rho. \tag{14}$$

Comparing the above equation with equation (4) obtained in the analysis of the motion of an actuating mechanism with an incompressible fluid we note that the right sides of the equations differ only by a constant multiplier $2GF/kl_0 + 2GF$. This multiplier, for $kl_0 \ll$ $\ll 2GF$, a condition which as a rule exists in practice, is close to unity. Therefore, the compressibility of the fluid in the force cylinder has a relatively small effect upon the motion of a piston with only one positional load.

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† Under this condition the deviation of the curve for the transitional piston process from the curve computed without taking into consideration the compressibility of the fluid will exceed the deviation obtained from the solution of equation (13).

‡ See English translation.

INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL (IFAC) AND ITS FIRST CONGRESS

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 934-937, June, 1960

I. Brief History of Its Development

The question of forming the International Federation of Automatic Control was first brought up in September, 1956, during the International Congress of Automatic Regulation in Heidelberg (West Germany).

The initiating group of scientists and engineers from among the participants in the Congress, at a special meeting, adopted a resolution signed by 26 representatives of various countries, among them representatives of the USSR, in which the formation of the International Federation of Automatic Control was approved.

It was pointed out in the resolution that the IFAC is created for the purpose: 1) of promoting the exchange of scientific information in the area of automatic control between the member-countries and 2) of organizing international congresses of automatics.

At the same meeting a preparatory committee was elected to compose a draft of rules and regulations of the IFAC and call a first organizational meeting of representatives of the interested countries, a General Assembly.

The First General Assembly of the IFAC, held in September, 1957, in Paris, approved the code of rules of the IFAC and elected the managing organ of the Federation, the Executive Council, consisting of 11 members and headed by G. Chestnut (USA), the president of the IFAC.

The General Assembly decided to call the First Congress of the IFAC in Moscow in 1960.

The Executive Council of the IFAC of the first staff did great work in the organizational strengthening of the Federation. Six special scientific-technical committees were created by it for the development of the international exchange of scientific information in the field of automatics. The Executive Council also established the Advisory Committee of the IFAC.

The Second General Assembly was called in September, 1959, in Chicago. It approved a new code of rules of the Federation and the draft of the scientific program of the First Congress of the IFAC, worked out by the Soviet Union's National Committee of Automatic Control [see Avtomat. i Telemekh. 22, 1 (1959)].*

In accordance with the Code of Rules of the General Assembly a new staff of the Executive Council was elected, as well as a new president of the IFAC, Professor A. M. Letov, a representative of the USSR.

The Executive Council now includes the following:

Prof. A. M. Letov, President (USSR)
Prof. E. Gereke, First Vice-President (Switzerland)
Prof. O. Benedict, Second Vice-President (Belgium)
Prof. M. Kyueno, Treasurer (Switzerland)
Prof. J. Howells, member of the Council (England)
Prof. P. Nowacki, member of the Council (Poland)
Prof. J. Evangelisti, member of the Council (Italy)
Prof. Ed Trnka, member of the Council (Czechoslovakia)
Dr. H. Balchen, member of the Council (Norway)
Prof. K. Kaneshige, member of the Council (Japan)
Dr. G. Chestnut, member of the Council as Past President (USA)

The Executive Council has designated Professor D. Eckman (USA) as chairman of the Advisory Comitteee and Dr. J. Loeb (France) as vice-chairman, Dr. G. Ruppel (West Germany) as honorary secretary of the IFAC and Professor V. Broid (France) as honorary editor of the "IFAC Bulletin."

The chairman of the scientific-technical committees of the IFAC are: Committee on Theory – Academician B. N. Petrov (USSR), Committee on Technical Resources – Dr. D. Boromissa (Hungary), Committee on the Utilization of Automatics – Dr. D. Moseley (USA), Committee on Education – Prof. A. Marino (Italy), Committee on Terminology and Designations – Prof. E. Gereke (Switzerland), Committee on Bibliography – Prof. V. Oppelt (West Germany).

The Second General Assembly noted the great work performed by the Executive Council of the first staff and issued a special resolution expressing appreciation to Dr. G. Chestnut and certain other workers in the IFAC for their contribution to the creation of the Federation. A resolution to hold the Second Congress of the IFAC in Switzerland in 1963 was adopted at the Assembly.

In the preceding period the activity of the IFAC was devoted primarily to preparations for the First Congress of the Federation.

In all, 23 countries are represented by their national organizations in the IFAC, and this number includes a majority of the countries most advanced in industrial respects.

On January 1, 1960 the IFAC became a member of the International Council of Scientific and Technical Societies.

II. The Soviet Union's National Committee of Automatic Control

The Soviet Union is represented in IFAC by its National Committee of Automatic Control, formed in the

*See English translation.

Academy of Sciences of the USSR. The representative of the National Committee of the USSR is Academician V. A. Trapeznikov.

The National Committee unites the most eminent specialists of the Soviet Union in automatics and in its work leans upon territorial groups formed in scientific and industrial centers: Leningrad, Kiev, Novosibirsk, Sverdlovsk, Tbilis, Erevan, Baku, and Ivanov.

A number of scientific-technical committees have been formed in the Soviet Union's National Committee of Automatic Control:

1) On the theory of automatic regulation and control – chairman, Prof. Ya. Z. Tsipkin; vice-chairman, Prof. A. A. Krasovskii; scientific secretary, A. N. Korolev, Candidate in Technical Sciences (C. T. S.).

2) On technical resources of automatics and remote control – chairman, Prof. B. S. Sotskov (Corresp. member); vice-chairman, D. I. Ageikin, C.T. S.; scientific secretary, I. E. Dekabrun, C.T.S.

3) On automatic electrical operation and its application to electrical machinery – chairman, Prof. M. G. Chilikin; vice-chairman, Prof. I. I. Petrov; scientific secretary, A. A. Sirotin, C.T.S.

4) On the automation of continuous productive processes in the metallurigcal, chemical, energy and other fields of industry – chairman, E. L. Stefani, C.T.S.; vice-chairman, A. Ya. Lerner, D.T.S.; scientific secretary, C. P. Khlebnikov, Engineer.

5) On the automation of productive processes in machine building – chairman Academician V. I. Dikushin; vice-chairman, Academician I. I. Artobolevskii; scientific secretary, V. M. Raskatov, Engineer.

6) On the automation of productive process in agriculture – chairman, Academician VASKhNIL I. A. Budzko; vice-chairman, N. N. Levykin, C.T.S.; scientific secretary, K. M. Poyarkov, Engineer.

7) On terminology – chairman, M. A. Gavrilov; vice-chairman, A. V. Khramoi, C.T.S.; scientific secretary, E. V. Babicheva, Engineer.

The Soviet Union's National Committee of Automatic Control gave its assent to the Executive Committee of the IFAC to hold the First International Congress of the Federation in Moscow, and, consequently, assumes responsibility for the organization of that Congress.

III. The First Congress of the IFAC

The First Congress of the IFAC will be held June 27 to July 7, 1960 on the premises of Moscow University on Lenin Heights.

An organizing committee has been formed to carry out the practical work in preparing for the Congress: Academician V. A. Trapeznikov (chairman), Academician, B. N. Petrov (vice-chairman), V. V. Karibskii, Engineer (vice-chairman), B. N. Naumov, C.T.S. (vice-chairman) and N. O. Biryukov, C.T.S. (scientific secretary).

The organizing committee is performing much preparatory work, with the active support of the national organization-members of the IFAC. Great interest in the coming Congress is observed on the part of specialists of many countries of the world. A great number of reports have been submitted to the organizing committee, from 21 countries, and from them 285 reports have been selected.

The reports selected for presentation to the Congress are distributed by country as follows:

Austria	5	Canada	1	France	9
England	28	China	6	West Germany	15
Belgium	4	Norway	3	Czechoslovakia	5
Hungary	5	Poland	11	Switzerland	1
E. Germany	3	Rumania	6	Sweden	1
India	1	USSR	81	Yugoslavia	2
Italy	7	USA	73	Japan	18

The reports have been divided into three groups:

I. Theory of automatic regulation and control.

II. Technical resources of automatics and remote control.

III. Applications of automatics.

Within each of these groups the reports have been subdivided according to related scientific indications into separate sections. The ten sections of Group I included 143 reports, the five sections of Group II include 56, and the six sections of Group III include 86 reports. Simultaneous work of up to 11 sections is provided for.

The following is an enumeration of the sections of the Congress, with an indication of the number of reports pertaining to them.

I. 1. Theory of continuous linear systems – 23
I. 2. Theory of continuous nonlinear systems – 14
I. 3. Theory of discrete systems – 25
I. 4. Stochastic problems – 18
I. 5. Theory of optimal systems – 12
I. 6. Theory of self-adjusting systems – 20
I. 7. Theory of structures and the construction of signals – 7
I. 8. Special mathematical problems – 13
I. 9. Modeling and experimental methods of investigation – 11
II. 1. Electrical and magnetic elements of systems of regulation – 9
II. 2. Electrical computing and modeling equipment, programming elements and machines – 10
II. 3. Counters, elements and systems of automatic control and remote control of productive processes – 8
II. 4. Pneumatic means of automatics and calculating technique – 11
II. 5. Instruments and equipment of automatic control – 18
III. 1. Automation of machine building – 10
III. 2. Automation of energy systems – 24

III. 3. Automation of the chemical and petroleum processing industry — 8

III. 4. Automated electrical operation and electrical machines — 12

III. 5. Automation of metallurgical processes — 17

III. 6. Unclassified problems — 9

An additional six reports were presented to combined sessions of some sections of Group III.

The names of the sections give a complete enough representation of the subject of the accepted reports. A characteristic feature should be noted — the great attention devoted in the reports to the application of computer technique for the control of complicated forms of technological processes, including the control of nuclear processes, the application of optimal, extreme and self-adjusting systems.

On the whole the reports contain very valuable scientific information which sheds light on the experience and the level of knowledge attained in the area of automatics, and creates the basis for scientific discussion of the most important current problems of automatics.

The reports were prepared in conformance with the motto: to theory — practical applicability; to technical resources — maximum reliability; to applications — the greatest effectiveness.

The publication of the reports in the form of brochures in the Russian and English languages enables all the participants in the Congress to share more fruitfully in the scientific discussion, since they will have become acquainted with the contents of the reports.

In the future the papers of the Congress will be published in a separate issue which can be purchased by any one who wishes to purchase it.

Besides the sectional sessions, two plenary sessions are included in the program of the Congress. At the first of these a report by Academician V. A. Trapeznikov on the basic tasks of contemporary automatics will be heard and discussed, and in the final session the work of the Congress will be summed up.

Within the framework of the Congress there will be organized an international exhibition of books and journals on questions of automatics, an exhibit of instruments and equipment of automatics and telemechanics based on new principles, and also a demonstration of scientific and popularized scientific films on automatics and telemechanics.

During the period of the Congress it is proposed to conduct a series of lectures on current problems of automatics by eminent foreign scientists for the Soviet community.

The program of the Congress provides for visits of its participants to cultural and to more than 30 scientific and educational institutions and industrial enterprises in Moscow, Leningrad, and Kiev.

The participants in the technical excursions will become acquainted with the work done in the Soviet Union in the creation and utilization of various systems of automatic regulation and control, its resources in computing and measuring techniques, remote and telemechanical systems, automated lines and automated industrial production, machine tools with program control, and also with the state of higher education in educational institutions.

During the working period of the Congress there will be a session of the leaders of the IFAC organs, the Executive Council and the scientific-technical committees, which have to consider a series of questions regarding the current activities of the Federation and its plans for the future.

IV. General Schedule of the Congress

June 24 — Sessions of the Executive Council and the scientific-technical committees of the IFAC

June 25 — General Assembly of the IFAC; sessions of the scientific-technical committees of the IFAC

June 27 — morning — official opening of the Congress of the IFAC, plenary session

June 27 — evening — sectional sessions

June 28 to July 1 — sectional sessions and technical excursions in Moscow

July 2 — morning — plenary session

July 2 — evening — session of the Executive Council of the IFAC

July 3 to July 7 — technical excursions in Moscow, Leningrad, and Kiev

The Moscow International Congress of Automatic Control, as the First Congress of the IFAC, is destined to play an important role in the matter of strengthening international scientific contacts between specialists in this field of science, which is the basis for the development of one of the fundamental and very perspective directions of new technique.

N. O. Biryukov

SIGNIFICANCE OF ABBREVIATIONS MOST FREQUENTLY ENCOUNTERED IN SOVIET TECHNICAL PERIODICALS

AN SSSR	*Academy of Sciences, USSR*
FIAN	*Physics Institute, Academy of Sciences USSR*
GITI	*State Scientific and Technical Press*
GITTL	*State Press for Technical and Theoretical Literature*
GOI	*State Optical Institute*
GONTI	*State United Scientific and Technical Press*
Gosénergoizdat	*State Power Press*
Gosfizkhimizdat	*State Physical Chemistry Press*
Goskhimizdat	*State Chemistry Press*
GOST	*All-Union State Standard*
Gostekhizdat	*State Technical Press*
GTTI	*State Technical and Theoretical Press*
IAT	*Institute of Automation and Remote Control*
IF KhI	*Institute of Physical Chemistry Research*
IFP	*Institute of Physical Problems*
IL	*Foreign Literature Press*
IPF	*Institute of Applied Physics*
IPM	*Institute of Applied Mathematics*
IREA	*Institute of Chemical Reagents*
ISN (Izd. Sov. Nauk)	*Soviet Science Press*
IYap	*Institute of Nuclear Studies*
Izd	*Press (publishing house)*
LÉTI	*Leningrad Electrotechnical Institute*
LFTI	*Leningrad Institute of Physics and Technology*
LIM	*Leningrad Institute of Metals*
LITMiO	*Leningrad Institute of Precision Instruments and Optics*
Mashgiz	*State Scientific-Technical Press for Machine Construction Literature*
MGU	*Moscow State University*
Metallurgizdat	*Metallurgy Press*
MOPI	*Moscow Regional Pedagogical Institute*
NIAFIZ	*Scientific Research Association for Physics*
NIFI	*Scientific Research Institute of Physics*
NIIMM	*Scientific Research Institute of Mathematics and Mechanics*
NIKFI	*Scientific Institute of Motion Picture Photography*
NKTM	*People's Commissariat of the Heavy Machinery Industry*
Obrongiz	*State Press of the Defense Industry*
OIYaI	*Joint Institute of Nuclear Studies*
ONTI	*United Scientific and Technical Press*
OTI	*Division of Technical Information*
OTN	*Division of Technical Science*
RIAN	*Radium Institute, Academy of Sciences of the USSR*
SPB	*All-Union Special Planning Office*
Stroiizdat	*Construction Press*
URALFTI	*Ural Institute of Physics and Technology*
TsNIITMASh	*Central Scientific Research Institute of Technology and Machinery*
VNIIM	*All-Union Scientific Research Institute of Metrology*

NOTE: Abbreviations not on this list and not explained in the translation have been transliterated, no further information about their significance being available to us – *Publisher.*

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